Transition density of charge-exchange processes

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The transition density between parent and analog states is studied with an eye on its role in charge-exchange nuclear reactions. The structure of the target nucleus is described in a perturbative approach, in which the Coulomb and asymmetry potentials mix the eigenstates of a charge-independent single-particle Hamiltonian. In this model formulae are derived for the transition density, the Coulomb displacement energy, and the neutron-proton density difference, and their relationship is used to estimate the transition density. This estimate shows that (i) the largest contribution comes from the density of the excess neutrons; (ii) the weight of the Coulomb-mixing effect is small up to excess neutron number 10, and grows rapidly beyond; (iii) the weight of the core polarization term induced by the excess neutrons is modest and is the same for all nuclei. It is indicated that the Coulomb effect may explain the departure from the Lane model of nucleon charge-exchange scattering found for heavy nuclei, whereas the core polarization may account for the observed anomalous dependence of the 0° pion charge-exchange cross section on the number of excess neutrons.

[NUCLEAR STRUCTURE Transition densities for charge exchange to IAS; Coulomb and core-polarization effects.]

I. INTRODUCTION

This paper is concerned with direct charge-exchange transitions, such as (p,n), (^{3}He,t), (π^+,π^0), (π^0,π^-), etc., that lead from a parent state Φ^p to its isobaric analog Φ^a. If there is no target-projectile antisymmetry, the transition density for such a process is given by

$$\Delta\rho(r)=\langle\Phi^a|\hat{\rho}^1_{-1}|\Phi^p\rangle\ , \tag{1}$$

where

$$\hat{\rho}^1_q=\sum_{i=1}^{A}\delta(\vec{r}-\vec{r}_i)\hat{t}_{iq}\ , \tag{2}$$

with $\hat{t}_{iq}$ being the q component of the nucleon isospin tensor $\hat{t}_i$, and A being the target mass number. If Φ^p is assumed to have pure isospin $T_0=\frac{1}{2}(N-Z)$ and

$$\Phi^a=T_0^{-1/2}\sum_i\hat{t}_{i,-1}\Phi^p\ ,$$

the transition density becomes proportional to the neutron-proton density difference $\rho_{\rm np}\equiv\rho_{\rm n}-\rho_{\rm p}$ in the target nucleus:

$$\Delta\rho=\tfrac{1}{2}T_0^{-1/2}\rho_{\rm np}\ . \tag{3}$$

If, furthermore, Φ^p consists of a single configuration of single-particle (sp) states $\phi_\alpha(\vec{r})$ whose space-spin parts are the same for neutrons and protons, then $\rho_{\rm np}$ reduces to the density of the neutron excess (ne),

$$\rho^{\rm ne}=\sum_{\alpha=2Z+1}^{A}|\phi_\alpha|^2\ ;$$

hence

$$\Delta\rho=\tfrac{1}{2}T_0^{-1/2}\rho^{\rm ne}\ . \tag{4}$$

In real nuclei the isospin is impure, thus Eq. (3) is not satisfied. Neither are Eqs. (3) and (4) equivalent, because $\rho_{\rm np}$ gains a large contribution from the self-conjugate core. Owing to these facts, the computation of the transition densities is not straightforward. Therefore, the isospin impurity is usually disregarded, and the transition density is either assumed to be given by Eq. (3) (Ref. 1) or Eq. (4) (Ref. 2).

The relationship between prescriptions (3) and (4) was first studied by Lovas, Brown, and Hodgson.[3] For (p,n) scattering from medium light nuclei we showed that prescriptions (3) and (4) give rather different results.[3] In a semimicroscopic model we estimated the core effects and found that the corrected transition density lies between those of Eqs. (3) and (4). We, however, neglected the component of Φ^a of isospin $T=T_0-1$.

