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Further tests of the multi-j supersymmetry scheme using transfer reactions

M. Vergnes, G. Berrier-Ronsin, and R. Bijker*
Institut de Physique Nucléaire, 91406 Orsay, France
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The experimental strengths extracted from several one neutron pickup and stripping reactions on the Pt isotopes are compared to predictions derived in the framework of the multi-j supersymmetry model. It appears that the model describes quite reasonably the experimental results for the reactions involving ^{195}Pt. However, due to a clear change of structure between ^{195}Pt and ^{197}Pt, an appreciable breaking is observed for the reactions involving ^{197}Pt.

[NUCLEAR STRUCTURE ^{194,195,196,197,198}Pt. Multi-j supersymmetry model; calculation of S for one nucleon transfer reactions; comparison with experimental results.]

The possibility of describing both even-even nuclei (bosonic spectra) and odd-A nuclei (fermionic spectra) in the common theoretical framework of a single group was first suggested by Iachello[1] in the particular case of a $j=\frac{3}{2}$ particle coupled to an O(6) core. This U(6/4) supersymmetry scheme has been tested and, although acceptable agreement has been observed in several nuclei of the Os, Ir, Pt, and Au region, breaking has been shown, particularly in single particle transfer reactions.[2] One of the difficulties of this first supersymmetry scheme was clearly the neglect of orbitals other than $j=3/2$. Recently, the first example of a supersymmetry based upon more than one orbital was proposed.[3] In this "multi-j supersymmetry" a fermion in the $j=1/2$, 3/2, and 5/2 orbitals is coupled to a U(6)$\supset$O(6) core. The model has just been tested[4] for excitation energies and some $B(E2)$ values in the cases of ^{195}Pt, ^{197}Pt and ^{199}Pt, and the results can be considered at least encouraging. The authors of Ref. 4 have suggested further testing of the model using the results of transfer reactions, and this is just what will be done in the present paper.

As far as ^{195}Pt is concerned, several one nucleon transfer reactions were performed, long before the supersymmetry scheme was proposed, going from even-even to odd-A nuclei: the ^{196}Pt$\rightarrow$^{195}Pt one neutron pickup reactions[5–7] and the ^{194}Pt(d,p)^{195}Pt reaction.[5] Going from odd-A to even-even nuclei, the ^{195}Pt(p,d)^{194}Pt reaction was studied more recently.[8] For ^{197}Pt, the ^{198}Pt$\rightarrow$^{197}Pt pickup reactions[6,7,9] and the ^{196}Pt(d,p)^{197}Pt reaction[9] have been performed. To the best of our knowledge, no result has been published so far for the ^{198}Pt(d,p)^{199}Pt reaction.

For single nucleon transfer reactions, selection rules on the quantum numbers[3] $(\sigma_1,\sigma_2,\sigma_3)$, (τ_1,τ_2), and L result from the transformation character of the transfer operator under the symmetry groups O(6), O(5), and O(3).

(i) For reactions between nuclei with the same boson number N and a number of fermions M equal to 0 or 1 $(N,M=1\leftrightarrow N,M=0)$, the one nucleon transfer operator can be approximated by[10]

$$P_+^{(j)}=\xi_j a_j^\dagger+\sum_{j'}\xi_{jj'}(s^\dagger\times\tilde{d}\times a_{j'}^\dagger)^{(j)} \quad . \tag{1}$$

For simplicity, only the first term with the selection rules

$$\Delta(\sigma_1,\sigma_2,\sigma_3)=(1,0,0) \quad ,$$

$$\Delta(\tau_1,\tau_2)=(1,0) \text{ and } \Delta L=2 \ , \ \text{ for } j=3/2 \text{ or } 5/2 \quad ,$$

$$\Delta(\tau_1,\tau_2)=(0,0) \text{ and } \Delta L=0 \ , \ \text{ for } j=1/2 \quad ,$$

is kept.

