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Reduced nuclear amplitudes in quantum chromodynamics

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We present a new formalism which systematically accounts for nucleon compositeness in nuclear scattering amplitudes, consistent with quantum chromodynamics and covariance. Reduced gauge-invariant nuclear amplitudes are defined which have elementary quantum chromodynamic scaling properties. The procedure is applied to the photodisintegration and electrodisintegration of the deuteron as a test of nuclear chromodynamics and as a method to isolate contributions of dibaryon resonances.

[NUCLEAR REACTIONS Treatment of nucleon compositeness; general formalism developed. Applied to d(γ,p)n, d(e,p)n, $E \gtrsim 1$ GeV; calculated $\sigma(\theta)$, background to dibaryon resonances. Compared $\sigma(\theta)$ with available data.]

I. INTRODUCTION

One of the most basic problems in the analysis of nuclear scattering amplitudes is how to consistently take into account the effects of the quark/gluon composite structure of nucleons. In nuclear physics the traditional method of treating nucleon dynamics has been to use an effective meson-nucleon local Lagrangian field theory. However, this method is sorely deficient for a number of reasons: (1) the wrong degrees of freedom are used, (2) neither the t^{-2} power-law falloff of nucleon form factors nor the t^{-1} falloff of pion form factors is naturally reproduced,[1] (3) nucleon pair terms are not correctly suppressed in intermediate states, and (4) a renormalizable (i.e., calculable) field theory of massive isovector mesons requires the full apparatus of non-Abelian gauge theories, including a spontaneous symmetry breaking mechanism. Models for nuclear scattering amplitudes based on the Born approximation and local meson-nucleon couplings have the wrong dynamical dependence in virtually every kinematical variable for composite hadrons. The inclusion of ad hoc form factors at each meson-nucleon or photon-nucleon vertex is unsatisfactory since one must understand the off-shell dependence in each leg while retaining gauge invariance. None of these traditional methods have any real predictive power.

In principle all nuclear scattering amplitudes could be calculated from quantum chromodynamics (QCD) in terms of the basic quark and gluon degrees of freedom. A method for computing large momentum transfer exclusive scattering amplitudes for hadrons and nuclei, starting with a Fock state wave function expansion on the light cone (equal $\tau = t + z$), has been developed.[2] At large momentum transfer one can readily derive QCD predictions for the leading fixed angle power-law scaling behavior and spin structure of hadronic and nuclear scattering matrix elements. However, the explicit evaluation of the multiquark and gluon hard scattering amplitudes needed for predicting the normalization and angular dependence for a nuclear process, even at leading order in α_s, requires the consideration of millions of Feynman diagrams. Beyond leading order, one must include contributions of nonvalence Fock states, wave function and binding corrections, and a rapidly expanding number of radiative corrections and loop diagrams.

In this paper we will discuss a new definition of nuclear scattering amplitudes which provides a simple method for identifying the dynamical effects of nucleon substructure, consistent with QCD and covariance. Although this technique cannot replace a full QCD calculation, it does provide a basis for constructing models for "reduced" nuclear scattering amplitudes consistent with QCD scaling laws and gauge invariance.

The basic idea for this method was given by Brodsky and Chertok.[3] Consider the deuteron form factor as measured in electron-deuteron elastic scattering. In general, a form factor $F(Q^2 = -q^2)$ is the probability amplitude that the target remains intact after absorbing four-momentum

q. To the extent that we can neglect its binding energy, the deuteron can be represented as two nucleons, each with an equal portion of the nuclear momentum. Therefore the deuteron form factor contains the probability that each nucleon remains intact after absorbing one-half of the momentum transfer. We thus define the "reduced" deuteron form factor

$$f_d(Q^2)=\frac{F_d(Q^2)}{F_p(Q^2/4)F_n(Q^2/4)} \quad (1.1)$$

which effectively removes the falloff of the measured form factor due to the internal degrees of freedom of the nucleons. It is defined separately for each helicity form factor.

The reduced form factor must still be a decreasing function of Q^2 since it still contains the probability that the scattered nucleons reform into the ground state deuteron. An important prediction of QCD is that, modulo logarithmic factors[4] that come from the running coupling constant and anomalous dimensions of the hadronic distribution amplitudes, the large Q^2 behavior is

$$f_d(Q^2)\sim\frac{\text{const}}{Q^2} . \quad (1.2)$$

Thus the reduced deuteron form factor and meson form factors (for helicity $\lambda=0$ to $\lambda'=0$) have the identical (monopole) scaling law. After removing the nucleon form factors, the nucleons are effectively reduced to pointlike spin $\frac{1}{2}$ fermions, so the reduced deuteron and meson form factors have the same dimensional scaling behavior $f\sim(1/Q^2)^{n-1}$, basically the slowest possible for two-particle composites. Similarly, if one defines for $A=3$:

