

## Electromagnetic and weak transitions in the seven-nucleon systems

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Electromagnetic  $M1$  and  $E2$  transitions in the seven-nucleon systems, the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  radiative-capture problem, and the electron-capture reaction of  ${}^7\text{Be}$  into  ${}^7\text{Li}$  are studied. The wave functions employed are from a previous single-channel resonating-group calculation which yielded satisfactory results for the  ${}^3\text{He} + \alpha$  and  ${}^3\text{H} + \alpha$  scattering cross sections and polarizations. With no adjustable parameters, the calculated  $B(M1)$  and  $B(E2)$  values for transitions between the ground and first excited states of  ${}^7\text{Li}$  or  ${}^7\text{Be}$  are found to agree rather well with values empirically determined. The  $B(E2)$  study shows, in particular, that the nuclei  ${}^7\text{Li}$  and  ${}^7\text{Be}$  must have strong collective features, which are describable in terms of  $\alpha$  plus three-nucleon clustering. The cross-section factor  $S(0)$ , important in astrophysical considerations, is determined to have a value between 0.5 and 0.6 keV b. For the electron-capture reaction, the calculated value of the branching ratio to the first excited state of  ${}^7\text{Li}$  is 10.31%, which is in very good agreement with the measured value of  $10.36 \pm 0.10\%$  reported prior to 1981.

NUCLEAR REACTIONS  ${}^3\text{He}(\alpha, \gamma)$ ,  ${}^7\text{Be}(e^-)$ ,  ${}^7\text{Li}(\gamma)$ ; calculated capture cross section,  $\log ft$ ,  $B(M1)$  and  $B(E2)$  with resonating-group wave functions.

### I. INTRODUCTION

The resonating-group method<sup>1,2</sup> (RGM) has been successfully employed, over the past thirty years, to study a large number of nuclear scattering and reaction problems. As is now well known, it is especially useful in treating those systems where strong core-exchange contributions are present.<sup>3,4</sup> By studying the nature of the exchange-kernel function which characterizes the nonlocal part of the internuclear interaction, it has been determined<sup>5</sup> that these are the systems in which the interacting nuclei have a small nucleon-number difference. Precisely because of this reason, the  ${}^3\text{H} + \alpha$  and  ${}^3\text{He} + \alpha$  systems have, in particular, received extensive attention, and the results obtained were found to agree well with measured values of the differential scattering cross section and polarization over a wide energy region.<sup>6</sup>

In the  ${}^3\text{H} + \alpha$  and  ${}^3\text{He} + \alpha$  systems, the resultant many-nucleon wave functions have further been used to compute the  $C0$  and  $C2$  charge form factors of  ${}^7\text{Li}$ ,<sup>7</sup> and the  $E1$  transition rate of the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  radiative capture reaction.<sup>8</sup> From these calculations, we have gained a better understanding of the collective nature of the seven-nucleon system and of the solar-neutrino problem.<sup>9</sup> In this investigation, we shall continue such studies using resonating-group wave functions by examining the following electromagnetic- and weak-transition problems: (i) the

$M1$ - $E2$  transitions between the ground and first excited states of  ${}^7\text{Li}$  and  ${}^7\text{Be}$ , (ii) a further examination of the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  reaction, and (iii) the branching ratio in the electron-capture reaction of  ${}^7\text{Be}$ .

The  $M1$ - $E2$  transition problem is considered in order to obtain additional understanding concerning the collective properties of the ground and first excited states of  ${}^7\text{Li}$  and  ${}^7\text{Be}$ . Empirically, it has been shown that the  $E2$  transition rate in  ${}^7\text{Li}$  is about 17 W.u.,<sup>10</sup> suggesting that the nucleus  ${}^7\text{Li}$  is highly deformed in these low excited states. It would certainly be interesting to see whether such large intrinsic deformation can be properly accounted for in a microscopic calculation which assumes an  ${}^3\text{H} + \alpha$  cluster configuration.

The  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  reaction has been considered previously<sup>8</sup>; we discuss it further here mainly for the following reason. Since the publication of our calculated results, careful measurements of the reaction rates have been performed,<sup>11-13</sup> and values for the zero-energy cross-section factor  $S(0)$  have been deduced. These measurements were of two types. The first type involves the observation of prompt  $\gamma$  rays (direct measurement), while the second type involves the measurement of the activity of  ${}^7\text{Be}$  by observing the 478-keV  $\gamma$  ray following the electron capture of  ${}^7\text{Be}$  to the first excited state of  ${}^7\text{Li}$  (activation measurement). The results for  $S(0)$  obtained empirically are shown in Table I. Here one finds that the empirical value

TABLE I. Comparison of empirical values of  $S(0)$ .

Reference	$S(0)$ (keV b)	Remark
Kräwinkel <i>et al.</i> (Ref. 11)	$0.31 \pm 0.03$	Direct measurement
Osborne <i>et al.</i> (Ref. 12)	$0.51 \pm 0.03$	Direct measurement
Osborne <i>et al.</i> (Ref. 12)	$0.56 \pm 0.03$	Activation measurement
Robertson <i>et al.</i> (Ref. 13)	$0.63 \pm 0.04$	Activation measurement

obtained by Kräwinkel *et al.* is singularly low. Since the value of  $S(0)$  is of critical importance in the solar-neutrino problem, we wish to determine if our theoretical analysis can help to resolve this inconsistency.

The determination of  $S(0)$  from the activation measurement depends critically upon the branching ratio  $\xi$  of the electron capture of  ${}^7\text{Be}$  to the first excited state of  ${}^7\text{Li}$ . This ratio has been carefully measured and the accepted value prior to 1982 was  $10.36 \pm 0.10\%$ .<sup>10,14</sup> Recently, however, there was a remeasurement of  $\xi$  by Rolfs *et al.*,<sup>15</sup> and the resultant value was  $15.4 \pm 0.8\%$ . This is an astonishingly larger value, in marked disagreement with the results reported previously. If the  $\xi$  value measured by Rolfs *et al.* should turn out to be indeed correct, then the values of  $S(0)$  obtained by Osborne *et al.* and by Robertson *et al.*, given in the last two rows of Table I, need to be appreciably reduced and the resultant  $S(0)$  values would be in better agreement with the value obtained by Kräwinkel *et al.* Thus, it is very important to settle the problem concerning the value of  $\xi$ , and our hope is that a theoretical study using our resonating-group wave functions will be useful in this respect.

The outline of this paper is as follows. In Sec. II, we give a brief description of the  ${}^7\text{Li}$  and  ${}^7\text{Be}$  resonating-group wave functions. The  $M1$ - $E2$  transition calculation, the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  capture reaction, and the  ${}^7\text{Be}$  electron-capture problem are then discussed, respectively, in Secs. III, IV, and V. Finally, in Sec. VI, we summarize the findings of this investigation and make some concluding remarks.

