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Sensitivity of N-d polarization observables on the off-shell behavior of the N-N interaction

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We demonstrate the off-shell sensitivity of second-order polarization observables in elastic N-d scattering by means of the spin-correlation parameter $C_{y,y}$. For the N-N interaction we start with the Paris potential and then generate on-shell equivalent separable potentials with different off-shell properties. The results for $E_d = 10$ and 20 MeV reported here show sizeable off-shell effects, which can be related to the different potential behavior in the inner region $r \lesssim 1.2$ fm.

NUCLEAR REACTIONS Off-shell effects in elastic N-d scattering; separable approximation of the Paris potential used together with on-shell equivalent transformations.

The driving impetus for studying composite nuclear systems has apparently always ensued from our inquiry about the fundamental nucleon-nucleon (N-N) interaction. N-N data alone do not allow for a complete determination of the pertinent transition operator both on and off energy shell. However, any problem involving the N-N system as a partial component can provide information about its off-shell behavior. Often electron- or photon-induced reactions on ^{2}H and ^{3}H (respectively, ^{3}He) have been considered as efficient testing grounds for probing off-shell properties. But similarly the 3-N or π-d system can be valuable in this respect. Of course, it has been a controversial issue—mainly as far as N-d scattering is concerned[1]—as to what extent the extracted information can really restrict the off-shell form of the N-N interaction. At present this question becomes increasingly important, since in searching for the "true" N-N force we are facing several realistic models, which are basically derived from first dynamical principles. Especially the introduction of quark degrees-of-freedom leads to N-N potentials which modify the intermediate- and short-range parts ($r \lesssim 1$ fm) of the interaction.[2] As a consequence these models also manifest a different off-shell behavior, the effect of which remains to be tested.

In this context we consider N-d scattering. Despite the great effort invested in this problem most results have been found to provide disappointingly weak evidence on the off-shell N-N interaction, the reason being that inadequate observables have been considered. Evidently sensible changes in the off-shell behavior are connected with variations of the intermediate- (and probably short-) range part of the N-N force, since the long-range part is well constrained by theory [the one-pion exchange potential (OPEP)] and experiment. The interior region, however, is predominantly coming in through N-N S waves, because in higher partial waves the centrifugal barrier prevents the particles from approaching as closely. Consequently, in order to detect off-shell effects, such N-d observables that are dominated by N-N S waves should be looked for. Here, we can follow the arguments by Saylor and Rad.[3] In particular, they demonstrated that aside from the differential cross section only second-order polarizations with the spins of projectile and target aligned are nonzero, if just S waves are used in the N-N subsystem. Furthermore, for such second-order polarizations it was found that the main structure of their angular distribution is determined already by the N-N S waves,[4] in contrast to first-order polarizations, whose structure appears with the predominance of higher N-N partial waves.

A first indication for significant off-shell effects was observed only recently for certain N-d spin-transfer coefficients.[5] Unfortunately this work is hampered by the use of N-N potentials that are not really on-shell equivalent. This motivated us to carry out a thorough study of the off-shell sensitivity of the second-order polarizations in question. We performed Faddeev calculations using strictly on-shell equivalent potentials with different kinds of off-shell behavior in N-N S waves.

We exemplify our results by means of the N-d spin-correlation parameter $C_{y,y}$. At the low energies considered

TABLE I. Parameters of PEST potentials. Dimensions are $(\beta) =$ fm^{-1} and $(C) =$ MeV$^{1/2}$ fm$^{-1/2}$.

$^1S_0$		$^3S_1$	
$\beta_1 = 1.8$	$C_1 = 120.367\,30$	$\beta_1 = 1.5$	$C_1 = 3.378\,646\,9$
$\beta_2 = 2.924\,108\,6$	$C_2 = -3381.1886$	$\beta_2 = 3.0$	$C_2 = -637.419\,08$
$\beta_3 = 3.883\,804\,7$	$C_3 = 29\,271.287$	$\beta_3 = 4.5$	$C_3 = 1750.2432$
$\beta_4 = 4.750\,228\,5$	$C_4 = -88\,383.471$	$\beta_4 = 6.0$	$C_4 = 3561.3535$
$\beta_5 = 5.553\,304\,8$	$C_5 = 106\,637.01$	$\beta_5 = 7.5$	$C_5 = -12\,939.749$
$\beta_6 = 6.309\,259\,4$	$C_6 = -44\,462.132$	$\beta_6 = 9.0$	$C_6 = 8656.6202$

TABLE II. Some N-N properties of the PEST interaction as compared to the original Paris potential. In ${}^1S_0$ the data represent purely nuclear p-p quantities. For the deuteron the binding energy for both potentials is $E_d = 2.2249$ MeV.

