

Partially separable *t* matrix for optical potential

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In a recent article, we have proposed a theory of three-body reactions. The theory is constructed in a manner in which, even for the breakup process, the numerical calculations need to be performed only in a finite spacial region, and in which the calculations are performed in a perturbative way, where the zeroth order term includes all pole terms. The use of the partially separable *t* matrix plays a central role in this theory. In the present paper, we extend the partially separable *t* matrix to the optical potential so that the theory of three-body reactions may be applicable to nuclear reactions.

[ NUCLEAR REACTIONS Off-shell *t* matrix with one nonseparable and one separable term. Optical potential. Many-level formula. ]

In a recent article,<sup>1</sup> we have proposed a method for treating three-body reactions for a local potential in coordinate space. The theory has the following advantages for practical applications: (i) The numerical calculations need to be performed only in a finite spacial region, and the contribution from large distances is calculated analytically, and (ii) the calculation is performed in a perturbative way, where the zeroth order term includes all physically important contributions.

Our theory is based on the Faddeev equation, in which the dynamical quantity is given by the kernel  $G_0t$ , where  $G_0$  denotes the Green's function in the three-body free space and  $t$  the two-body scattering matrix embedded in the three-body space. The result of calculations of the Faddeev equation depends therefore on how we handle this kernel. In Ref. 1, we use this kernel, expressed as a sum of one separable and one nonseparable term.<sup>2,3</sup> Each term is regular at the origin. All physically important poles are involved in the separable term and not in the nonseparable term. This property makes the perturbative calculation of the nonseparable contributions feasible. The contribution from large distances is involved only in the separable term in a simple manner. As a result, this contribution can be calculated analytically.

In Ref. 1, we have assumed a real potential for the purpose of clear presentation of the theory. However, for applications to a wide class of nuclear reactions, we should extend the expressions in Refs. 2 and 3 to be applicable to an optical potential. This is done in the present article. We follow Ref. 3 for notations.

For real potentials  $V_1$  and  $V_2$ , let us define an optical potential  $W$  by

$$W = V_1 + iV_2 . \tag{1}$$

We define the Sturm-Liouville function  $\psi_n$  and the corresponding eigenvalue  $\lambda_n$  by

$$PG_0V_1\psi_n = \lambda_n\psi_n \quad (n = 1, \dots, \infty) . \tag{2}$$

As in Ref. 3, we make use of those Sturm-Liouville states whose eigenvalues become unity or very close to unity at some energies. Let  $\psi_n$  ( $n = 1, \dots, N$ ) denote the Sturm-Liouville functions belonging to this set, and normalized as

$$\langle j | V_1 | \psi_n \rangle = -\lambda_n \quad (n = 1, \dots, N) . \tag{3}$$

Here the notation  $|j\rangle$  represents the spherical Bessel func-

tion. We define a function  $\hat{\psi}$  by

$$\hat{\psi} = \sum_{n=1}^N \alpha_n^{(N)} \hat{\psi}_n , \tag{4}$$

with

$$\alpha_n^{(N)} = \frac{\lambda_n^{N-1}}{\prod_{m \neq n} (\lambda_n - \lambda_m)} , \quad \alpha_1^{(1)} = 1 . \tag{5}$$

Using this function, we define a Green's function  $\hat{g}$  by

$$\hat{g} = PG_0 + |\hat{\psi}\rangle \langle j| . \tag{6}$$

Here  $P$  denotes the principal value of Cauchy. We note that  $\hat{g}$  thus defined is real, regular at the origin, and vanishes at large distances from the origin.<sup>2,3</sup> By the way, we remark here that the Green's function used in the Jost solution is real and vanishes at large distances, but irregular at the origin. This last property of the Jost's Green's function prevents it from applying to the Faddeev equation. Our Green's function  $\hat{g}$  is freed from this difficulty. The usual Green's function  $G_0$  is expressed in terms of  $\hat{g}$  as

$$G_0 = \hat{g} - |\hat{\psi} + ikj\rangle \langle j| . \tag{7}$$

[See Eqs. (15) and (29) in Ref. 3.]

The *t* matrix for the optical potential  $W$  is defined by

$$t = W + WG_0t . \tag{8}$$

Similarly, we define a matrix  $\tau$  in terms of the Green's function  $\hat{g}$  by

$$\tau = W + W\hat{g}\tau . \tag{9}$$

If we use Eqs. (7) and (8), we readily see that the *t* matrix is expressed in terms of the  $\tau$  matrix as

$$t = \tau - \tau |\hat{\psi} + ikj\rangle \frac{1}{1 + \langle j | \tau | \hat{\psi} + ikj \rangle} \langle j | \tau . \tag{10}$$

If we make use of Eqs. (8) and (10), we can express  $G_0t$  in terms of a wave matrix  $\Omega_\tau$  defined by

$$\tau = W\Omega_\tau \tag{11}$$

as

$$G_0t = (\Omega_\tau - 1) - \Omega_\tau |\hat{\psi} + ikj\rangle \times \frac{1}{1 + \langle j | W\Omega_\tau | \hat{\psi} + ikj \rangle} \langle j | W\Omega_\tau . \tag{12}$$

Thus the  $t$  matrix and  $G_0 t$  are expressed as a sum of one nonseparable and one separable term. Since  $\Omega_\tau$  is given by

$$\Omega_\tau = 1 + \hat{g} W \Omega_\tau, \quad (13)$$

and  $\hat{g}$  vanishes at the origin, each of the terms  $(\Omega_\tau - 1)$  and the separable term on the right hand side of Eq. (12) vanishes at the origin. Also, by the nature of  $\hat{g}$  that vanishes asymptotically, the nonseparable term vanishes at large distances from the origin and the function  $\Omega_\tau |\hat{\psi} + ikj\rangle$  asymptotically behaves as the spherical Hankel function (as shown in Ref. 3).