## II. RESONATING-GROUP WAVE FUNCTIONS

The  ${}^7\text{Be}$  bound-state wave function is chosen to have an  $\alpha + {}^3\text{He}$  cluster configuration. It is specified by the relative orbital angular momentum  $l$  ( $l=1$ ) and total angular momentum  $J$  with  $z$  component  $M$ . Its explicit form is

$$\psi_J^M = C_J \mathcal{A} \left[ \phi_\alpha \phi_h \frac{1}{R} f_{Jl}(R) \mathcal{Y}_{JlS}^M Z(\vec{\mathbf{R}}_{\text{c.m.}}) \right], \quad (1)$$

where  $\mathcal{A}$  is an antisymmetrization operator,  $C_J$  is a normalization factor,  $Z(\vec{\mathbf{R}}_{\text{c.m.}})$  is a normalized spherically-symmetric function describing the total c.m. motion, and  $\mathcal{Y}_{JlS}^M$  is a spin-isospin-angle function appropriate for  $T = \frac{1}{2}$  and  $S = \frac{1}{2}$ . The normalized functions  $\phi_\alpha$  and  $\phi_h$  represent the internal spatial structures of the  $\alpha$  and  ${}^3\text{He}$  clusters; they are assumed to have the lowest configurations in harmonic-oscillator wells having width parameters  $\alpha_A$  and  $\alpha_B$  equal to 0.514 and 0.367 fm<sup>-2</sup>, respectively [see Eqs. (4)–(7) of Ref. 8]. These particular values are chosen in order to yield correct  $\alpha$ -particle and  ${}^3\text{He}$  rms

matter radii in accordance with data from electron-scattering experiments.<sup>16</sup>

The relative-motion function  $f_{Jl}(R)$  is determined by solving an integrodifferential equation obtained from a projection procedure (for details, see Ref. 1). The nucleon-nucleon potential used has been described previously.<sup>7,8</sup> It involves an exchange-mixture parameter  $u$  and spin-orbit depth parameters  $V_\lambda$  and  $V_{\lambda\tau}$ . These parameters are chosen to have the values given by Eq. (12) of Ref. 8, such that the  ${}^3\text{He}$  separation energies in the ground and first excited states of  ${}^7\text{Be}$  are well reproduced.

The  ${}^7\text{Be}$  continuum function is similarly chosen to have an  $\alpha + {}^3\text{He}$  cluster structure. It is discussed in Sec. II A of Ref. 8 and, hence, will not be further described here.

For the  ${}^7\text{Li}$  bound state, the wave function  $\tilde{\psi}_J^M$  used has the same expression as that given by Eq. (1), except that the normalization factor and the relative-motion function will be denoted as  $\tilde{C}_J$  and  $\tilde{f}_{Jl}(R)$ , respectively, and the three-nucleon (triton) internal spatial function will be denoted as  $\phi_t$ . This latter function is also assumed to have a simple Gaussian form but with a slightly different width parameter equal to 0.378 fm<sup>-2</sup> [see Eq. (7) of Ref. 7]. Also, it should be mentioned that, in the  ${}^7\text{Li}$  case, we have made a very minor adjustment of the nucleon-nucleon parameters  $u$ ,  $V_\lambda$ , and  $V_{\lambda\tau}$  in order to obtain accurate values for the  ${}^3\text{H}$  separation energies in the ground and first excited states.

For  ${}^7\text{Be}$  the relative-motion functions  $f_{3/2,1}$  or  $f_G$  of the ground state and  $f_{1/2,1}$  or  $f_E$  of the first excited state are plotted in Fig. 1 of Ref. 8, while for  ${}^7\text{Li}$  the relative-motion function  $\tilde{f}_{3/2,1}$  or  $\tilde{f}_G$  is depicted in Fig. 1 of Ref. 7. The  ${}^7\text{Li}$  first excited-state function  $\tilde{f}_{1/2,1}$  or  $\tilde{f}_E$ , which was not needed in either Ref. 7 or Ref. 8, will not be shown here because, as expected, it has a behavior quite similar to those of the other relative-motion functions.

## III. ELECTROMAGNETIC TRANSITIONS

Electromagnetic  $M1$  and  $E2$  transitions between the ground  $\frac{3}{2}^-$  and the first excited  $\frac{1}{2}^-$  states of  ${}^7\text{Li}$  and  ${}^7\text{Be}$  provide information about the structure of these nuclei, and can be best discussed in terms of the reduced transition probabilities  $B(M1; \frac{3}{2}^- \rightarrow \frac{1}{2}^-)$  and  $B(E2; \frac{3}{2}^- \rightarrow \frac{1}{2}^-)$ . In this section, we shall utilize the RGM wave functions, described in Sec. II, to compute these  $B$  values and compare them with empirical values determined from measurements of the transition widths.

Magnetic  $M1$  transitions proceed via the operator

$$\mathcal{M}(M1, \mu) = \left( \frac{3}{4\pi} \right)^{1/2} \mu_N \sum_{i=1}^7 [g_i(i)l_{i\mu}^* + g_s(i)s_{i\mu}^*], \quad (2)$$

which is a tensor operator of rank 1, with  $\mu$  denoting its spherical components ( $\mu = -1, 0, \text{ and } 1$ ).<sup>17</sup> It consists of a sum over the orbital angular-momentum operators

$$\vec{l}_i = \vec{p}_i \times \frac{1}{i} \vec{\nabla}_{\vec{p}_i} \quad (\vec{p}_i = \vec{r}_i - \vec{R}_{c.m.}) \quad (3)$$

and the spin angular-momentum operators  $\vec{s}_i$  of all the nucleons involved, multiplied with the appropriate  $g$  factors

$$\begin{aligned} g_l(p) &= 1, & g_s(p) &= 5.5856, \\ g_l(n) &= 0, & g_s(n) &= -3.8262, \end{aligned} \quad (4)$$

for the protons and neutrons. The quantity  $\mu_N$  in Eq. (2) represents the nuclear magneton; it is equal to  $e\hbar/2M_p c$  with  $M_p$  being the proton mass.

The reduced probability for a transition from an initial state with total angular momentum  $J_i$  to a final state with total angular momentum  $J_f$  is given by

$$\begin{aligned} B(M1; J_i \rightarrow J_f) &= \sum_{\mu M_f} |\langle \psi_{J_f}^{M_f} | \mathcal{M}(M1, \mu) | \psi_{J_i}^{M_i} \rangle|^2 \\ &= \frac{1}{2J_i + 1} |\langle \psi_{J_f} | \mathcal{M}(M1) | \psi_{J_i} \rangle|^2, \end{aligned} \quad (5)$$

$$\mathcal{M}_{sp}(\vec{R}', \vec{R}'') = \langle \phi_\alpha \phi_h \delta(\vec{R} - \vec{R}') \xi_S^{M_S} Z(\vec{R}_{c.m.}) | \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N \sum_{i=1}^7 g_s(i) s_{iz} | \mathcal{A}[\phi_\alpha \phi_h \delta(\vec{R} - \vec{R}'') \xi_S^{M_S} Z(\vec{R}_{c.m.})] \rangle, \quad (8)$$