$$f_{^3\text{He}}(Q^2)=\frac{F_{^3\text{He}}(Q^2)}{[F_N(Q^2/9)]^3}, \quad (\lambda=\tfrac{1}{2} \text{ to } \lambda'=\tfrac{1}{2}) \quad (1.3)$$

then QCD predicts that the reduced ^{3}He (and triton) form factor scales at large Q^2 in the same way as a nucleon form factor:

$$f_{^3\text{He}}(Q^2)\sim F_N(Q^2)\sim(1/Q^2)^2 . \quad (1.4)$$

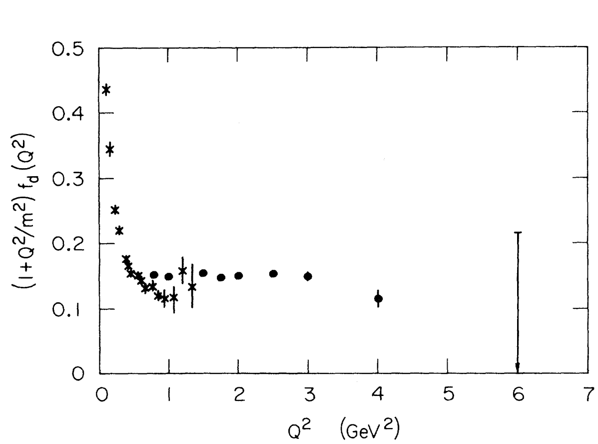

FIG. 1. Comparison of deuteron form factor data with the QCD prediction of Eq. (1.5) of the text. The data are from Ref. 5.

A comparison of data[5] with the QCD prediction

$$(1+Q^2/m_0^2)f_d(Q^2)\simeq\text{const} \quad (1.5)$$

is shown in Fig. 1. (Here $m_0^2=0.3$ GeV^2, as predicted in Ref. 3, although any value $m_0^2\leq 1$ GeV^2 is irrevelant for the comparison.) The results show that QCD works remarkably well down to scales of order $Q^2\sim 1$ GeV^2.

One can compare the definition (1.1) for the reduced deuteron form factor with the standard "impulse approximation" form

$$F_d(Q^2)=F_d^{\text{body}}(Q^2)F_N(Q^2) , \quad (1.6)$$

where $F_N(Q^2)$ is the on-shell form factor for the struck nucleon and $F_d^{\text{body}}(Q^2)$ is defined to represent the remaining structure of the nucleus. In fact, as discussed in Ref. 3, this approximation is incorrect since the struck nucleon has at least one leg off shell and the off-shell form factor has a completely different dynamical dependence than does the on-shell form factor of QCD.

The idea of "reducing" nuclear form factors leads to a general treatment of nuclear amplitudes that is discussed in Sec. II. The method is applied to deuteron disintegration in Sec. III, where we consider photodisintegration, dibaryon resonances, and a specific model for the reduced background amplitude. Section IV contains a summary of our results and some additional remarks. Details of the model for the reduced deuteron disintegration amplitudes are given in the Appendix. As an aside, the specifics of this model have relevance for the calculation of higher twist effects in electroproduction.

II. GENERAL TREATMENT OF REDUCED NUCLEAR AMPLITUDES

We can go beyond the case of nuclear form factors and define reduced nuclear scattering amplitudes in general. If we consider a generic process with amplitude $\mathcal{M}(s,t)$ that involves A ingoing and outgoing nucleons and transfers, in the zero binding limit, momentum q_i to nucleon i, then the reduced amplitude is defined as

$$m(s,t)=\mathcal{M}(s,t)\left[\prod_{i=1}^{A} F_N(\hat{t}_i=q_i^2)\right]^{-1} . \quad (2.1)$$

For example, the reduced amplitude for the photodisintegration (or electrodisintegration) of the deuteron would be written as

$$m_{\gamma d\to np}=\frac{\mathcal{M}_{\gamma d\to np}}{F_n(\hat{t}_n)F_p(\hat{t}_p)} , \quad (2.2)$$

where

$$\hat{t}_n=(p_n-\tfrac{1}{2}p_d)^2 , \quad (2.3a)$$

$$\hat{t}_p=(p_p-\tfrac{1}{2}p_d)^2 , \quad (2.3b)$$

with p_n, p_p, and p_d the momenta of the neutron, proton, and deuteron, respectively.

The nominal fixed-angle scaling behavior of the reduced amplitude is predicted by dimensional counting rules.[6] Modulo logarithms they give

$$m \sim p_T^{4-n} f\left[\frac{\text{invariants}}{s}\right], \tag{2.4}$$

where $p_T^2 = tu/s$ is the transverse momentum and n is the number of "elementary" fields in the external state (ingoing and outgoing photons, leptons, gluons, quarks, or reduced nucleons). Thus for deuteron photodisintegration the reduced amplitude scales as

$$m_{\gamma d \to np} \sim p_T^{-1} f(\theta_{\text{c.m.}}), \tag{2.5}$$

the angle $\theta_{\text{c.m.}}$ being that of the proton direction with respect to the beam direction of the c.m. frame. This is the same QCD scaling as that for $\mathcal{M}_{\gamma M \to q_1 \bar{q}_2}$; here M is a meson with constituents q_1 and $\bar{q}_2$.