Auerbach and Nguyen Van Giai examined the same problem in the context of pion charge exchange.[4] They pointed out that, because of the extreme sensitivity of such a transition to the nuclear surface, the differences between prescriptions (3) and (4) result in even larger discrepancies in the cross sections. Subsequently, Auerbach and Yeverechyahu demonstrated this by actual calculations for the (π^+,π^0) cross section.[5]

Auerbach and Van Giai[4] advocate that Eq. (4) is the correct prescription, at least for nuclei of small T_0. In their considerations they have duly taken into account the $T=T_0-1$ component of Φ^a. Their reasoning relies on three premises: (a) the core contribution to $\rho_{\rm np}$ is brought about by the Coulomb self-polarization of the core; (b) the core effect on $\Delta\rho$ is similar to that on the Coulomb displacement energy ΔE; and (c) as was demonstrated numerically for the case of ^{41}Ca-^{41}Sc (Ref. 6), the core effect on ΔE is very small.

In this paper we shall examine the validity of these

statements and, ultimately, of the prescription given by Eq. (4). First we shall set up a perturbative model, which is likely to be realistic for a broad class of nuclei (Sec. II A), and introduce simplifications so that Eqs. (3) and (4) are exactly valid (Sec. II B). Then we drop the simplifying assumptions (Sec. II C), and assess the size of the various correction terms (Sec. III). As a by-product, we shall obtain formulae for various contributions to the Coulomb displacement energy as well. The perturbative framework we shall use is very appropriate for exploring trends of behavior of many nuclei at a level of considerable generality. For actual calculations, however, there are more powerful methods. In drawing the conclusions (Sec. IV) we shall also discuss some points concerning the practical calculation of the transition densities.

II. TRANSITION DENSITY AND COULOMB DISPLACEMENT ENERGY

A. Basic model

The nuclear model to be outlined treats the interaction terms that distinguish between neutrons and protons as first-order perturbations. As a zero-order approximation to start with, we wish to choose a state in which $\rho_{\rm np}=\rho^{\rm ne}$ is satisfied, and, for simplicity, we take a single-configuration state. (If natural orbitals[7] are used, the latter restriction is not necessary.) To generate such a state, we define an average sp field, which includes the terms of the nuclear as well as the Coulomb force that are scalar in the isospin space of any one nucleon. A state $|\,T_0T_0\rangle$ of the A-particle system of isospin $T=T_0$ and isospin projection $T_Z=T_0$ in this field will be considered as the vacuum, on which Φ^p and Φ^a are built.

We further assume that the rest of the interaction can be represented by sp potential terms that are rank-one tensors in the isospin space of any single particle. The perturbation potential is thus a combination of the sp Coulomb potential terms V_C and asymmetry (or symmetry) potentials U:

$$\mathscr{C}^1_0=-\sum_{i=1}^{A} V_C(r_i)\hat{t}_{i0}$$

$$=-\int V_C(r)\hat{\rho}^1_0(\vec{r},\vec{r}_1,\ldots,\vec{r}_A)d\vec{r}\,, \tag{5a}$$

$$\mathscr{U}^0_0=\sum_{j=1}^{A}\sum_{i\neq j}^{A} U(r_j)\,\vec{t}_i\cdot\vec{t}_j\,, \tag{5b}$$

where the indices $\lambda\mu$ of $\mathscr{C}^\lambda_\mu$ and $\mathscr{U}^\lambda_\mu$ indicate the tensorial character in the isospin space of the A nucleons. The perturbation $\mathscr{C}^1_0$ causes the Coulomb self-polarization, whereas the role of $\mathscr{U}^0_0$ is twofold: while it represents the isovector core polarization (cp) effect induced by the ne, it also distorts the density of the ne itself. Since $\mathscr{U}^0_0$ is isoscalar in the A-nucleon space, it does not mix the *total* isospin, and the Coulomb displacement energy can be written as

$$\Delta E=\langle\Phi^a|\,\mathscr{C}^1_0\,|\Phi^a\rangle-\langle\Phi^p|\,\mathscr{C}^1_0\,|\Phi^p\rangle\,. \tag{6}$$