$$\mathcal{M}_{orb}(\vec{R}', \vec{R}'') = \langle \phi_\alpha \phi_h \delta(\vec{R} - \vec{R}') \xi_S^{M_S} Z(\vec{R}_{c.m.}) | \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N \sum_{i=1}^7 g_l(i) l_{iz} | \mathcal{A}[\phi_\alpha \phi_h \delta(\vec{R} - \vec{R}'') \xi_S^{M_S} Z(\vec{R}_{c.m.})] \rangle, \quad (9)$$

where  $\xi_S^{M_S}$  is a spin-isospin function having  $M_S$  for the  $z$  component of the spin angular momentum. The evaluation of the spin kernel  $\mathcal{M}_{sp}(\vec{R}', \vec{R}'')$  is quite simple; only the unpaired nucleon in the  ${}^3\text{He}$  cluster contributes and the result is

$$\mathcal{M}_{sp}(\vec{R}', \vec{R}'') = M_S P_{sp}(\vec{R}', \vec{R}''), \quad (10)$$

with

$$P_{sp}(\vec{R}', \vec{R}'') = \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N g_s(n) \mathcal{N}(\vec{R}', \vec{R}''), \quad (11)$$

where  $\mathcal{N}(\vec{R}', \vec{R}'')$  denotes the norm kernel. This latter kernel may be written in the form

$$\begin{aligned} \mathcal{M}_{orb}(\vec{R}', \vec{R}'') &= \langle \phi_\alpha \phi_h \delta(\vec{R} - \vec{R}') \xi_S^{M_S} Z(\vec{R}_{c.m.}) | \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N \\ &\quad \times \sum_{i=1}^7 g_l(i) \left[ \vec{r}_i \times \frac{1}{i} \vec{\nabla}_{\vec{r}_i} \right]_z | \mathcal{A}[\phi_\alpha \phi_h \delta(\vec{R} - \vec{R}'') \xi_S^{M_S} Z(\vec{R}_{c.m.})] \rangle, \end{aligned} \quad (13)$$

where the operator involved consists now of a sum of one-body terms. By employing the complex-generator-coordinate technique,<sup>2,18</sup> or by first computing the corresponding GCM (generator-coordinate method<sup>19</sup>) kernel and then transforming into the RGM basis,<sup>20</sup> the final re-

sult for  $\mathcal{M}_{orb}$  in the  ${}^7\text{Be}$  case is

$$\begin{aligned} \vec{\mu} &= \left[ \frac{4\pi}{3(2J+1)} \right]^{1/2} C(J1J; J0J) \langle \psi_J | \mathcal{M}(M1) | \psi_J \rangle. \\ & \quad (6) \end{aligned}$$

As is noted, both of these quantities are related to the reduced matrix element of the magnetic dipole operator

$$\begin{aligned} \langle \psi_{J_f} | \mathcal{M}(M1) | \psi_{J_i} \rangle &= \frac{(2J_f + 1)^{1/2} (-1)^\mu}{C(J_i 1 J_f; M_i, -\mu, M_f)} \\ &\quad \times \langle \psi_{J_f}^{M_f} | \mathcal{M}(M1, \mu) | \psi_{J_i}^{M_i} \rangle, \end{aligned} \quad (7)$$

which is independent of the magnetic quantum numbers  $M_i$ ,  $M_f$ , and  $\mu$ . For the evaluation of this reduced matrix element, one can, therefore, choose these quantum numbers in any convenient way, e.g.,  $\mu = 0$  and  $M_i = M_f = M = \frac{1}{2}$ .

Since wave functions having the form of Eq. (1) will be used in this investigation, one needs to derive first the expressions of the following two kernel functions:

$$\mathcal{N}(\vec{R}', \vec{R}'') = \sum_{x=0}^3 \mathcal{N}_x(\vec{R}', \vec{R}''), \quad (12)$$

with  $x$  being the number of nucleons interchanged between the  $\alpha$  and the  ${}^3\text{He}$  clusters. The expressions for  $\mathcal{N}_x$  are given in the Appendix for the general case where the bra and ket sides contain three-nucleon functions of different width parameters. These expressions reduce, of course, to the expressions given by Eqs. (33)–(42) in Ref. 7. The computation of the orbital kernel  $\mathcal{M}_{orb}(\vec{R}', \vec{R}'')$  is more complicated, but may be simplified by making use of the spherically symmetric nature of  $Z(\vec{R}_{c.m.})$ . Using this latter property, one obtains, then,

sult for  $\mathcal{M}_{orb}$  in the  ${}^7\text{Be}$  case is

$$\mathcal{M}_{orb}(\vec{R}', \vec{R}'') = \left[ \vec{R}' \times \frac{1}{i} \vec{\nabla}_{\vec{R}'} \right]_z P_{orb}(\vec{R}', \vec{R}''), \quad (14)$$

where

$$P_{\text{orb}}(\vec{R}', \vec{R}'') = \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N \left[ \frac{25}{42} \mathcal{N}_0 + \frac{4(17-4\delta)}{21(6-\delta)} \mathcal{N}_1 + \frac{9-32\delta}{21(1-2\delta)} \mathcal{N}_2 + \frac{16}{21} \mathcal{N}_3 \right], \quad (15)$$

with

$$\delta = \frac{4\alpha_A \alpha_B}{(\alpha_A + \alpha_B)^2}. \quad (16)$$

In the  ${}^7\text{Li}$  case, the procedure is, of course, exactly the same. The resultant expressions for the corresponding kernels  $\tilde{P}_{\text{sp}}$  and  $\tilde{P}_{\text{orb}}$  are

$$\tilde{P}_{\text{sp}}(\vec{R}', \vec{R}'') = \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N g_s(p) \mathcal{N}(\vec{R}', \vec{R}'') \quad (17)$$

$$\langle \psi_{J_f}^M | \mathcal{N}(M 1, 0) | \psi_{J_i}^M \rangle = \sum_m C(1S J_f; m, M - m, M) C(1S J_i; m, M - m, M)$$

$$\times C_{J_f} C_{J_i} \int f_{J_f, l}(R') [m p_l^{\text{orb}}(R', R'') + (M - m) p_l^{\text{sp}}(R', R'')] f_{J_i, l}(R'') dR' dR''. \quad (20)$$

For the ground and first excited states, all the Clebsch-Gordan coefficients involved can be easily calculated and we obtain the simple expressions

$$B(M 1; \frac{3}{2} \rightarrow \frac{1}{2}) = \frac{1}{3} C_{1/2}^2 C_{3/2}^2 \left\{ \int f_{1/2, 1}(R') [p_1^{\text{orb}}(R', R'') - p_1^{\text{sp}}(R', R'')] f_{3/2, 1}(R'') dR' dR'' \right\}^2 \quad (21)$$

for the reduced transition probability in the  ${}^7\text{Be}$  case and

$$\tilde{\mu} = \left[ \frac{4\pi}{3} \right]^{1/2} \tilde{C}_{3/2}^2 \int \tilde{f}_{3/2, 1}(R') [\tilde{p}_1^{\text{orb}}(R', R'') + \frac{1}{2} \tilde{p}_1^{\text{sp}}(R', R'')] \tilde{f}_{3/2, 1}(R'') dR' dR'' \quad (22)$$

for the magnetic moment in the  ${}^7\text{Li}$  case.