We can motivate the definition of the reduced amplitude by returning to the basic definition of hadronic matrix elements in τ-ordered perturbation theory[7]:

$$\mathcal{M} = \int \prod [dx][d^2k_\perp]\Psi'(x_f, k_{\perp f}) \times T(x_f, x_i; k_{\perp f}, k_{\perp i})\Psi(x_i, k_{\perp i}), \tag{2.6}$$

where the Ψ are the equal $\tau = t + z$ wave functions and T is the momentum-space quark-gluon scattering amplitude. A sum over the Fock state amplitudes and quark and gluon helicities is understood. In the zero nuclear binding energy limit the nuclear Fock state wave function reduces to the product of wave functions for collinear nucleons with the nuclear momentum partitioned among the nucleons in proportion to each nucleon mass. Thus one is evidently neglecting corrections of order $2m_N \Delta\epsilon_{\text{BE}}/\mu^2$, where m_N is the nuclear mass, $\Delta\epsilon_{\text{BE}}$ the nuclear binding energy, and μ^2 a hadronic scale parameter, as well as contributions from higher Fock states in the nucleus, e.g., the hidden-color six-quark configurations.

At this stage of approximation one must compute the corresponding multinucleon scattering amplitude, e.g., the amplitude for the elastic electron-deuteron scattering process

$$e + p(\tfrac{1}{2}p) + n(\tfrac{1}{2}p) \to e' + p'(\tfrac{1}{2}p + \tfrac{1}{2}q) + n'(\tfrac{1}{2}p + \tfrac{1}{2}q). \tag{2.7}$$

If the momentum transfer occurs rapidly compared to the scale of hadronic binding then one can argue (as in the Chou-Yang model of elastic scattering[8]) that the probability amplitude for transferring the required momentum $\hat{t}_i$ to each nucleon is proportional to its elastic form factor. Since Sudakov effects always suppress near on-shell (long-distance) momentum transfer mechansims from pinch singularities[9] and end point regions of phase space,[10,11] one can argue that large momentum transfer is always local in QCD. Thus this assumption is justified, with corrections of order μ^2/q^2. A specific diagram which explicitly exhibits the factorization intrinsic to the reduced deuteron form factor is shown in Fig. 2.

As an application of nuclear amplitude reduction, we consider deuteron disintegration. The reduced amplitude is defined in (2.2). Both the scaling behavior (2.5) and a model for the angular dependence are discussed in the next section.

Some other processes[12] that might be profitably treated with our reduction method are $pp \to d\pi^+$,[13] $pd \to {}^3H\pi^+$, and $\pi^\pm d \to \pi^\pm d$. The reduced amplitudes have the same QCD scaling behavior as the amplitudes for $q\bar{q} \to M\pi$, $qqq \to B\pi$, and $\pi M \to \pi M$, respectively, where B represents a baryon. From (2.4) we find the scaling to be

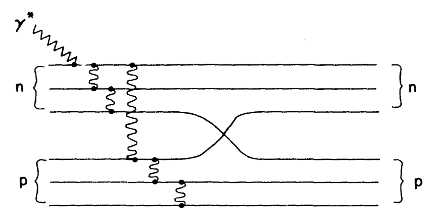

FIG. 2. A deuteron form factor diagram that exhibits factorization.

$$m_{pp \to d\pi^+} \sim p_T^{-2} f(t/s), \tag{2.8a}$$

$$m_{pd \to {}^3H\pi^+} \sim p_T^{-4} f(t/s), \tag{2.8b}$$

$$m_{\pi d \to \pi d} \sim p_T^{-4} f(t/s). \tag{2.8c}$$

III. A MODEL FOR REDUCED DEUTERON DISINTEGRATION AMPLITUDES

A. Photodisintegration

The asymptotic scaling law (2.5) is a remarkably simple form. The scaling holds for the hadron helicity conserving amplitude with $\lambda_n + \lambda_p = \lambda_d$, independent of the photon helicity. Amplitudes with $\lambda_n + \lambda_p \neq \lambda_d$ should be suppressed by a power of μ^2/p_T^2. One could hope that the simple scaling

$$p_T m_{\gamma d \to np} \simeq \text{const} \tag{3.1}$$

at fixed $\theta_{\text{c.m.}}$ will hold for $p_T^2 \geq 1$ GeV^2 since the scaling (1.5) (see Fig. 1) begins in this region.[4] In terms of the differential cross section, (2.2) and (2.5) become

$$\left.\frac{d\sigma}{d\Omega_{\text{c.m.}}}\right|_{\gamma d \to np} \sim \frac{1}{s - m_d^2} F_p^2(\hat{t}_p) F_n^2(\hat{t}_n) \frac{1}{p_T^2} f^2(\theta_{\text{c.m.}}). \tag{3.2}$$

A comparison of this form with present low energy data[14] is shown in Fig. 3. The form factors were computed from the usual dipole formula[3]

$$F_N(\hat{t}_i) = \frac{\text{const}}{(1 - \hat{t}_i/0.71\ \text{GeV}^2)^2}. \tag{3.3}$$

Although the results are encouraging, the available energies are too low to make a detailed check of the prediction.