(ii) For reactions between nuclei with different boson numbers $(N,M=1\leftrightarrow N+1,M=0$; same supermultiplet), the transfer operator can be approximated by[10]

$$P_+^{(j)}=\theta_j(s^\dagger\times\tilde{a}_j)^{(j)}+\sum_{j'}\theta_{jj'}(d^\dagger\times\tilde{a}_{j'})^{(j)} \quad . \tag{2}$$

Assuming that for a $j=3/2$ transfer

$$\theta_{3/2,1/2}=\theta_{3/2}$$

and

$$\theta_{3/2,3/2}=-(7/3)^{1/2}\theta_{3/2,5/2}=\theta'_{3/2}$$

and that for a $j=5/2$ transfer

$$\theta_{5/2,1/2}=-\theta_{5/2}$$

and

$$\theta_{5/2,3/2}=\tfrac{1}{2}\theta_{5/2,5/2}=\theta'_{5/2} \quad ,$$

the transfer operator can be written as

$$P_+^{(3/2)}=\theta_{3/2}[(s^\dagger\times\tilde{a}_{3/2})^{(3/2)}+(d^\dagger\times\tilde{a}_{1/2})^{(3/2)}]+\theta'_{3/2}[(d^\dagger\times\tilde{a}_{3/2})^{(3/2)}-(3/7)^{1/2}(d^\dagger\times\tilde{a}_{5/2})^{(3/2)}] \quad , \tag{3}$$

$$P_+^{(5/2)}=\theta_{5/2}[(s^\dagger\times\tilde{a}_{5/2})^{(5/2)}-(d^\dagger\times\tilde{a}_{1/2})^{(5/2)}]+\theta'_{5/2}[(d^\dagger\times\tilde{a}_{3/2})^{(5/2)}+2(d^\dagger\times\tilde{a}_{5/2})^{(5/2)}] \quad . \tag{4}$$

The selection rules for the matrix elements of $P_+^{(3/2)}$ and $P_+^{(5/2)}$ are now given by

$$\Delta(\sigma_1,\sigma_2,\sigma_3)=(2,0,0) \ ;$$

$\Delta(\tau_1,\tau_2)=(1,0)$ for the first terms and (2,0) for the last terms of Eqs. (3) and (4); and finally $\Delta L=2$. Similarly, assuming $\theta_{1/2,3/2}=(2/3)^{1/2}\theta_{1/2,5/2}=\sqrt{2}\theta_{1/2}$, the $j=1/2$ transfer operator can be written as

$$P_{+}^{(1/2)}=\theta_{1/2}[(s^{\dagger}\times\tilde{a}_{1/2})^{(1/2)}+\sqrt{2}(d^{\dagger}\times\tilde{a}_{3/2})^{(1/2)}+\sqrt{3}(d^{\dagger}\times\tilde{a}_{5/2})^{(1/2)}] \tag{5}$$

with the selection rules

$$\Delta(\sigma_1,\sigma_2,\sigma_3)=(0,0,0)\ ,$$
$$\Delta(\tau_1,\tau_2)=(0,0)\ ,$$
$$\Delta L=0\ .$$

It is possible, using the operators written above, to analytically[11] compute the strengths

$$|\langle f\,||\,P_{+}^{(j)}\,||\,i\rangle|^2$$

for single nucleon transfer. These computed strengths will be compared to, respectively, the spectroscopic factors S_f for the pickup reactions and the strengths G_f, defined as

$$G_f=[(2J_f+1)/(2J_i+1)]S_f'\ ,$$

for the stripping reactions.

In Tables I—III our experimental pickup results[7,8] and the (d,p) results of Yamazaki *et al.*[5,9] are compared to the predictions of the multi-j supersymmetry model, using for the levels of ^{195}Pt and ^{197}Pt the quantum numbers (σ_1,σ_2) and (τ_1,τ_2) suggested by the authors of Ref. 4.