A comparison between calculated and empirical values for  $B(M 1; \frac{3}{2} \rightarrow \frac{1}{2})$  in both  ${}^7\text{Li}$  and  ${}^7\text{Be}$  is shown in Table II. To obtain the empirical values, we have used the experimental data, given in Ref. 10, for the  $M 1$  transition

TABLE II. Reduced transition probabilities  $B(M 1; \frac{3}{2} \rightarrow \frac{1}{2})$  and  $B(E 2; \frac{3}{2} \rightarrow \frac{1}{2})$ .

Nucleus	Reduced transition probability	Theory	Experiment
${}^7\text{Li}$	$B(M 1) (\mu_N^2)$	2.17	2.49 ± 0.12
	$B(E 2) (e^2 \text{fm}^4)$	7.55	8.3 ± 0.6 7.4 ± 0.1
${}^7\text{Be}$	$B(M 1) (\mu_N^2)$	1.58	1.87 ± 0.25
	$B(E 2) (e^2 \text{fm}^4)$	21.76	

and

$$\tilde{P}_{\text{orb}}(\vec{R}', \vec{R}'') = \left[ \frac{3}{4\pi} \right]^{1/2} \mu_N \left[ \frac{17}{42} \mathcal{N}_0 + \frac{58-5\delta}{21(6-\delta)} \mathcal{N}_1 + \frac{2(6-5\delta)}{21(1-2\delta)} \mathcal{N}_2 + \frac{5}{21} \mathcal{N}_3 \right]. \quad (18)$$

With the familiar angular-momentum decomposition

$$P_{\text{orb}}(\vec{R}', \vec{R}'') = \frac{1}{R' R''} \sum_{l'm} p_l^{\text{orb}}(R', R'') \times Y_l^m(\hat{R}') Y_l^{m*}(\hat{R}'') \quad (19)$$

and a similar expression for  $P_{\text{sp}}(\vec{R}', \vec{R}'')$  involving  $p_l^{\text{sp}}(R', R'')$ , the matrix element on the right-hand side of Eq. (7) can be straightforwardly evaluated. The result is

widths of the  $\frac{1}{2}^-$  first excited states. Here one notes that the calculated values are about 15% too small, a discrepancy which may arise from the fact that meson-exchange effects have not been considered in our calculation.

The calculated value for the  ${}^7\text{Li}$  magnetic moment is

$$\tilde{\mu} = 3.148 \mu_N. \quad (23)$$

This is in good agreement with the value of  $3.149 \mu_N$  obtained in a GCM calculation<sup>21</sup> employing equal width parameters (i.e.,  $\alpha_A = \alpha_B$ ) for the  $\alpha$  and  ${}^3\text{H}$  clusters, and with the value of  $3.14 \mu_N$  obtained in a shell-model calculation with effective interaction.<sup>22</sup> The experimental value<sup>10</sup> is  $3.256 \mu_N$ , which is about 3% larger than our calculated result.

For electric  $E\lambda$  transitions, the central quantity to be calculated is

$$Q = \left\langle \psi_{J_f}^{M_f} \left| \sum_{i=1}^7 e \frac{1+\tau_{iz}}{2} \rho_i^\lambda Y_\lambda^{\mu_\lambda^*}(\hat{\rho}_i) \right| \psi_{J_i}^{M_i} \right\rangle, \quad (24)$$

which can also be written as

$$Q = \frac{(2\lambda+1)!!}{4\pi i^\lambda (\lambda!)^2} \lim_{q \rightarrow 0} \frac{\partial^\lambda}{\partial q^\lambda} \int Y_\lambda^{\mu*}(\hat{q}) \langle \psi_{J_f}^{M_f} | \sum_{i=1}^7 e^{\frac{1+\tau_{iz}}{2}} \exp(i\vec{q} \cdot \vec{\rho}_i) | \psi_{J_i}^{M_i} \rangle d\hat{q}. \quad (25)$$

This latter form is convenient, since the matrix element appearing in the integrand is the type which has been considered in Refs. 7 and 8. Thus, the computation follows essentially the same procedure as described in these references and we can easily obtain a general expression for the reduced transition probability  $B(E\lambda; J_i \rightarrow J_f)$ .

Calculated values of  $B(E2; \frac{3}{2} \rightarrow \frac{1}{2})$  are compared with experimental results<sup>10</sup> also in Table II. From this comparison, one finds that, in contrast to the  $M1$  case, there is a good agreement. This may indicate that meson-exchange contributions are probably not significant in a low-order electric transition.<sup>17</sup>

We have also computed the value of  $B(E2; \frac{3}{2} \rightarrow \frac{1}{2})$  in the  ${}^7\text{Li}$  case using oscillator shell-model functions of the highest spatial symmetry and the configuration  $(1s)^4(1p)^3$ . These functions are described in case (i) of a previous publication.<sup>23</sup> The resultant value is  $1.75 e^2 \text{fm}^4$ , which is more than a factor of 4 smaller than the value listed in Table II. This is convincing evidence that the nucleus  ${}^7\text{Li}$  does have a strong collective nature which may be described in terms of  ${}^3\text{H} + \alpha$  clustering.

#### IV. ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$ RADIATIVE CAPTURE REACTION

A microscopic discussion of the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  capture reaction has been given previously.<sup>8</sup> We consider it further here, mainly because detailed results of recent measurements have appeared in the literature,<sup>11-13</sup> and it would be interesting to compare these results with our calculated values. In addition, we wish to take this opportunity to improve the previous calculation<sup>8</sup> in two minor respects. First, there was a small numerical-accuracy problem which needs to be corrected. Second, the quantity 163.78 (in units of  $\text{keV}^{1/2}$ ), appearing in Eq. (48) of Ref. 8, was simply taken from the paper of Parker and Kavanagh<sup>24</sup> and is inconsistent with the value of  $\hbar^2/2\bar{\mu} = 12.095 \text{ MeV fm}^2$  used in the calculation ( $\bar{\mu}$  is the resonating-group reduced mass of the  ${}^3\text{He} + \alpha$  system, equal to  $\frac{12}{7}$  times the

nucleon mass<sup>25</sup>). This inconsistency has the unfortunate consequence that the cross-section factor  $S(E)$  has no longer a linear dependence on  $E$  in the very low energy region.<sup>26</sup>

With improved numerical accuracy, the results obtained for the capture cross sections  $\sigma_{3/2}$  and  $\sigma_{1/2}$  to the ground and first excited states, respectively, the total capture cross section  $\sigma_t$ , and the branching ratio  $\tilde{\rho} = \sigma_{1/2}/\sigma_{3/2}$  are listed in Table III. By comparing with the cross-section values given in Table II of Ref. 8, we note that our present improvement results in a small change of about 2%.