We have not yet specified the form of f^2; however, it is easy to construct a model for the reduced amplitude which is gauge invariant and has the correct helicity and scaling form. As a prototype for the reduced amplitude we propose the amplitude for the photodisintegration of a polarized meson M into its constituent quarks q_1 and $\bar{q}_2$. We will only use the lowest order QCD diagrams for this process. An actual calculation of the hard-scattering amplitude for $\gamma d \to pn$ includes a coherent sum of such amplitudes with varied charge assignments and additional gluon lines attached. For the model, the charge assignments, e_1 for q_1 and $-e_2$ for $\bar{q}_2$, can be varied as parameters. The quark masses are taken to be zero. A computation of the squared amplitude summed over final spins (see the Appendix) then gives[15]

$$f^2(\theta_{\rm c.m.})=N\frac{(ue_1+te_2)^2}{tu}\begin{cases}1, & \text{transverse}\\ \dfrac{t^2+u^2}{4s^2}, & \text{longitudinal,}\end{cases} \tag{3.4}$$

where N is a normalization constant with dimensions $\mathrm{GeV}^2/\mathrm{sr}$ and "transverse" indicates an average over the two possible helicities. In the limit of $\sqrt{s} \gg m_d$ we find

$$f^2(\theta_{\rm c.m.})\simeq N\frac{[(2e_1-1)+\cos\theta_{\rm c.m.}]^2}{1-\cos^2\theta_{\rm c.m.}}\begin{cases}1, & \text{transverse}\\ \frac{1}{8}(1+\cos^2\theta_{\rm c.m.}), & \text{longitudinal}\end{cases} \tag{3.5}$$

with the charges normalized by $e_1-e_2=1$. This, when combined with (3.2), provides a one-parameter model for the asymptotic behavior of deuteron photodisintegration away from the beam axis. The actual angular distribution predicted by QCD from the coherent sum over the many diagrams of the type illustrated in Fig. 2 is undoubtedly more complicated than that given by the above model. Nevertheless, Eq. (3.5) should be representative of the scaling and functional dependence predicted by QCD for the reduced photodisintegration amplitude.

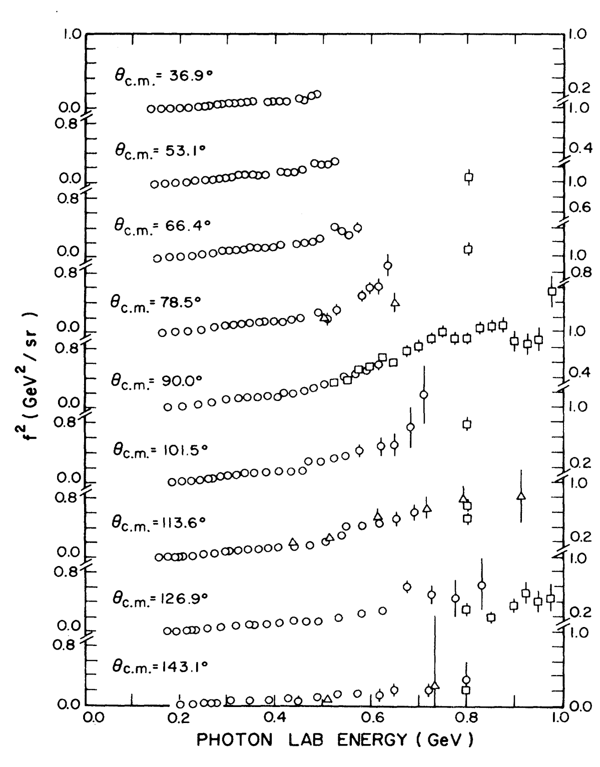

FIG. 3. Comparison of deuteron photodisintegration data with the prediction (3.2) of the text. The angle $\theta_{\rm c.m.}$ is that of the proton direction with respect to the beam in the c.m. frame. The predicted scaling requires $f^2(\theta_{\rm c.m.})$ to be independent of energy at any fixed angle. The data are from Ref. 8.