As shown in Table I, the agreement for the $^{196}\text{Pt}\rightarrow{}^{195}\text{Pt}$ and $^{194}\text{Pt}\rightarrow{}^{195}\text{Pt}$ reactions is quite good: The allowed transitions are strong and the forbidden ones are either not observed or are quite weak. More quantitatively, if we define the symmetry breaking as

$$\sum_f |S_f^{\text{exp}}-S_f^{\text{th}}| \Big/ \sum_f S_f^{\text{exp}}$$

for pickup (pu) reactions and as

$$\sum_f |G_f^{\text{exp}}-G_f^{\text{th}}| \Big/ \sum_f G_f^{\text{exp}}$$

for stripping reactions, the breaking is only 20.3% in the first case (for 11 transitions) and 14% in the second case (also for 11 transitions). Moreover, it is possible to define "parameter-free" ratios. For example (Table I), the ratio R(pu) of the spectroscopic factors for the transfer to the $J^{\pi}=3/2^-$ levels at, respectively, 99. and 211.4 keV and the ratio R'(pu) of the spectroscopic factors for the transfer to the $J^{\pi}=5/2^-$ levels at, respectively, 129.8 and 239.3 keV, are both predicted—independent of any parameter—to be

$$R(\text{pu})=R'(\text{pu})=[8N(N+2)]/[(N+3)(N+5)]\ .$$

With $N=6$, the ratios are both equal to 3.88. The experimental values are 4.65 and 5.53. Similarly, the ratios of the strengths G to the same levels are predicted to be

$$R(\text{strip})=R'(\text{strip})$$
$$=[N(N+2)(N+5)]/[2(N+3)]=29.3\ .$$

The experimental value (Table I) of R'(strip) is 19. Although the agreement is only qualitative, it should be remarked that the important variation predicted—independent of any parameter—by the model for R' between the stripping and the pickup reactions is indeed qualitatively observed experimentally.

It is interesting to compare the experimental results for the ^{195}Pt(p,d)^{194}Pt reaction to the predictions of the model, because the three parameters $\xi_{1/2}{}^2$, $\xi_{3/2}{}^2$, and $\xi_{5/2}{}^2$ have al-

TABLE I. One nucleon transfer strengths to the $J^{\pi}=1/2^-$, $3/2^-$ and $5/2^-$ levels of ^{195}Pt.

E_{ex} [a] (keV)	J^{π} [a]	Quantum numbers[a] (σ_1,σ_2)	(τ_1,τ_2)	Pickup S_{exp} [b]	S_{th} [c]	Class[d]	Stripping G_{exp} [e]	G_{th} [f]	Class[d]
0.	$1/2^-$	(7,0)	(0,0)	1.08	1.0	*A*	0.54	0.5	*A*
99.	$3/2^-$	(6,1)	(1,0)	1.21	1.13	*A*	0.68	0.725	*A*
129.8	$5/2^-$	(6,1)	(1,0)	2.27	2.19	*A*	1.52	1.57	*A*
199.5	$3/2^-$	(6,1)	(1,1)	0.15	0	*F*	0.04	0	*F*
211.4	$3/2^-$	(7,0)	(1,0)	0.26	0.29	*A*	0.18	0.025	*A*
239.3	$5/2^-$	(7,0)	(1,0)	0.41	0.56	*A*	0.08	0.054	*A*
389.1	$5/2^-$	(6,1)	(1,1)	not seen	0	*F*	not seen	0	*F*
419.7	$3/2^-$	(7,0)	(2,0)	not seen	0	*F*	not seen	0.04	*A*
455.3	$5/2^-$	(7,0)	(2,0)	0.07	0	*F*	not seen	0.0	*A*
524.8	$3/2^-$	(6,1)	(2,0)	not seen	0	*F*	0.08	0.08	*A*
590.9	$(1/2^-)$	(5,0)	(0,0)	<0.02	0.49	*A*	not seen	0	*F*

[a]From Ref. 4.
[b]From the (p,d) and (d,t) results of Ref. 7.
[c]These S_{th} depend on three parameters (see the text). The values chosen are $\xi_{1/2}{}^2=0.8$, $\xi_{3/2}{}^2=0.37$, and $\xi_{5/2}{}^2=0.48$.
[d]*A* means allowed, *F* means forbidden. The forbiddenness results from the $\Delta(\tau_1,\tau_2)$ selection rules. The quantum numbers (σ_1,σ_2) and (τ_1,τ_2) for the even-even targets are (6,0) and (0,0) for ^{196}Pt and (7,0) and (0,0) for ^{194}Pt.
[e]From the S values of Ref. 5 ($G=2S$).
[f]These G_{th} depend on five parameters (see the text). The values chosen are $\theta_{1/2}{}^2=0.036$, $\theta_{3/2}{}^2=\theta_{3/2}'^2=0.045$, $\theta_{5/2}{}^2=0.065$, and $\theta_{5/2}'{}^2=0$.