Consistent with the value of the reduced mass  $\bar{\mu}$  used in the resonating-group formulation, the cross-section factor  $S(E)$  is defined as

$$S(E) = E\sigma_t(E) \exp(164.514/\sqrt{E}), \quad (26)$$

where  $E$  is expressed in units of keV. It should be remarked here<sup>25</sup> that the resonating-group reduced mass  $\bar{\mu}$  is very close to, but not exactly equal to, the experimental reduced mass  $\bar{\mu}_{\text{ex}}$  obtained by using measured values for the masses of the  $\alpha$  and the  ${}^3\text{He}$  nuclei. At very low energies much below the Coulomb barrier, this will cause a small underestimate in the calculated capture cross section (about a few % at  $E=0.1 \text{ MeV}$ ). However, the calculated value of  $S(E)$  should be quite reliable, since this underestimate in  $\sigma_t$  is compensated by a correspondingly larger value of the exponential factor appearing in the right-hand side of Eq. (26).<sup>27</sup>

In Figs. 1 and 2, we show a comparison of our calculated results for  $S(E)$  and the branching ratio with the experimental data given in Ref. 11 (labeled as Münster-Stuttgart or MS data) and Ref. 12 (labeled as Cal Tech or CT data), respectively. As is seen from these figures, the cross-section factor  $S(E)$  does depend linearly on  $E$  in the very low energy region and there is a good agreement between the calculated and experimental values for the branching ratio. On the other hand, the calculated values

TABLE III. Calculated results for capture cross sections and branching ratio.

$E$ (MeV)	$\sigma_{3/2}$ ( $\mu\text{b}$ )	$\sigma_{1/2}$ ( $\mu\text{b}$ )	$\sigma_t$ ( $\mu\text{b}$ )	$\tilde{\rho}$
0.10	0.000 332 <sup>a</sup>	0.000 137 <sup>a</sup>	0.000 469 <sup>a</sup>	0.414
0.15	0.004 40	0.001 82	0.006 22	0.414
0.20	0.0194	0.008 01	0.0274	0.414
0.30	0.102	0.0424	0.145	0.415
0.40	0.257	0.107	0.364	0.416
0.50	0.457	0.191	0.648	0.417
0.75	1.045	0.441	1.486	0.422
1.00	1.615	0.689	2.304	0.427
1.41	2.470	1.072	3.542	0.434
1.70	3.038	1.330	4.368	0.438
2.06	3.718	1.639	5.357	0.441
2.65	4.787	2.122	6.909	0.443
3.30	5.907	2.621	8.528	0.444
4.00	7.082	3.135	10.217	0.443

<sup>a</sup>The calculated cross-section values at 0.10 MeV are slightly less accurate than those at higher energies.

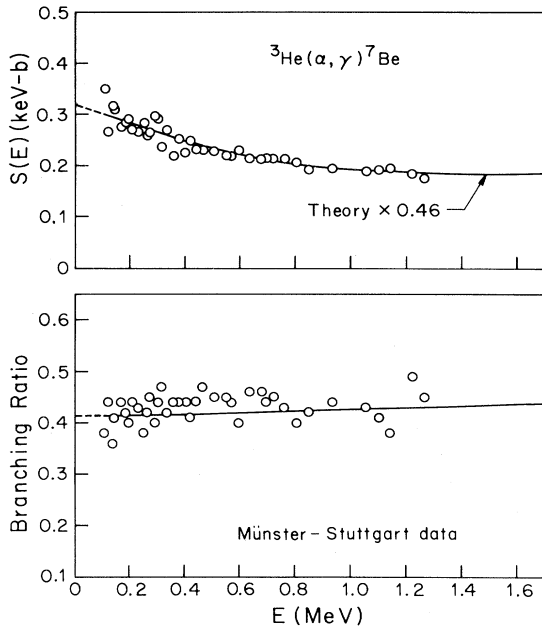


FIG. 1. Comparison of calculated and experimental  $S$  factors and branching ratios. Experimental data shown are those of Ref. 11.

of  $S(E)$  can fit the MS and CT data only if these values are multiplied by different normalizing factors of 0.46 and 0.75, respectively.

In the low-energy region,  $S(E)$  can be characterized by the equation<sup>26</sup>

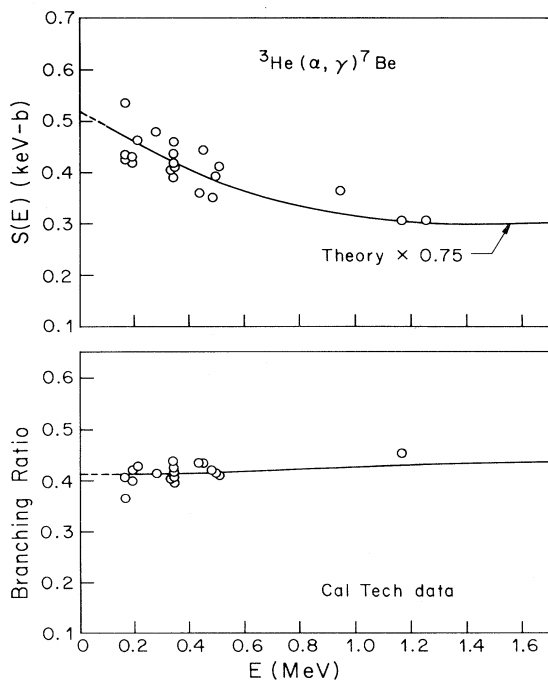


FIG. 2. Comparison of calculated and experimental  $S$  factors and branching ratios. Experimental data shown are those of Ref. 12.

$$S(E) = S(0) \exp(aE + bE^2), \quad (27)$$

from which the important astrophysical quantity  $S(0)$  can be determined. Using the values of  $S(E)$  calculated at 0.15, 0.20, and 0.30 MeV, we obtain

$$\begin{aligned} S(0) &= 0.698 \text{ keV b}, \\ a &= -0.607 \text{ MeV}^{-1}, \\ b &= -0.0527 \text{ MeV}^{-2}. \end{aligned} \quad (28)$$

With these parameter values, Eq. (27) is found to be well satisfied for energies up to about 0.5 MeV.

