Differential muon-capture rate and the deuterium form factors in the timelike region

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The differential muon-capture rate, $d\Gamma/dE_n$, is obtained over the total allowed range of E_n, the neutron energy for both F_A, the axial current form factor, analytically continued to the timelike region, and for F_A under the condition that $F_A(q^2, \text{timelike}) = F_A(q^2, \text{spacelike})$. It is found that a difference of approximately 10% exists in the extreme timelike region. The contribution from the timelike region to the muon-capture rate, Γ, is calculated and found to be negligible.

NUCLEAR REACTIONS Muon-capture $^2\text{H}(\mu^-, \nu_\mu)nn$, Γ, $d\Gamma/dE_n$ calculated under different assumptions for form factor behavior in timelike region.

I. INTRODUCTION

Recently there has been some interest[1–3] in the question of the behavior of the nuclear axial current form factor, $F_A(q^2)$, in the timelike region. The usual procedure has been to assume analytic continuation for $F_A(q^2)$ to the timelike region, but some evidence exists suggesting a falloff of $F_A(q^2)$ in the timelike region in a form consistent with

$$F_A(q^2, \text{timelike}) \simeq F_A(q'^2, \text{spacelike}),$$

where $q'^2 = -q^2$, and q^2 is in the range appropriate to muon capture. One possibility for studying this question is to examine the differential muon-capture rate in deuterium, $d\Gamma/dE_n$, where E_n is the neutron energy. For this process,

$$\mu^- + {}^2\text{H} \rightarrow n + n + \nu_\mu,$$

q^2 varies from $-m_\mu{}^2$ to $+m_\mu{}^2$, although the matrix elements are largest in the spacelike region.

The use of deuterium in a reaction of this kind has both advantages and disadvantages. A substantial amount of work has been done on deuterium, and from pion-photoproduction data[4] the axial current form factor in the spacelike region can be obtained. Furthermore, over the range of q^2 considered here, the capture rate is dominated by the axial current form factor.[5] On the other hand, the presence of two neutrons and a neutrino makes the final state a difficult one with which to work. Nevertheless, the result of a careful measurement of $d\Gamma/dE_n$ might be very useful.

In Sec. II we discuss the matrix elements occurring in these calculations. In Sec. III we present values for $d\Gamma/dE_n$ over the allowed energy range of E_n for both analytically continued form factors and for mirrorlike form factors

$$F_A(q'^2, \text{spacelike}) = F_A(q^2, \text{timelike}),$$

[with $q'^2 = -q^2$]. The contribution from the timelike to the capture rate Γ is also calculated. In Sec. IV we discuss our results.

II. MATRIX ELEMENTS

The matrix elements for muon capture in deuterium

$$\mu^- + {}^2\text{H} \rightarrow n + n + \nu_\mu,$$

is given to the lowest order in $G\,(=1.02 \times 10^{-5} m_p{}^{-2})$, the weak coupling constant, by

$$\langle n,n,\nu \,|\, H_w(0) \,|\, {}^2\text{H},\mu^- \rangle = G \cos\theta_C \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_\mu \langle nn \,|\, J_\lambda^+(0) \,|\, {}^2H \rangle,$$

where θ_C is the Cabibbo angle ($\cos\theta_C = 0.98$), and

$$J^\lambda(0) = V^\lambda(0) - A^\lambda(0)$$

is the weak hadronic current, $V^\lambda(0)$ being the vector part and $A^\lambda(0)$ the axial part, respectively. Thus it is necessary to obtain $\langle nn \,|\, V_\lambda^+(0) \,|\, {}^2\text{H} \rangle$ and $\langle nn \,|\, A_\lambda^+(0) \,|\, {}^2\text{H} \rangle$. The form of these matrix elements has been obtained by the author[6–9] in previous papers and is given by

$$\langle nn \mid V_\lambda^+(0) \mid {}^2\mathrm{H}\rangle = \eta\bar{u}(p_1)\left[\frac{F_1}{M_d{}^2}\epsilon_{\lambda\nu\rho\sigma}\xi^\nu Q^\rho d^\sigma + \frac{F_2}{M_d}\epsilon_{\nu\rho\sigma\lambda}\gamma^\nu\xi^\rho q^\sigma\right]\gamma^5 v(p_2), \tag{3a}$$

$$\langle nn \mid A_\lambda^+(0) \mid {}^2\mathrm{H}\rangle = \eta\bar{u}(p_1)\left[F_A\xi_\lambda + F_P\frac{\xi\cdot Q q_\lambda}{M_d{}^2}\right]\gamma_5 v(p_2), \tag{3b}$$