		a (fm)	r (fm)	δ (deg) $E_{\text{lab}} = 5$ MeV	 $E_{\text{lab}} = 10$ MeV	 $E_{\text{lab}} = 50$ MeV
${}^1S_0$	Paris potential	−17.543	2.868	59.93	56.91	38.10
	PEST	−17.534	2.783	59.82	56.13	33.74
${}^3S_1$	Paris potential	5.427	1.789	118.04	102.42	62.27
	PEST	5.422	1.785	117.82	101.82	58.58

here $C_{y,y}$ is essentially governed by N-N S waves, like the spin-transfer coefficient in Ref. 5. This is especially true for the angular region around $\theta_{\text{c.m.}} \approx 90°$ where we can compare our results with existing experimental data.[6–8] Of course, at present their accuracy is not so good, but this situation is likely to be improved in the near future.[9]

We prefer to begin with a N-N potential, the on- and off-shell behaviors of which could be considered realistic. Thus we chose the Paris potential[10] and cast it into a separable representation by means of the Ernst-Shakin-Thaler (EST) method.[11] The resulting S-wave potentials, called PEST, quite accurately reproduce the Paris on- and off-shell transition operators at $E = -E_d$ (deuteron pole) for ${}^3S_1$ and at $E = 0$ for ${}^1S_0$; in a broad neighborhood of these energies they furnish an excellent approximation.[12] The analytic form of the PEST potentials in either partial wave is

$$V(p',p) = -g(p')g(p),$$
$$g(p) = \sum_{i=1}^{6} \frac{C_i}{p^2+\beta_i^2}\,, \tag{1}$$

with the parameters given in Table I. By construction the on-shell properties of PEST (see Table II) are either equal to the Paris potential or do not deviate much up to $E_{\text{lab}}^{\text{N-N}} \approx 50$ MeV (i.e., the N-N subsystem domain is most relevant for the 3-N calculation performed). In the same energy region the off-shell behavior of PEST is practically equivalent to the Paris potential; small differences occur only at unimportant high off-shell momenta $p > 8$ fm^{-1} (see Fig. 1).

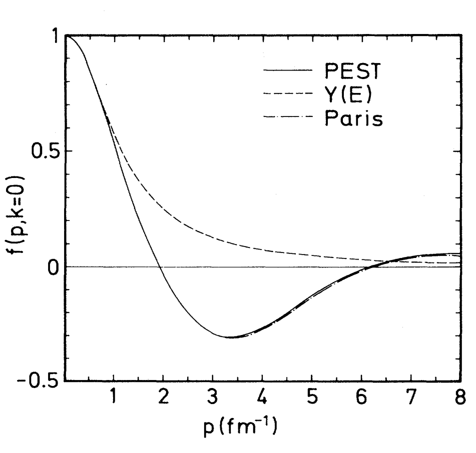

FIG. 1. Noyes-Kowalski half-off-shell functions for the ${}^1S_0$ N-N state at zero energy.

The $l = 0$ PEST interaction yields n-d spin correlations $C_{y,y}$ as shown in Fig. 2 for deuteron laboratory energies of 10 and 20 MeV. Because of the arguments given above, the results represent a good estimate of what can be expected from the true Paris potential. The experimental data quoted belong to p-d measurements and therefore still

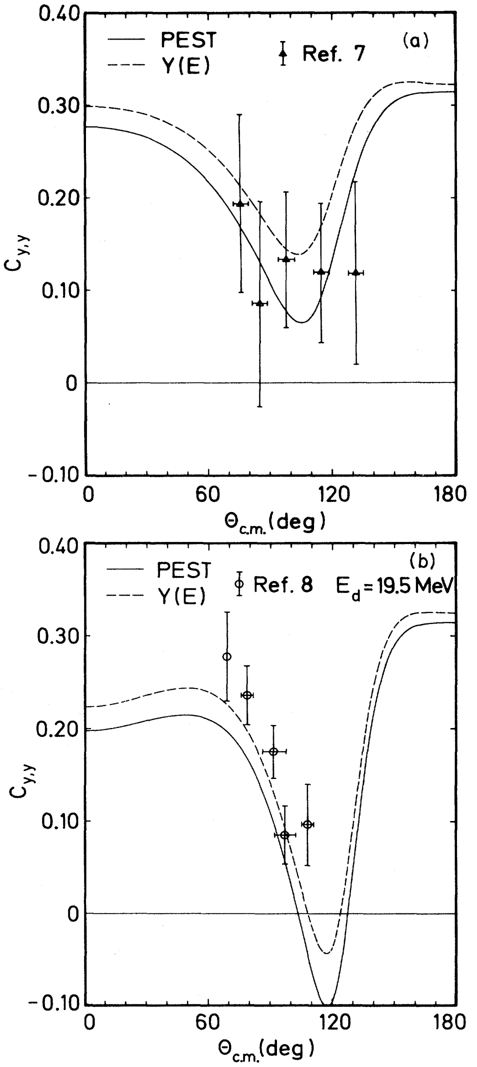

FIG. 2. Spin-correlation parameter for the ${}^1\text{H}(\vec{d},d){}^1\text{H}$ reaction at (a) $E_d = 10$ and (b) $E_d = 20$ MeV laboratory energy.