The MS and CT cross-section values agree in energy dependence but not in magnitude. Thus, the important question is the following: Which of these two data sets is more reliable? To answer this question from the theoretical viewpoint, one should note that, as has been pointed out in Ref. 8, our present calculation suffers from two defects: (i) the resonating-group wave function used has a single,  ${}^3\text{He} + \alpha$  cluster configuration, and (ii) the calculated rms radius of  ${}^7\text{Be}$  is likely 2% to 3% too large. Without a further detailed investigation, it is clearly not possible to predict definitively the consequences of these defects. However, we notice that a recent experimental study<sup>28</sup> in the neighboring nucleus  ${}^6\text{Li}$  did yield a value for the  $d + \alpha$  spectroscopic factor equal to about  $0.85 \pm 0.04$ . Since the nuclei  ${}^6\text{Li}$  and  ${}^7\text{Be}$  have similar separation energies and reduced masses with respect to  $d + \alpha$  and  ${}^3\text{He} + \alpha$  clustering, one may reasonably maintain that a similar value (or a slightly larger value due to tighter internal binding of  ${}^3\text{He}$ ) for the  ${}^3\text{He} + \alpha$  spectroscopic factor in  ${}^7\text{Be}$  could be a good estimate. As for the effect arising from the larger  ${}^7\text{Be}$  size, we can only speculate that it may cause an overestimate of the capture cross section, but by no more than 10%. Thus, the value of  $S(0)$  obtained in this calculation, equal to 0.698 keV b, is very likely too large. With improved  ${}^7\text{Be}$  bound-state and continuum wave functions, we believe that this value will be reduced by roughly 15% to 30%, i.e., in the range from about 0.5 to 0.6 keV b. We realize, of course, that our argument, based mainly on our understanding and experience with the seven-nucleon system, is quite crude. Even so, however, it is our opinion that the present investigation favors the  $S(0)$  value determined from the CT data. The MS measurement may suffer from an overall normalization uncertainty and should be further examined.

It should be remarked that, although the capture cross section may become substantially reduced when improved  ${}^7\text{Be}$  wave functions, consisting of both  ${}^3\text{He} + \alpha$  and other cluster configurations, are employed in the calculation, the rates of  $M1$  and  $E2$  transitions between the ground and first excited states may be less affected by such an improvement. The reason for this is as follows. At very low energies, the radiative capture takes place mainly in the region outside of nuclear interaction. On the other hand, electromagnetic transitions between bound states are mainly influenced by the behavior of the wave function in the region of strong interaction, where the Pauli principle has the effect of reducing greatly the differences between apparently different cluster structures.

### V. ELECTRON CAPTURE OF ${}^7\text{Be}$

The nucleus  ${}^7\text{Be}$  in its ground state is unstable. It decays by electron capture to the ground or first excited state of  ${}^7\text{Li}$ . Because of the similarity in the nuclear structures of these two nuclei, both weak transitions are superallowed.

Experimental data on the half-life  $\tau_{1/2}$  and the branching ratio  $\xi$  or, alternatively, the decay constants  $\tilde{\lambda}$  and  $\tilde{\lambda}^*$  to the ground and first excited states of  ${}^7\text{Li}$ , respectively, are commonly presented in terms of  $ft$  and  $(ft)^*$  values by the expression<sup>29</sup>

$$ft = \frac{\pi}{2} q^2 P \frac{\ln 2}{\tilde{\lambda}}, \quad (29)$$

and a similar expression for  $(ft)^*$  involving  $q^*$  and  $\tilde{\lambda}^*$ . In the above equation, natural units  $\hbar=c=m_e=1$  ( $m_e$  is the electron mass) are adopted, and  $q$  and  $q^*$  represent neutrino momenta equal to 1.687 and 0.751 for transitions to the two states of  ${}^7\text{Li}$ . The quantity  $P$  represents the atomic-physics part of the capture problem; it is related to the probability densities of the orbital electrons at the nucleus, corrected by exchange and overlap considerations.<sup>30</sup> This latter quantity must be determined theoretically. With a multiconfigurational Hartree-Fock approach,<sup>31</sup> the latest value calculated by Chen and Crasemann is  $P=7.221 \times 10^{-5}$ . Using this value, together with  $\tau_{1/2}=53.29$  d and  $\xi=10.36\%$ ,<sup>10</sup> we obtain

$$\log ft = 3.220, \quad \log (ft)^* = 3.454, \quad (30)$$

which are somewhat different from the values quoted in Ref. 10, presumably due to different choices in the value of  $P$ .

Our motivation in studying this particular problem is that the branching ratio  $\xi$  to the first excited state of  ${}^7\text{Li}$  is important in the determination of the  $S(0)$  value from the activation measurement of the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  capture reaction, and a recent experimental investigation by Rolfs *et al.*<sup>15</sup> has yielded a result for  $\xi$  equal to 15.4%, in violent disagreement with the value of  $10.36 \pm 0.10\%$  obtained prior to 1982 by many experimental groups. To resolve this controversy, we shall use the resonating-group wave functions to calculate the value of  $\xi$ . At the same time, we shall of course compare the calculated  $ft$  and  $(ft)^*$  values with experiment in order to gain confidence in our result.

From the nuclear-physics viewpoint, allowed electron-capture reactions are governed by the Fermi ( $F$ ) and Gamow-Teller ( $GT$ ) operators or moments<sup>17</sup>

$$\mathcal{M}(F) = \frac{g_V}{\sqrt{4\pi}} \sum_i \tau_i^-, \quad (31)$$

$$\begin{aligned} K_F(\vec{R}', \vec{R}'') &= \langle \phi_\alpha \phi_t \delta(\vec{R} - \vec{R}') \tilde{\xi}_S^{M_S} Z(\vec{R}_{c.m.}) | \mathcal{M}(F) | \mathcal{A}[\phi_\alpha \phi_h \delta(\vec{R} - \vec{R}'') \tilde{\xi}_S^{M_S} Z(\vec{R}_{c.m.})] \rangle \\ &= \frac{g_V}{\sqrt{4\pi}} \mathcal{N}(\vec{R}', \vec{R}''), \end{aligned} \quad (41)$$

$$\begin{aligned} K_{GT}(\vec{R}', \vec{R}'') &= \langle \phi_\alpha \phi_t \delta(\vec{R} - \vec{R}') \tilde{\xi}_S^{M_S} Z(\vec{R}_{c.m.}) | \mathcal{M}(GT, 0) | \mathcal{A}[\phi_\alpha \phi_h \delta(\vec{R} - \vec{R}'') \tilde{\xi}_S^{M_S} Z(\vec{R}_{c.m.})] \rangle \\ &= \frac{g_A}{\sqrt{4\pi}} 2M_S \mathcal{N}(\vec{R}', \vec{R}''). \end{aligned} \quad (42)$$

$$\mathcal{M}(GT, \mu) = \frac{g_A}{\sqrt{4\pi}} \left[ 2 \sum_i \tau_i^- s_{i\mu}^* \right], \quad (32)$$

where  $\tau_i^-$  is an operator transforming a proton into a neutron, and  $g_V$  and  $g_A$  are vector and axial-vector coupling constants, given by<sup>32,33</sup>

$$g_V = 1.4128(5) \times 10^{-49} \text{ ergs cm}^3, \quad (33)$$

and

$$g_A = -1.255(6) g_V. \quad (34)$$

In addition, it is noted that, while the operator responsible for the Fermi transition is independent of the nucleon spin, the operator responsible for the Gamow-Teller transition is proportional to the spin operator of the decaying nucleon.