TABLE II. One nucleon transfer strengths ($l=1$) for the reaction ^{195}Pt(p,d)^{194}Pt.

E_{exc}[a] (keV)	$J^{\pi a}$	Quantum numbers[b] (σ_1,σ_2)	(τ_1,τ_2)	S_{exp}[a]	S_{th}[c]	Class[b]
0.	0_1^+	(7,0)	(0,0)	0.43	0.5	*A*
328.5	2_1^+	(7,0)	(1,0)	0.05	0.022	*A*
622.	2_2^+	(7,0)	(2,0)	0.13	0.045	*A*
1267.2	0_2^+	(7,0)	(3,0)	0.028	<u>0</u>	*F*

[a]From Ref. 8.
[b]The quantum numbers (σ_1,σ_2) and (τ_1,τ_2) for the target are (7,0) and (0,0). The forbiddenness results from the $\Delta(\tau_1,\tau_2)$ selection rules.
[c]These S_{th} are determined by the values of the three parameters $\xi_{1/2}{}^2$, $\xi_{3/2}{}^2$, and $\xi_{5/2}{}^2$ given in Table. I.

ready been fixed in the above study of the inverse reaction ^{194}Pt(d,p)^{195}Pt. The 0^+ levels of ^{194}Pt can only be populated in the pickup reaction by a pure $l=1$ transfer, but a mixture of $l=1+3$ is allowed for the 2^+ levels. Owing to the kinematical conditions, the $l=3$ part of the mixture corresponds to small cross sections (as compared to the $l=1$ part) and is not well determined experimentally. Accordingly, only the well determined $l=1$ spectroscopic factors are compared to the model predictions in Table II. The agreement, although not perfect, is still acceptable, the breaking being 33% (four transitions). To get a more general impression, we can calculate the breaking for the three reactions analyzed here and involving ^{195}Pt: The total breaking is 19% for 29 transitions.

If we now turn to reactions involving ^{197}Pt, the choice is reduced because ^{197}Pt is not stable. Our results for the ^{198}Pt$\rightarrow$^{197}Pt pickup reactions[7] and the results of Yamazaki *et al.*[9] for the ^{196}Pt(d,p)^{197}Pt reaction are compared to the predictions of the model in Table III. For the stripping reaction the strengths given in column nine have been computed using the same parameters $\theta_{1/2}{}^2$, $\theta_{3/2}{}^2$, and $\theta_{5/2}{}^2$ as in Table I. The breaking is then 52% (for seven transitions). However, ^{196}Pt and ^{197}Pt do not belong to the same supermultiplet as ^{194}Pt and ^{195}Pt, and the parameters do not have to be the same. The agreement can be improved by an appropriate choice of the three parameters (see column eight of Table III), and the breaking is reduced to 29%. The most striking individual breaking is observed for the transition—classified as strictly forbidden—to the 98.6 keV level, experimentally observed to be strong. For the pickup reaction, the spectroscopic factors given in column six have been computed using the same parameters $\xi_{1/2}{}^2$, $\xi_{3/2}{}^2$, and $\xi_{5/2}{}^2$ as in Table I. The breaking is then 60% (for seven transitions). However, ^{198}Pt and ^{197}Pt do not belong the the same supermultiplet as ^{196}Pt and ^{195}Pt, and as before, the parameters do not have to be the same. The agreement can be improved by an appropriate choice of the three parameters (see column five of Table III), and the breaking is reduced to 39%. The most striking point is, however, the fact that the two transitions to the levels at 98.6 and 131.2 keV—classified as strictly forbidden—are in fact experimentally observed to be strong. It should also be remarked that the ratios R(pu) of the spectroscopic factors for the transfer to the $J^\pi=\frac{5}{2}^-$ levels at, respectively, 53 and 297 keV, and R'(pu) of the spectroscopic factors for the transfer to, the $J^\pi=\frac{3}{2}^-$ levels at, respectively, 71.4 and 268.9 keV, both predicted—independent of any parameter—to be equal to 3.5 (here

TABLE III. One nucleon transfer strengths to the $J^\pi=1/2^-$, $3/2^-$, and $5/2^-$ levels of ^{197}Pt.