The simple model given in (3.5) makes apparent the need for data at higher energies. The points plotted in Fig. 4 were extracted by inspection from the data in Fig. 3 under the assumption that scaling had begun. The error bars reflect the range of values that would be consistent with the data. The empirical form $\sin^4\theta_{\rm c.m.}$ fits the points fairly well but does not agree with (3.5). In particular, (3.5) is unbounded at one or both endpoints. Of course the physical cross section is not unbounded at either endpoints; its rise is curtailed by mass terms dropped in our approximations. However, the $\sin^4\theta_{\rm c.m.}$ behavior of the data is not compatible with any rise at all. If the $\gamma M \to q\bar{q}$ model is a good guide, then a sign that experimental energies are approaching the true scaling limit would be that the value of $f^2(\theta_{\rm c.m.})$ near the backward or forward direction has become large relative to the values at wider angles.

B. Dibaryon resonances

An interesting feature of QCD is the possible occurrence of resonances in the dibaryon system corresponding to six-quark Fock states which are dominantly hidden color, i.e., orthogonal to the usual n-p and Δ-Δ configura-

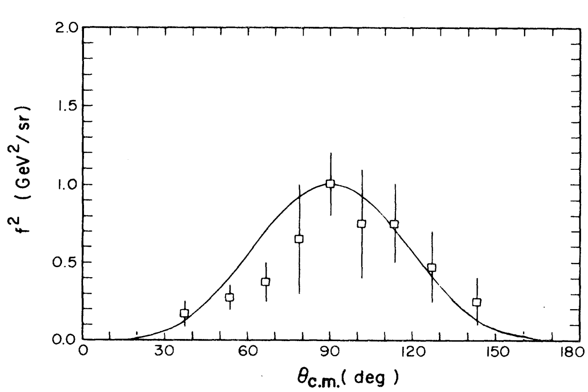

FIG. 4. Values of $f^2(\theta_{\rm c.m.})$ extracted by inspection from the data presented in Fig. 3 with the assumption that scaling has begun in each data set. The solid line represents $\sin^4\theta_{\rm c.m.}$ which was chosen empirically to summarize the extracted values.

tions. Signals for such resonances could appear in photodisintegration or electrodisintegration of the deuteron at fixed $\hat{s}=M^2$ in a specific partial wave in the full amplitude. The virtual photon probe may enhance the signal since it is sensitive to off-shell configurations in the nuclear target. Analyses[16] of deuteron photodisintegration data have suggested the presence of dibaryon resonances with masses at 2.26 and 2.38 GeV, although definitive results have been elusive. The isolation of possible dibaryon contributions from the hard-scattering background is clearly interesting and important. It would be useful to have a specific model of the hard-scattering continuum since this would permit a more precise separation of the resonance and background contributions. Given the correct kinematic regime, the reduced amplitude technique leads directly to just such a model.

As an application of this approach we treat deuteron electrodisintegration. We have already discussed photodisintegration, but for that process the resonances occur at energies where the asymptotic form (3.2) does not apply. In electrodisintegration, however, the kinematics of resonance production are consistent with large transfers of momentum for the nucleons. The methods of the previous sections should then be applicable.

We write the full disintegration amplitude as the sum of a dibaryon resonance amplitude $\mathcal{M}_{\rm DB}$ and a background amplitude $\mathcal{M}_{\rm BG}$:

$$\mathcal{M}_{\rm ed\to epn}=\mathcal{M}_{\rm DB}+\mathcal{M}_{\rm BG}\ . \qquad (3.6)$$

As discussed in Sec. II, the hard-scattering background amplitude factorizes into a reduced amplitude $m_{\rm BG}$ and the appropriate nucleon form factors,

$$\mathcal{M}_{\rm BG}\simeq m_{\rm BG}F_{\rm p}(\hat{t}_{\rm p})F_{\rm n}(\hat{t}_{\rm n})\ . \qquad (3.7)$$

From (2.4) we find that the nominal scaling behavior for the reduced amplitude is

$$m_{\rm BG}\sim p_T^{-2}f\left[\frac{\text{invariants}}{s}\right]. \qquad (3.8)$$

As a model for the reduced electrodisintegration amplitude we suggest the natural extension of the model for photodisintegration, that is, the electrodisintegration of a polarized meson into its constituent quarks. This model is developed in the following subsection and the Appendix.

In general, one would expect the dibaryon resonance and the continuum hard-scattering contributions to the electroproduction amplitude to have quite different q^2 dependence. On one hand, the resonance contribution, if it is dominated by soft hadronic physics, would be expected to have a characteristic vector meson-dominated falloff in q^2:

$$\mathcal{M}_{\rm DB}\propto(1-q^2/m_v^2)^{-1}$$

independent of p_T^2. On the other hand, the q^2 dependence of $\mathcal{M}_{\rm BG}$ is minimal for $|q^2|\ll p_T^2$ and p_T^2 large, at least for the contribution from transversely polarized photons. These characteristics in q^2 should be useful in separating possible resonance from the continuum.