The  $ft$  value is related to the reduced transition probabilities  $B(F)$  and  $B(GT)$  through the equation<sup>17</sup>

$$ft = D \frac{g_V^2}{4\pi} [B(F) + B(GT)]^{-1}, \quad (35)$$

with

$$D = \frac{2\pi^3 \ln 2}{g_V^2} = 6164 \text{ s}. \quad (36)$$

These reduced transition probabilities are given by

$$B(F; J \rightarrow J) = \frac{1}{2J+1} | \langle \tilde{\psi}_J | \mathcal{M}(F) | \psi_J \rangle |^2, \quad (37)$$

$$B(GT; J_i \rightarrow J_f) = \frac{1}{2J_i+1} | \langle \tilde{\psi}_{J_f} | \mathcal{M}(GT) | \psi_{J_i} \rangle |^2, \quad (38)$$

which depend on the reduced matrix elements

$$\begin{aligned} \langle \tilde{\psi}_J | \mathcal{M}(F) | \psi_J \rangle &= (2J+1)^{1/2} \langle \tilde{\psi}_J^M | \mathcal{M}(F) | \psi_J^M \rangle, \quad (39) \\ \langle \tilde{\psi}_{J_f} | \mathcal{M}(GT) | \psi_{J_i} \rangle &= \frac{(2J_f+1)^{1/2} (-1)^{\mu}}{C(J_i 1 J_f; M_i, -\mu, M_f)} \\ &\quad \times \langle \tilde{\psi}_{J_f}^{M_f} | \mathcal{M}(GT, \mu) | \psi_{J_i}^{M_i} \rangle. \end{aligned} \quad (40)$$

As in the case of the magnetic-dipole transition described in Sec. III, we can again choose the magnetic quantum numbers as  $\mu=0$  and  $M_i=M_f=M=\frac{1}{2}$ . With such a choice, the kernel functions  $K_F$  and  $K_{GT}$  required in the calculation, corresponding to the Fermi and Gamow-Teller moments, are then

Here one sees that both  $K_F$  and  $K_{GT}$  are proportional to the generalized norm kernel  $\mathcal{N}(\vec{R}', \vec{R}'')$ . This latter kernel depends on three different width parameters  $\alpha_A$ ,  $\alpha_B$ , and  $\alpha'_B$  characterizing the  $\alpha$  particle, the triton, and the  ${}^3\text{He}$  nucleus, respectively. Its explicit expression is given in the Appendix.

By performing an angular-momentum expansion of  $\mathcal{N}(\vec{R}', \vec{R}'')$ , we obtain the partial-wave norm kernels  $\kappa_l(R', R'')$ . Using these kernels and the resonating-group wave functions of Sec. II, we then find

$$\langle \tilde{\psi}_J^M | \mathcal{M}(F) | \psi_J^M \rangle = \frac{g_V}{\sqrt{4\pi}} \tilde{C}_J C_J \int \tilde{f}_{Jl}(R') \kappa_l(R', R'') f_{Jl}(R'') dR' dR'' , \quad (43)$$

$$\begin{aligned} \langle \tilde{\psi}_{J_f}^M | \mathcal{M}(GT, 0) | \psi_{J_i}^M \rangle &= \frac{g_A}{\sqrt{4\pi}} \sum_m 2(M-m) C(ISJ_f; m, M-m, M) \\ &\times C(ISJ_i; m, M-m, M) \tilde{C}_{J_f} C_{J_i} \int \tilde{f}_{J_f l}(R') \kappa_l(R', R'') f_{J_i l}(R'') dR' dR'' . \end{aligned} \quad (44)$$

Substituting these equations into Eqs. (37)–(40) finally yields

$$B(F; \frac{3}{2} \rightarrow \frac{3}{2}) = \frac{g_V^2}{4\pi} \eta^2 , \quad (45)$$

$$B(GT; \frac{3}{2} \rightarrow \frac{3}{2}) = \frac{g_A^2}{4\pi} \frac{5}{3} \eta^2 , \quad (46)$$

$$B(GT; \frac{3}{2} \rightarrow \frac{1}{2}) = \frac{g_A^2}{4\pi} \frac{4}{3} \eta^{*2} , \quad (47)$$

where

$$\begin{aligned} \eta &= \tilde{C}_{3/2} C_{3/2} \int \tilde{f}_{3/2,1}(R') \kappa_1(R', R'') \\ &\quad \times f_{3/2,1}(R'') dR' dR'' , \end{aligned} \quad (48)$$

$$\begin{aligned} \eta^* &= \tilde{C}_{1/2} C_{3/2} \int \tilde{f}_{1/2,1}(R') \kappa_1(R', R'') \\ &\quad \times f_{3/2,1}(R'') dR' dR'' , \end{aligned} \quad (49)$$

represent the overlaps between the  ${}^7\text{Be}$  and  ${}^7\text{Li}$  radial wave functions.

Let us assume for the moment that both  $\eta$  and  $\eta^*$  are equal to 1. Under this assumption, the ratio

$$\frac{\tilde{\lambda}^*}{\tilde{\lambda}} = \frac{q^{*2}}{q^2} \frac{B(GT; \frac{3}{2} \rightarrow \frac{1}{2})}{B(F; \frac{3}{2} \rightarrow \frac{3}{2}) + B(GT; \frac{3}{2} \rightarrow \frac{3}{2})} \quad (50)$$

becomes 0.1148 and the corresponding branching ratio  $\xi$  is 10.30%, in good agreement with the experimental value of  $10.36 \pm 0.10\%$ . In addition, it is important to note that  $\tilde{\lambda}^*/\tilde{\lambda}$  and  $\xi$  do not depend on the electron probability  $P$ . These quantities are, therefore, not subject to the uncertainty associated with the theoretical determination of this particular probability.

It is clear that, to obtain the value of  $\xi$  equal to 15.4% claimed by Rolfs *et al.*,<sup>15</sup> the nuclear overlaps must be significantly different from 1. However, this does not turn out to be the case in our calculation. Using the RGM wave functions, we find that the overlaps are<sup>34</sup>

$$\eta = 0.9990, \quad \eta^* = 0.9996 , \quad (51)$$

which are very close to 1. With these values, the branching ratio is only slightly increased to 10.31%. In fact, it is very difficult to expect a value for  $\xi$  appreciably different from this result. Even though there may exist some uncertainties in the calculated transition rates resulting from the lack of consideration of meson-exchange effects,<sup>35</sup> we

must emphasize that the branching ratio is nearly free of such uncertainties. For instance, an arbitrary increase of the axial-vector coupling constant  $g_A$  by as large as 10% enhances the branching ratio to only 10.77%. In any case, we do not expect the meson-exchange contributions to have a major significance, since the calculated values of  $\log ft$  and  $\log(ft)^*$  are, respectively, equal to 3.231 and 3.468, which agree quite well with empirical values given in Eq. (30). Thus, it is our opinion that the value of  $10.36 \pm 0.10\%$  obtained from older measurements is reliable and a value of  $\xi$  around 15% is very unlikely.