In Refs. 2 and 3, all poles on the real momentum axis were shifted to some points in the complex momentum plane, and given as the zeros of the denominator of Eqs. (10) and (12). This was done so that the nonseparable term may not diverge on the real momentum axis and, at the same time, the separable term may accommodate physically important contributions. Since Eqs. (10) and (12) are expressed, as in Ref. 3, in terms of  $\hat{\psi}$  and  $\hat{g}$ , this property is kept also in these equations. This is seen in the following manner.

$$1 + \langle j | W \Omega_\tau | \hat{\psi} + ikj \rangle = \prod_{n=1}^N (1 - \lambda_n) - k \left\langle j \left| \frac{1}{1 - V_1 \hat{g}} V_2 \Omega_\tau \right| j \right\rangle + i \left\{ k \langle j | V_1 \hat{\omega} | j \rangle + \left\langle j \left| \frac{1}{1 - V_1 \hat{g}} V_2 \Omega_\tau \right| \hat{\psi} \right\rangle \right\}. \quad (17)$$

Clearly, all roots of the equation  $1 + \langle j | W \Omega_\tau | \hat{\psi} + ikj \rangle = 0$  are complex and not real. These roots are resonances. As in Ref. 3, the separable term of Eq. (10) or Eq. (12) represents the many-level formula in one term. The physical meaning of each term of Eq. (17) may be readily seen if we assume that  $V_2$  is small, and take it into account only to the first order term

$$\text{Eq. (17)} \sim \prod_{n=1}^N (1 - \lambda_n) - k \langle \hat{\omega} j | V_2 | \hat{\omega} j \rangle + i (k \langle j | V_1 | \hat{\omega} j \rangle + \langle \hat{\omega} j | V_2 | \hat{\omega} \hat{\psi} \rangle). \quad (18)$$

If only one resonance  $E_n$  is important, as in the case of low energy neutron scattering, we can further approximate the right hand side of Eq. (18) by using Eq. (49) of Ref. 3, and obtain

$$\text{Eq. (18)} \sim \left[ -\frac{\partial \lambda_n}{\partial E} \right]_{E=E_n} (E - E_n - \Delta E_n + i \Gamma_n), \quad (19)$$

We define the wave matrix  $\hat{\omega}$  for the real potential  $V_1$  by

$$\hat{\omega} = 1 + \hat{g} V_1 \hat{\omega}. \quad (14)$$

If we subtract Eq. (14) from Eq. (13), we can express  $\Omega_\tau$  in terms of  $\hat{\omega}$  as

$$\begin{aligned} \Omega_\tau &= \hat{\omega} + \frac{1}{1 - \hat{g} V_1} \hat{g} V_2 \Omega_\tau = \hat{\omega} (1 + \hat{g} V_2 \Omega_\tau) \\ &= \hat{\omega} \frac{1}{1 - \hat{g} V_2 \hat{\omega}} = \frac{1}{1 - \hat{\omega} \hat{g} V_2} \hat{\omega}. \end{aligned} \quad (15)$$

It was shown in Ref. 3 [Eq. (41)] that  $\hat{\omega}$  does not involve any singularity on the real momentum axis. Due to this property, we see from Eq. (15) that  $\Omega_\tau$  is free from the singularity also on this axis.

In Ref. 3, the equation

$$1 + \langle j | V_1 \hat{\omega} | \hat{\psi} \rangle = \prod_{n=1}^N (1 - \lambda_n) \quad (16)$$

was derived. If we use Eqs. (15) and (16) in the denominator of Eq. (10) or Eq. (12), it reads

where  $\Delta E_n$  and  $\Gamma_n$  denote the energy shift and width, respectively. These are given by

$$\Delta E_n = k \langle \hat{\omega} j | V_2 | \hat{\omega} j \rangle \left[ \left[ \frac{\partial \lambda_n}{\partial E} \right]_{E=E_n} \right]^{-1} \quad (20)$$

and

$$\begin{aligned} \Gamma_n &= (k \langle j | V_1 | \hat{\omega} j \rangle \\ &+ \langle \hat{\omega} j | V_2 | \hat{\omega} \hat{\psi} \rangle) \left[ \left[ -\frac{\partial \lambda_n}{\partial E} \right]_{E=E_n} \right]^{-1}. \end{aligned} \quad (21)$$

These expressions describe the energy shift and the broadening of the width of a resonance state due to the imaginary part of the optical potential.

In conclusion, all properties required in Ref. 3 are satisfied by Eqs. (10) and (12) for the optical potential, and the Faddeev equation is ready for use in a three-body model of nuclear reactions, if we follow the method of Ref. 1.

<sup>1</sup>T. Sasakawa, J. Horáček, and T. Sawada, Phys. Rev. C **27**, 18 (1983).

<sup>2</sup>T. Sasakawa and T. Sawada, Prog. Theor. Phys. Suppl.

**61**, 1 (1977).

<sup>3</sup>T. Sasakawa, H. Okuno, S. Ishikawa, and T. Sawada, Phys. Rev. C **26**, 42 (1982).