C. Electrodisintegration

To model the reduced background amplitude for deuteron electrodisintegration we will assume that it is a single-photon exchange process. The square of the photon emission amplitude will be written as $E_{\alpha\beta}$ and the square of the absorption amplitude, summed over final spins, as $F^{\alpha\beta}$. Thus we have

$$\sum_{\substack{\text{final}\\ \text{hadronic}\\ \text{spins}}}|m_{\rm BG}|^2\simeq\frac{1}{(q^2)^2}E_{\alpha\beta}F^{\alpha\beta}\ . \qquad (3.9)$$

Just as for photodisintegration we choose to model $F^{\alpha\beta}$ by the lowest order QCD contributions to the process $\gamma^*M\to q_1\bar{q}_2$, where the photon now has mass q^2. The results of the calculation are given in (A4), (A13), and (A14).

As an example we treat the case of a longitudinal deuteron and unpolarized electrons, for which we easily find

$$E_{\alpha\beta}\sim{\rm Tr}\{\not p_e\gamma_\alpha\not p'_e\gamma_\beta\}\ . \qquad (3.10)$$

Upon substitution of (3.10) and (A4), Eq. (3.9) becomes, in the limit $\hat{s}\ll s$,

$$\sum|m_{\rm BG}|^2\sim -F_1+\{5+2c+c^2\}\frac{sF_2}{8(1-c)}+\{a^2-b^2\}\frac{sF_3}{2(1-c)}$$
$$+\{(1-c)(a-b)[2b+a(1+c)+(a-b)(1-c)]-2(a-b)^2(1+c)-ab(1-c)^2\}\frac{s^2F_5}{8(1-c)^2}\ , \qquad (3.11)$$

where the F_i are given in (A14),

$$a=(E_{\rm p}-\vec{p}_{\rm p}\cdot\hat{p}_{\rm e})/s^{1/2}\ , \qquad (3.12a)$$

$$b=(E_{\rm p}-\vec{p}_{\rm p}\cdot\hat{p}'_{\rm e})/s^{1/2}\ , \qquad (3.12b)$$

$$c=\hat{p}_{\rm e}\cdot\hat{p}'_{\rm e}\ , \qquad (3.12c)$$

$\hat{p}_{\rm e}$ is the beam direction, $\hat{p}'_{\rm e}$ the direction of the outgoing electron, and $(E_{\rm p},\vec{p}_{\rm p})$ the four-momentum of the proton, all in the c.m. frame. The invariants used to define the F_i are, in the same limit, given by

$$Q^2\simeq\tfrac{1}{2}s(1-c)\ , \qquad (3.13a)$$

$$\hat{t}\simeq-\tfrac{1}{2}s(1-c)-s(a-b)\ , \qquad (3.13b)$$

$$\hat{u}\simeq s(a-b)\ . \qquad (3.13c)$$

The expression in (3.11) should describe the background near a resonance. For a transverse deuteron the background amplitude is suppressed by additional factors of $(\hat{s}/s)^{1/2}$ that come from angular momentum effects.[2]

IV. CONCLUSION

The reduced amplitude method discussed in Sec. II is very general. The principal formulas, Eqs. (2.1) and (2.4), give an accurate estimate of the leading QCD behavior of hadron helicity-conserving amplitudes. Comparison with experiment should provide a new test of QCD. These formulas also imply constraints on low energy models since one expects a synthesis[4] of QCD and nuclear physics. Our results suggest the possibility that fully analytic nuclear amplitudes can be constructed which at low momentum transfer fit standard electromagnetic and chiral boundary conditions and low energy theorems, while satisfying the scaling law and anomalous dimension structure predicted by QCD at high momentum transfer.

An application to deuteron disintegration and a model for its angular dependence were described in Sec. III. The prediction for the photodisintegration differential cross section is contained in (3.2) and (3.5). The general form for the square of the electrodisintegration amplitude is given by (3.9), (A4), (A13), and (A14). This latter result provides a new means for understanding the background to dibaryon resonances. Equation (3.11) supplies a specific prediction for this background.

The predictions made for deuteron disintegration apply to an energy domain that is as yet uncharted by coincidence experiments. With the advent of intermediate-energy cw electron beams[17] this should soon not be the case. Some other nuclear processes that are of interest in the context of the reduced amplitude method are mentioned at the end of Sec. II. We urge experimentalists to pursue the acquisition of data at the largest possible energy and momentum transfer in order to test the scaling behavior predicted by QCD.