## VI. CONCLUSION

In this investigation, we have studied several electromagnetic and weak-transition problems in the seven-nucleon systems with wave functions obtained from a single-channel resonating-group calculation which yielded successful results for the  ${}^3\text{H} + \alpha$  and  ${}^3\text{He} + \alpha$  scattering cross sections and polarizations. The main purposes are to achieve a better understanding of the clustering features in the nuclei  ${}^7\text{Li}$  and  ${}^7\text{Be}$ , to obtain a good estimate of the zero-energy cross-section factor  $S(0)$  in the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  capture reaction, and to resolve the discrepancy between measured values for the branching ratio to the  ${}^7\text{Li}$  first excited state by the electron capture of  ${}^7\text{Be}$ .

The essential findings of this investigation are as follows:

(i) The calculated values of  $B(M1)$  and  $B(E2)$  for transitions between the ground and first excited states agree rather well with the values determined empirically. The  $B(M1)$  study yields an indication that meson-exchange effects may not be insignificant in  $M1$  transitions, while the  $B(E2)$  study shows clearly that the nuclei  ${}^7\text{Li}$  and  ${}^7\text{Be}$  must have strong collective features, describable in terms of  ${}^3\text{H} + \alpha$  and  ${}^3\text{He} + \alpha$  nucleon clustering.

(ii) Based on the calculated result and upon making some crude but reasonable corrections, we can make the statement that the cross-section factor  $S(0)$  has, likely, a value between 0.5 and 0.6 keV b. This suggests that the empirical  $S(0)$  value determined by Osborne *et al.*<sup>12</sup> is reliable, while that determined by Kräwinkel *et al.*<sup>11</sup> may need to be further examined.

(iii) The calculated value for the  ${}^7\text{Be}$  electron-capture branching ratio to the first excited state of  ${}^7\text{Li}$  is 10.31%. This agrees very well with the measured value of  $10.36 \pm 0.10\%$  (Ref. 10) reported prior to 1982, but is in complete variance with the recent value of 15.4% reported



by Rolfs *et al.*<sup>15</sup>

In conclusion, it is our opinion that the present study establishes convincingly the usefulness of resonating-group wave functions in describing the properties of light nuclear systems with respect to electromagnetic and weak interactions. Thus, as future projects, it would be interesting to use similar wave functions<sup>36,37</sup> to examine other electromagnetic problems such as the  $d + \alpha$  and  $p + {}^7\text{Be}$  radiative capture reactions, which are important from astrophysical viewpoints.<sup>28,38</sup> In addition, it will of course be desirable to refine our present calculation by further adopting more flexible resonating-group wave functions which take into account the specific distortion of the clusters in the strong-interaction region<sup>25</sup> and, thereby, yield a better description of the clustering properties of the nuclei under consideration. Especially for the  ${}^3\text{He}(\alpha, \gamma){}^7\text{Be}$  capture reaction, such a refined calculation should be carried out, since an accurate determination of the  $S(0)$  value is clearly a necessary step in any future attempt to resolve the puzzling solar-neutrino problem.

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#### APPENDIX: EXPRESSION FOR $\mathcal{N}_x$

The norm kernel for the  $\alpha + 3N$  system is given by

$$\mathcal{N}(\vec{R}', \vec{R}'') = \langle \phi_\alpha \phi_3 \delta(\vec{R} - \vec{R}') \xi_S^{M_S} Z(\vec{R}_{c.m.}) | \mathcal{A}[\phi_\alpha \phi_3' \delta(\vec{R} - \vec{R}'') \xi_S^{M_S} Z(\vec{R}_{c.m.})] \rangle.$$

Here we consider the general case where the three-nucleon spatial functions  $\phi_3$  and  $\phi_3'$  on the bra and ket sides are characterized by different width parameters equal to  $\alpha_B$  and  $\alpha_B'$ , respectively. By using standard techniques, one can easily integrate over all spatial coordinates, sum over all spin-isospin coordinates, and obtain

$$\mathcal{N}(\vec{R}', \vec{R}'') = \sum_{x=0}^3 \mathcal{N}_x(\vec{R}', \vec{R}''),$$

where

$$\begin{aligned} \mathcal{N}_0(\vec{R}', \vec{R}'') &= \left[ \frac{4\alpha_B \alpha_B'}{(\alpha_B + \alpha_B')^2} \right]^{3/2} \delta(\vec{R}' - \vec{R}''), \\ \mathcal{N}_x(\vec{R}', \vec{R}'') &= \left[ \frac{4\alpha_B \alpha_B'}{(\alpha_B + \alpha_B')^2} \right]^{3/2} (-1)^x P_x \\ &\quad \times \exp(-A_x \vec{R}'^2 - B_x \vec{R}''^2 - C_x \vec{R}' \cdot \vec{R}''), \end{aligned}$$

with

$$\begin{aligned} P_1 &= 3 \left[ \frac{12}{7} \right]^3 \left[ \frac{12\alpha_A}{17\pi} \right]^{3/2} \left[ \frac{17(\alpha_B + \alpha_B')}{16\alpha_A + 9(\alpha_B + \alpha_B')} \right]^{3/2}, \\ P_2 &= 3 \left[ \frac{12}{7} \right]^3 \left[ \frac{3\alpha_A}{5\pi} \right]^{3/2} \left\{ \frac{10\alpha_A(\alpha_B + \alpha_B')^2}{(\alpha_A + \alpha_B)(\alpha_A + \alpha_B')[4\alpha_A + 3(\alpha_B + \alpha_B')]} \right\}^{3/2}, \\ P_3 &= \left[ \frac{12}{7} \right]^3 \left[ \frac{4\alpha_A}{3\pi} \right]^{3/2} \left[ \frac{2\alpha_A(\alpha_B + \alpha_B')}{(\alpha_A + \alpha_B)(\alpha_A + \alpha_B')} \right]^3, \\ A_1 &= \frac{6}{49} \frac{72\alpha_A^2 + \alpha_A(41\alpha_B + 153\alpha_B') + 72\alpha_B\alpha_B'}{16\alpha_A + 9(\alpha_B + \alpha_B')}, \\ A_2 &= \frac{6}{49} \frac{6\alpha_A^2 + \alpha_A(17\alpha_B + 45\alpha_B') + 6\alpha_B\alpha_B'}{4\alpha_A + 3(\alpha_B + \alpha_B')}, \quad A_3 = \frac{150}{49} \alpha_A, \\ B_1 &= A_1 \quad \text{with } \alpha_B \leftrightarrow \alpha_B', \quad B_2 = A_2 \quad \text{with } \alpha_B \leftrightarrow \alpha_B', \quad B_3 = A_3, \\ C_1 &= -\frac{288}{49} \frac{3\alpha_A^2 + 2\alpha_A(\alpha_B + \alpha_B') + 3\alpha_B\alpha_B'}{16\alpha_A + 9(\alpha_B + \alpha_B')}, \quad C_2 = -\frac{72}{49} \frac{\alpha_A^2 - 3\alpha_A(\alpha_B + \alpha_B') + \alpha_B\alpha_B'}{4\alpha_A + 3(\alpha_B + \alpha_B')}, \quad C_3 = \frac{288}{49} \alpha_A. \end{aligned}$$

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