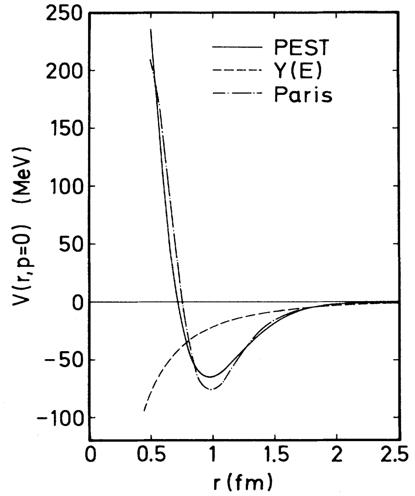

FIG. 3. Static limit of configuration-space transforms according to Eq. (4).

contain Coulomb effects. We could have easily implemented Coulomb corrections in our calculations by an approximate procedure,[7,13] but we preferred to deal with bare n-d quantities. Since we are concerned with purely hadronic off-shell effects in this work, we looked to avoid complications with electromagnetic interactions and also with the question of charge independence (symmetry).

In the next step we carried out the same calculation with a separable interaction $Y(E)$ of a Yamaguchi type:

$$V(p',p;E)=g(p')\lambda(E)g(p)\ ,$$
$$g(p)=\frac{1}{p^2+\gamma^2}\ , \tag{2}$$

with

$$\lambda^{-1}(E)=\frac{g^2(k)}{R^{\mathrm{PEST}}(k)}+\frac{2\mu}{\hbar^2}\int_0^\infty dp\,\frac{p^2g^2(p)}{k^2-p^2}\ ,$$
$$E=\frac{\hbar^2k^2}{2\mu} \tag{3}$$

where $R^{\mathrm{PEST}}(k)$ is the on-shell R matrix of the PEST potential. While $Y(E)$ ensures on-shell equivalence via the energy-dependent strength parameter $\lambda(E)$,[14] its off-shell behavior is rather different from PEST (or the Paris potential), cf. Fig. 1. The freedom in choosing the parameter γ, which determines the shape of the potential, is restricted by the requirement that the long-range N-N wave function remains unaltered. The corresponding values of γ, which are thus mainly determined by the Paris effective-range parameters, are found to be $\gamma(^1S_0)=1.149\,265\ \mathrm{fm}^{-1}$ and $\gamma(^3S_1)=1.392\,689\ \mathrm{fm}^{-1}$.

As is seen from Fig. 2, the different off-shell behavior induces a sizeable change in the observable $C_{y,y}$. We also examined the influence which comes from $^1S_0$ and $^3S_1$ N-N states separately. This study revealed that most of the effect observed in Fig. 2 must be attributed to the $^1S_0$ partial wave. Concentrating on this particular state we employed even further types of on-shell equivalent potentials. Thereby we could relate the differences in $C_{y,y}$ to the form of the half-shell function $f(p,k)$ around $p\approx 2\ \mathrm{fm}^{-1}$. Especially the occurrence of the first zero in $f(p,k)$ has a striking influence on $C_{y,y}$. If the long-range part is not subject to alterations, different models digress in their results for $C_{y,y}$ as to whether they possess this zero or not.

Let us translate these properties to configuration space. The nonlocality of the separable potentials can be cast into momentum dependence by a Hankel transform

$$V(r,p)=\left[\left[\frac{2}{\pi}\right]^{1/2}\int_0^\infty dp'p'^2j_0(p'r)g(p')\right]\lambda g(p)=\tilde{g}(r)\lambda g(p)\ . \tag{4}$$

For our potentials the static limit of this form is depicted in Fig. 3. Now it turns out that the first zero of $f(p,k)$ is connected with the change from long-range attraction to intermediate-range repulsion as appearing in PEST and the Paris potential at around 0.8 fm. Most meson-exchange models roughly resemble this behavior; it is connected with the predominance of ρ exchange over π exchange in this region. But there we also expect the interface to the quark description (bag models) of the N-N interaction. Any sensitivity of nuclear observables to this domain is therefore highly welcomed.

The N-d spin correlation parameter $C_{y,y}$ seems to be such an observable. With these characteristics it adds to other (tensor) polarizations occurring in π-d reactions[15] and also in e-d scattering.[16] Our theoretical results clearly demonstrate the off-shell sensitivity of $C_{y,y}$. So far the precision of experimental data does not allow for a decisive constraint on particular off-shell features which could be traced to specific potential properties. But as soon as more accurate measurements will become available, second-order polarization observables will prove themselves to be a valuable testing ground for off-shell effects. Thus the 3-N system will finally serve its desired purpose.

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[1] See, e.g., the review by A. W. Thomas, in *Proceedings of the Eighth International Conference on Few-Body Systems and Nuclear Forces II, Graz, 1978,* edited by H. Zingl, M. Haftel, and H. Zankel (Springer, Berlin, 1978).

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