where

$$\eta = [m^2/(E_1E_2)]^{1/2}(\pi)^{-1/2}(2d_0)^{-1/2};$$

m and M_d are the neutron and deuteron mass, respectively; d_μ is the deuteron four-momentum; E_1 and E_2 are the neutron energies; ξ_μ is the deuteron polarization vector; and E_1, E_2, n_{1_μ}, n_{2_μ} are the neutron energies and four-momenta, respectively, and

$$\begin{aligned} Q_\mu &= n_{1_\mu} + n_{2_\mu}, \\ q_\mu &= n_{1_\mu} + n_{2_\mu} - d_\mu, \\ P_\mu &= n_{1_\mu} - n_{2_\mu}. \end{aligned} \tag{4}$$

Recent work[10] has indicated that in the timelike region the pion exchange current contributes substantially to the transition matrix element. Elementary particle model form factors in principle include all hadronic contributions to the weak current. In the deuterium case treated here, Eqs. (3a) and (3b), this must be qualified. For example, the axial current matrix element [Eq. (3b)], after consideration of parity, is described by 12 independent form factors. In the usual case of a single hadron in the initial and final states, this number could be reduced by going to a frame in which the momenta of the hadrons are colinear and applying helicity arguments. Here this is not possible. The form factors in Eqs. (3a) and (3b) were chosen[6] such that in the q^2 range appropriate to muon capture, they would yield a result with no corrections greater than the order $(p/M_d)^2$, where p is the neutron momentum and M_d the deuteron mass. In the worst case, namely zero neutrino momentum, $(p/M_d)^2 \sim 0.03$. Thus the choice given by Eqs. (3a) and (3b) should be sufficient for our purpose.

The matrix elements Eqs. (3a) and (3b) are then completely determined by the form factors F_1, F_2, F_A, and F_P. We determine F_P from F_A by using partial conservation of axial-vector current (PCAC) in the form[11] suggested by Nambu

$$F_P = -M_d{}^2F_A/(q^2 - m_\pi{}^2), \tag{5}$$

and note that only the combination $F_1 - F_2$ occurs in the matrix element [Eq. (1)] squared. We previously[12] determined the form factors F_A from pion-photoproduction data and found that

$$|F_A|^2 = |\mathscr{F}_A|^2 f_A{}^2, \tag{6a}$$

where

$$|\mathscr{F}_A|^2 = \frac{(3.61\times 10 + 6.13\times 10^{-1}Q_0)}{((Q_0-0.11)^2 + 6.76\times 10^{-2})}[1.0 - e^{-6.7\times 10^{-7}(q^2+1.6\times 10^4)^2}]$$
$$\times[1.0 + 1.57\times 10^2 e^{-9.49\times 10^{-10}(q^2+0.97\times 10^5)^2}][1.0 - 1.71 e^{-2.83\times 10^{10}(q^2+1.05\times 10^5)^2}] \tag{6b}$$

$$f_A = 1/(1.0 - q^2/M_A{}^2)^2, \quad M_A = 0.912 \text{ GeV}. \tag{6c}$$

We also had previously determined[12] $F_1 - F_2$ from photodisintegration and electrodisintegration data and had found

$$|F_1 - F_2|^2 = (f_1(q^2))^2(\mathscr{F}_1 - \mathscr{F}_2)^2, \tag{7a}$$

$$M_V = 0.84 \text{ GeV},$$

$$|\mathscr{F}_1 - \mathscr{F}_2| = \frac{2^{1/2}(1.05 + 1.41\times 10^4 q\cdot d)}{(5.21\times 10^{-4}q\cdot d - 2.26)^2 + 1.8}\left\{1 - \frac{1}{M_d{}^4}\left[0.75q^2 - q\cdot d - \left[\frac{q\cdot d}{M_d}\right]^2\right]^2 R(q^2,\cos\theta)\right\}, \tag{7b}$$

$$R(q^2,\cos\theta) = [1.0 + 2.9\cos^2\theta + q^2(1.73\times 10^{-5} + 5.02\cos^2\theta) + q^4(3.27\times 10^{-9} + 9.48\times 10^{-9}\cos^2\theta)]$$
$$\times\frac{1. + 0.12e^{(-9.8\times 10^{-9}(q^2+0.02\times 10^6)^2)}}{1.0 + \{9.90\times 10^2 + 2.2\times 10^3[1 - e^{-5.5\times 10^{-12}q^4}]\}q^4}, \tag{7c}$$

where θ is the angle between p and q. With these form factors the matrix element, Eq. (1) is completely determined and found to be