ACKNOWLEDGMENTS

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APPENDIX

Consider the process

$$\gamma^* M \to q_1 \bar{q}_2 , \tag{A1}$$

where the photon may be off shell with mass q^2 and M is a polarized meson with constituents q_1 and $\bar{q}_2$. Let q be the photon momentum, p the meson momentum, and p_1 and p_2 the final momenta. We will assume all masses other than $q^2=-Q^2$ to be zero. The charges of the constituents are to be denoted by e_1 and $-e_2$. The usual Mandelstam invariants are defined as

$$\hat{s}=(q+p)^2,\quad \hat{t}=(p_1-q)^2,\quad \hat{u}=(p_2-q)^2 \tag{A2}$$

and related by

$$\hat{s}+\hat{t}+\hat{u}+Q^2=0 . \tag{A3}$$

The square of the amplitude for the process, when summed over final spins and when, in the case of transverse polarization, summed over the two helicity states of M, is written as $F^{\alpha\beta}$. Gauge invariance requires that it be of the form

$$\begin{aligned} F^{\alpha\beta}= {} & [q^\alpha q^\beta - q^2 g^{\alpha\beta}]F_1+[q\cdot p(q^\alpha p^\beta+q^\beta p^\alpha - q\cdot p g^{\alpha\beta}) - q^2 p^\alpha p^\beta]F_2 \\ & +[q\cdot p_1(q^\alpha p_1^\beta + q^\beta p_1^\alpha - q\cdot p_1 g^{\alpha\beta}) - q^2 p_1^\alpha p_1^\beta]F_3 \\ & +[q\cdot p_1(q^\alpha p^\beta - q^\beta p^\alpha) - q\cdot p(q^\alpha p_1^\beta - q^\beta p_1^\alpha)+q^2(p^\alpha p_1^\beta - p^\beta p_1^\alpha)]F_4 \\ & +[q\cdot p q\cdot p_1(p^\alpha p_1^\beta+p^\beta p_1^\alpha) - (q\cdot p_1)^2 p^\alpha p^\beta - (q\cdot p)^2 p_1^\alpha p_1^\beta]F_5 , \end{aligned} \tag{A4}$$

where the F_i are functions of $\hat{s}$, $\hat{t}$, and Q^2.

To estimate[18] the F_i we use the lowest order QCD diagrams, which are shown in Fig. 5. The $Mq_1\bar{q}_2$ vertex for a meson of spin J and helicity h is described by a factor[19]

$$\int [dx]\Phi(x)\chi^{Jh} , \tag{A5}$$

with

$$[dx]=dx_1 dx_2 \delta(1-x_1-x_2) \tag{A6}$$

and

$$\chi^{Jh}=\sum_{s_1,s_2} N_{s_1 s_2}^{Jh} x_1^{-1/2} x_2^{-1/2} u(x_1 p, s_1)\bar{v}(x_2 p, s_2) . \tag{A7}$$

For the massless case considered here, one can use[19]

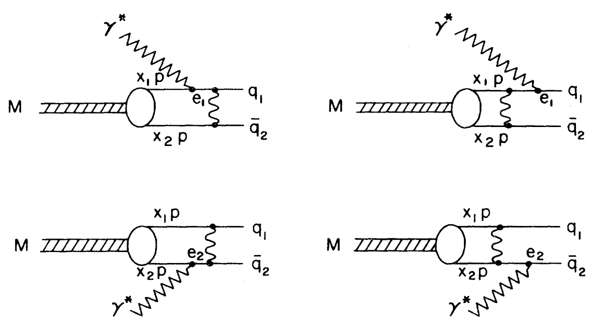

FIG. 5. Lowest order QCD diagrams for $\gamma^* M \to q_1\bar{q}_2$, where M is a bound state of q_1 and $\bar{q}_2$.

$$\chi^{Jh}=\begin{cases}\gamma_5 \not{p}/\sqrt{2}, & J=0\\ \left.\begin{array}{ll}\not{p}/\sqrt{2}, & h=0\\ \mp\not{\epsilon}_\pm \not{p}/\sqrt{2}, & h=\pm 1\end{array}\right\} & J=1\end{cases} \tag{A8}$$

where $\epsilon_\pm=\mp(1/\sqrt{2})(0,1,\pm i,0)$ in a frame with $p=(\,|\vec{p}\,|,0,0,|\vec{p}\,|\,)$. In writing the final formulas we will assume that the wave function Φ obeys the symmetry

$$\Phi=\Phi\,|_{x_1\leftrightarrow x_2}\,. \tag{A9}$$

It is useful to define the integrals

$$I=\int \frac{[dx]\Phi}{x_2\left[x_1\dfrac{\hat{s}}{s}-x_2\dfrac{Q^2}{s}\right]} \tag{A10}$$

and

$$I'=\int \frac{[dx]x_1\Phi}{x_2\left[x_1\dfrac{\hat{s}}{s}-x_2\dfrac{Q^2}{s}\right]}\underset{Q^2=0}{\longrightarrow}\tfrac{1}{2}I\,, \tag{A11}$$

where $\sqrt{s}$ is the c.m. energy of the process for which (A1) is a subprocess and $Q^2=0$, $s=\hat{s}$ is the photodisintegration limit. These integrals appear in the following combinations:

$$I_1=|I\,|^2\,, \tag{A12a}$$

$$I_2=I'I^*+II'^*-II^*\underset{Q^2=0}{\longrightarrow}0\,, \tag{A12b}$$

$$I_3=|I'|^2-\tfrac{1}{4}|I\,|^2\underset{Q^2=0}{\longrightarrow}0\,, \tag{A12c}$$