E_{ex}[a] (keV)	$J^{\pi a}$	Quantum numbers[a] (σ_1,σ_2)	(τ_1,τ_2)	S_{exp}[b]	Pickup S_{th}[c]	S_{th}[d]	G_{exp}[e]	Stripping G_{th}[f]	G_{th}[g]	Class[h]
0	$1/2^-$	(6,0)	(0,0)	0.67	0.707	1.03	0.26	0.252	0.43	*A*
53.	$5/2^-$	(5,1)	(1,0)	2.6	2.475	2.16	1.26	1.286	1.393	*A*
71.4	$3/2^-$	(5,1)	(1,0)	0.65	0.66	1.11	0.28	0.30	0.643	*A*
98.6	$3/2^-$	(5,1)	(1,1)	1.0	<u>0</u>	<u>0</u>	0.48	<u>0</u>	<u>0</u>	*F*
131.2	$1/2^-$	(5,1)	(1,1)	0.42	<u>0</u>	<u>0</u>	0.08	<u>0</u>	<u>0</u>	*F*
268.9	$3/2^-$	(6,0)	(1,0)	$\lesssim$0.05	0.19	0.318	not seen	0.014	0.03	*A*
297.	$5/2^-$	(6,0)	(1,0)	0.23	0.707	0.61	not seen	0.06	0.064	*A*

[a]From Ref. 4.
[b]From the (d,t) results of Ref. 7.
[c]Values obtained with $\xi_{1/2}{}^2=0.55$, $\xi_{3/2}{}^2=0.22$, and $\xi_{5/2}{}^2=0.55$.
[d]Values obtained using the same parameters as in Table I (see the text).
[e]From the S values of Ref. 9 ($G=2S$).
[f]Values obtained with $\theta_{1/2}{}^2=\theta_{3/2}{}^2=0.021$ and $\theta_{5/2}{}^2=0.06$ (the primed parameters are not involved for the levels discussed here).
[g]Values obtained using the same parameters as in Table I (see the text).
[h]The forbiddenness results from the $\Delta(\tau_1,\tau_2)$ selection rules. The quantum numbers (σ_1,σ_2) and (τ_1,τ_2) are (5,0) and (0,0) for ^{198}Pt and (6,0) and (0,0) for ^{196}Pt.

$N=5$), are indeed experimentally found to be[7] $R=11.3$ and $R'>13$. In summary, the general agreement with the model is worse for reactions involving ^{197}Pt: The total breaking for the two reactions analyzed here is 36% for 14 transitions, and three transitions classified as strictly forbidden are experimentally observed as strong.

To conclude, it appears that, even if it is possible to make a one-to-one correspondence[4] between the low-lying levels of ^{195}Pt and ^{197}Pt, there is an important change of structure[7] between ^{195}Pt and ^{197}Pt: The $p_{3/2}$ and $f_{5/2}$ pickup strengths, each mainly concentrated on one level in ^{195}Pt, are split between two levels in ^{197}Pt [this change of structure is also shown by the severe fragmentation of the $L=0$ strength, recently observed[12] in the ^{195}Pt(t,p)^{197}Pt reaction]. Accordingly, the multi-j supersymmetry scheme, which quite reasonably describes the present experimental results for transfer reactions involving ^{195}Pt, appears to be appreciably broken for the reactions involving ^{197}Pt. A similar deterioration of the agreement with increasing mass had also been observed[13] in the comparison of the results of the Ir(t,α)Pt reactions with the U(6/4) supersymmetry scheme.

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*Permanent address: Kernfysisch Versneller Instituut, Rijksuniversiteit Groningen, The Netherlands.

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