$$|M|^2=\frac{2}{6m_\mu m^2}\left[F_A\frac{(m_\mu+M_d)(m_\mu+M_d-\nu)}{2}\right.$$
$$\left.\times\left[3m_\mu\nu+\frac{2m_\mu{}^2\nu^2}{(m_\mu{}^2-2m_\mu\nu-m_\pi{}^2)}+\frac{\nu^3 m\mu^3}{(m_\mu{}^2-2\nu m\mu-m_\pi{}^2)^2}\right]+(F_1-F_2)^2m_\mu\nu^3\right]. \quad (8)$$

we note again that at the values of q^2 appropriate to muon capture, the matrix element, Eq. (8), is dominated by the axial current form factor F_A.

III. CALCULATIONS OF THE MUON CAPTURE AND DIFFERENTIAL MUON-CAPTURE RATES

The differential muon-capture rate is given by

$$d\Gamma=\frac{m^2m_\nu m_\mu|\Psi(0)|^2}{2M_d(2\pi)^5}|M|^2\frac{d^3\nu}{\nu}\frac{d^3p_{n_1}}{E_{n_1}}\frac{dp_{n_2}}{E_{n_2}}$$
$$\times\delta^4(p_{n_{1_\mu}}+p_{n_{2_\mu}}+\nu_\mu-(d_\mu+\mu_\mu)) . \quad (9)$$

This expression can be completely integrated to yield Γ, the muon-capture rate, or partially integrated to yield $d\Gamma/dE_n$, where E_n is either of the neutrons. We wish to examine the behavior of the axial current form factor F_A in the timelike region, here $m_\mu{}^2\geq q^2\geq 0$. As mentioned before, the usual assumption made is that the form factor may be analytically continued to the timelike region. However, some evidence[12] supports the possibility that in ^{12}C and ^{6}Li

$$F_A(m_\pi{}^2)=F_A(-m_\pi{}^2), \quad (10)$$

which is consistent with the assumption

$$F_A(q^2)\cong F_A(q'^2) \quad (11)$$

for q^2 timelike with $q'^2=-q^2$. The usual form factor for the axial current form factor in the spacelike region is the dipole fit

$$F_A(q^2)=F_A(0)/(1-q^2/M_A{}^2)^2 . \quad (12)$$

The form factor can depend only on q^2 as long as the initial and final hadronic states are single particle states. In the case under consideration here, F_A is a function of q^2, Q^2, and $P\cdot d$, because the final state of the hadronic weak current matrix element contains two particles in the final state rather than just one. We can therefore, with respect to Eq. (11), make two assumptions for the deuteron case. Either Eq. (11) can be used directly or one can assume that Eq. (11) merely represents an adjustment in the pole given in Eq. (12), and that the rest of $F_A(q^2, Q^2, P\cdot d)$ is unchanged. This would be equivalent to considering in Eq. (6c)

$$f_A(q^2>0)=f_A(0)/(1+q^2/M_A{}^2)^2 . \quad (13)$$

We refer to analytic continuation as assumption I, the formula given by Eq. (11) as assumption II, and the formula given by Eq. (13) as assumption III. Thus, assumption II refers to replacing timelike q^2 by $-q^2$ in F_A and assumption III refers to making this replacement in the dipole part only. Making use of Eqs. (1), (3), (5)—(9), and (13), and with the help of a Univac 1100/80A computer, we obtain the

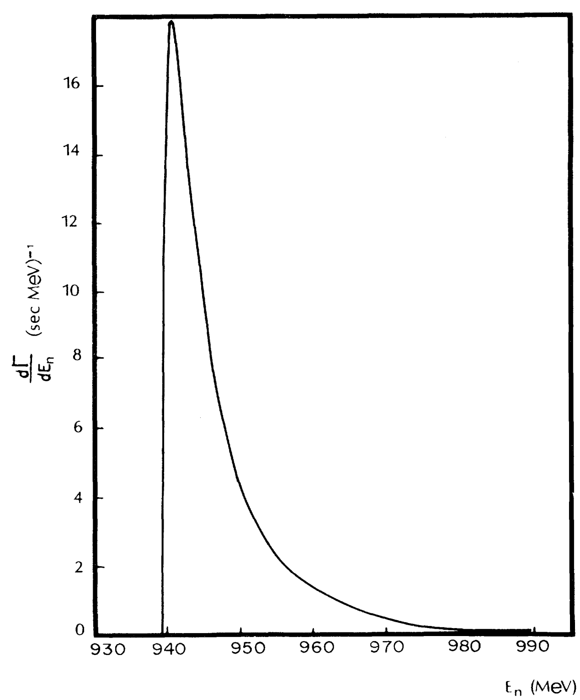

FIG. 1. Plot of the differential muon-capture rate as a function of neutron energy. The curve is plotted under the assumption of analytic continuation for F_A in the timelike region.