$$I_4=I'I^*-II'^*\underset{Q^2=0}{\longrightarrow}0\,. \tag{A12d}$$

In the transverse case we find

$$F_i=0,\quad i\neq 2\,, \tag{A13a}$$

$$F_2\sim\frac{4\hat{s}}{s^2}I_1\left[\frac{e_1}{\hat{t}}+\frac{e_2}{\hat{u}}\right]^2, \tag{A13b}$$

and in both the longitudinal and scalar cases we obtain

$$F_1\sim\frac{\hat{u}}{s^2}\left\{I_1\left[(2\hat{u}+\hat{s}-Q^2)\frac{e_1e_2}{\hat{t}\hat{u}}-(\hat{s}-Q^2)\frac{e_2^2}{\hat{u}^2}\right]+I_2(\hat{s}+Q^2)\left[\frac{e_1e_2}{\hat{t}\hat{u}}-\frac{e_2^2}{\hat{u}^2}\right]\right\}, \tag{A14a}$$

$$\begin{aligned}F_2\sim\frac{1}{s^2(\hat{s}+Q^2)}&\left\{I_1\left[\tfrac{1}{2}(\hat{u}^2+\hat{t}^2)\left[\frac{e_1}{\hat{t}}+\frac{e_2}{\hat{u}}\right]^2-\tfrac{1}{2}\hat{t}(\hat{u}+\hat{t})\left[\frac{e_1^2}{\hat{t}^2}-\frac{e_2^2}{\hat{u}^2}\right]\right.\right.\\ &\left.-\tfrac{1}{2}Q^2\left[2\hat{u}\frac{e_1^2}{\hat{t}^2}+(\hat{u}-3\hat{t}-2Q^2)\frac{2e_1e_2}{\hat{t}\hat{u}}+2(\hat{t}+2Q^2)\frac{e_2^2}{\hat{u}^2}\right]\right]\\ &\left.+I_2\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]\left[\hat{u}(\hat{s}-\hat{u}+2Q^2)\frac{e_1}{\hat{t}}+\hat{t}(\hat{t}+Q^2)\frac{e_2}{\hat{u}}\right]-2I_3\hat{u}(\hat{s}+Q^2)\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]^2\right\},\end{aligned} \tag{A14b}$$

$$\begin{aligned}F_3\sim\frac{1}{s^2(\hat{t}+Q^2)}&\left[I_1\left\{\tfrac{1}{2}\hat{s}(\hat{u}+\hat{t})\left[\frac{e_1^2}{\hat{t}^2}-\frac{e_2^2}{\hat{u}^2}\right]-\hat{s}\hat{u}\left[\frac{e_1}{\hat{t}}+\frac{e_2}{\hat{u}}\right]^2\right.\right.\\ &\left.+\tfrac{1}{2}Q^2\left[(3\hat{s}+2\hat{u}-Q^2)\frac{e_1^2}{\hat{t}^2}-2(\hat{s}+3\hat{u}-Q^2)\frac{2e_1e_2}{\hat{t}\hat{u}}+(\hat{s}+2\hat{u}-3Q^2)\frac{e_2^2}{\hat{u}^2}\right]\right\}\\ &\left.-I_2(\hat{s}+Q^2)\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]\left[(\hat{u}-2Q^2)\frac{e_1}{\hat{t}}-(\hat{u}-\hat{s}-3Q^2)\frac{e_2}{\hat{u}}\right]-2I_3(\hat{s}+Q^2)^2\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]^2\right],\end{aligned} \tag{A14c}$$

$$F_4\sim\frac{1}{s^2}I_4\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]\left[\hat{u}\frac{e_1}{\hat{t}}+\hat{t}\frac{e_2}{\hat{u}}\right], \tag{A14d}$$

$$F_5 \sim -\frac{4}{s^2(\hat{s}+Q^2)(\hat{t}+Q^2)}\left[I_1 \left\{ \tfrac{1}{2}\hat{t}(\hat{s}-Q^2)\left[\frac{e_1}{\hat{t}}+\frac{e_2}{\hat{u}}\right]^2 + \tfrac{1}{2}Q^2\left[(\hat{u}+\hat{t}-2Q^2)\frac{2e_1e_2}{\hat{t}\hat{u}} - 2(\hat{s}-Q^2)\frac{e_2^2}{\hat{t}^2}\right]\right\} \right.$$
$$\left. + I_2\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]\left\{(\hat{s}+2Q^2)e_1 - [(\hat{s}+Q^2)(\hat{t}-Q^2)+Q^2t]\frac{e_2}{\hat{u}}\right\} - 2I_3\hat{t}(\hat{s}+Q^2)\left[\frac{e_1}{\hat{t}}-\frac{e_2}{\hat{u}}\right]^2 \right]. \quad \text{(A14e)}$$

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