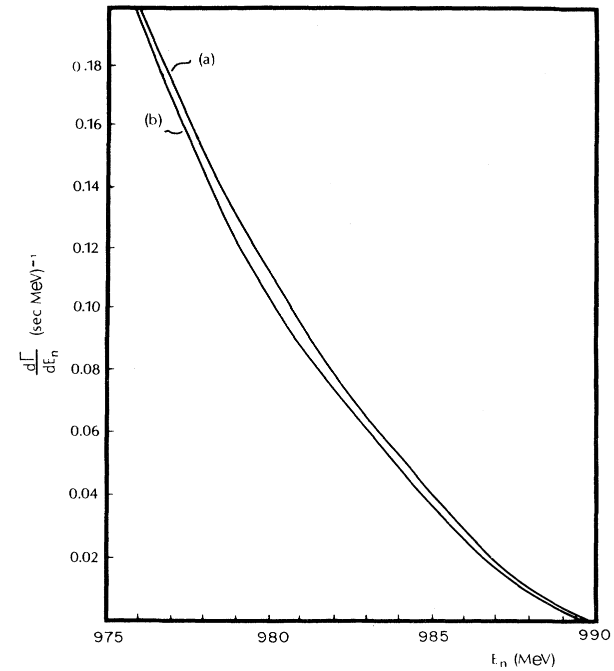

FIG. 2. Plot of the differential muon capture rate as a function of neutron energy for large neutron energy. Curve (a) is the rate under the assumption of analytic continuation for F_A. Curve (b) is the rate assuming

$$F_A(q^2,\ \text{timelike})=F_A(q'^2,\ \text{spacelike})$$

with $q'^2=-q^2$ (assumption II of the text).

following:

$$\Gamma=154.53\ \text{sec}^{-1}\ \text{(assumption I)}, \tag{14a}$$

$$\Gamma=154.27\ \text{sec}^{-1}\ \text{(assumption II)}, \tag{14b}$$

$$\Gamma=154.27\ \text{sec}^{-1}\ \text{(assumption III)}. \tag{14c}$$

We also obtain the differential cross-section $d\Gamma/dE_n$, which is plotted in Fig. 1. In Fig. 2 we show the large E_n part of the differential cross section under the three assumptions. We note that the results for assumptions II and III differ at most by 0.3%, and so we have not distinguished them in the plot.

IV. CONCLUSION

As can be seen from Eqs. (14a)—(14c), it is essentially impossible to distinguish among the three assumptions for $F_A(q^2,\ q^2>0)$ in the muon-capture rate. Experimental measurements[13–15] yield values which are essentially the doublet capture rate Γ_d. The most recent measurement is [15]

$$\Gamma_d=451\pm70\ \text{sec}^{-1}. \tag{15}$$

The quartet rate has not been measured but recent theoretical calculations place it in the range[16]

$$\Gamma_q\simeq 6-10\ \text{sec}^{-1}. \tag{16}$$

The total capture rate Γ is related to Γ_d and Γ_q by

$$\Gamma=\tfrac{1}{3}\Gamma_d+\tfrac{2}{3}\Gamma_q. \tag{17}$$

Making use of Eq. (15) and taking[12] $\Gamma_q=7.3\ \text{sec}^{-1}$, one obtains

$$\Gamma\simeq 150+20\%\ \text{sec}^{-1}$$

This error would be difficult to reduce greatly at present.

The situation for the differential cross section is somewhat better. For values of E_n corresponding to large contributions from the timelike q^2 region, there is about a 10% difference in the magnitude of $d\Gamma/dE_n$ between the assumption (I) of analytic continuation, and that of either assumption II or III. Because the effects of the difference between assumptions II and III are at most 0.3%, it is not possible to distinguish between these two possibilities at this time by this method.

It might therefore be very useful to have careful measurements of $d\Gamma/dE_n$ over its entire range of E_n. From the small E_n part of the spectrum which is dominated by $q^2>0$ contributions, F_A in the spacelike region could be determined and analytically continued to see if it agreed with F_A determined in the large E_n region dominated by timelike q^2 values.

Recently, some work[10] was done indicating that in the extreme timelike region, contributions from the pion exchange currents become important. This might provide a theoretical reason as to why the behavior in the timelike region might not be a simple analytic continuation from the spacelike region. More experimental and theoretical work is clearly necessary to clarify the situation.

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