In (5) the interaction between each nucleon pair is counted double in order to represent the one-particle one-hole (1p1h) excitation effect of the two-particle interaction in terms of the one-particle interactions $\mathscr{C}^1_0$ and $\mathscr{U}^0_0$ properly.[8]

Finally, we assume that the effect of the perturbation terms (5) consists in mixing $|\,T_0T_0\rangle$ with the isovector monopole states $[\mathscr{M}^1|\,T_0\rangle]^T_{T_Z}$, where $[\,]^T_{T_Z}$ signifies isospin coupling. The monopole state is characterized by the excitation

$$\mathscr{M}^1_\tau=\sum_{nn'ljm} c_{nn'lj}\langle jmj,-m\,|\,00\rangle[a^\dagger_{n'ljm(1/2)}\tilde{a}_{nlj,-m,(1/2)}]^1_\tau\,, \tag{7}$$

where $a^\dagger_{n'ljm(1/2)t_Z}$ and $\tilde{a}_{nlj,-m,(1/2)t'_Z}$ are particle and hole creation operators, respectively, and $c_{nn'lj}$ can be determined through a Tamm-Dancoff diagonalization. The overall factor of $c_{nn'lj}$ is chosen so that $c_{nn'lj}$ may be real and normalized as

$$\sum_{nn'ljm}^{(0)}\frac{c^2_{nn'lj}}{2j+1}+\frac{1}{2}\left[\sum_{nn'ljm}^{(1)}\frac{c^2_{nn'lj}}{2j+1}+\sum_{nn'ljm}^{(2)}\frac{c^2_{nn'lj}}{2j+1}\right]=1\,, \tag{8}$$

where the summations $\sum^{(\alpha)}$ run over excitations

$$nljm\rightarrow n'ljm\quad(n'>n)$$

of type α defined in Fig. 1. (Excitations of type 3 are not included since even the heaviest nuclei have too small ne to allow such excitations over their basic configuration.)

Now, with the admixtures kept only up to first order, Φ^p and Φ^a may be written as

$$\Phi^p=|\,T_0T_0\rangle+a_{T_0T_0}[\mathscr{M}^1|\,T_0\rangle]^{T_0}_{T_0}+a_{T_0+1,T_0}[\mathscr{M}^1|\,T_0\rangle]^{T_0+1}_{T_0}\,,$$

$$\Phi^a=|\,T_0,T_0-1\rangle+a_{T_0-1,T_0-1}[\mathscr{M}^1|\,T_0\rangle]^{T_0-1}_{T_0-1}+a_{T_0,T_0-1}[\mathscr{M}^1|\,T_0\rangle]^{T_0}_{T_0-1}+a_{T_0+1,T_0-1}[\mathscr{M}^1|\,T_0\rangle]^{T_0+1}_{T_0-1}\,, \tag{9}$$

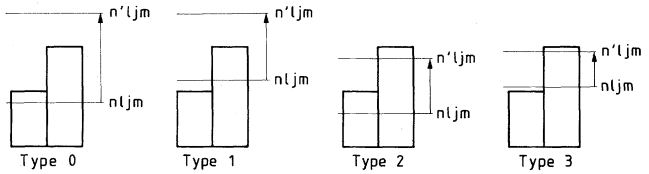

FIG. 1. Visualization of the four types of the ph excitations. The oblongs on the left and right represent the sp states filled by protons and neutrons, respectively.

where

$$a_{TT_Z}=-E_T^{-1}[\langle T_0 \mid \mathscr{M}^{1\dagger}]^T_{T_Z}(\mathscr{C}^1_0+\mathscr{U}^0_0) \mid T_0T_Z\rangle \quad (10)$$

with E_T being the excitation energy of $[\mathscr{M}^1 \mid T_0\rangle]^T_{T_Z}$. The terms multiplied by $a^2_{TT_Z}$ can justly be ignored if Φ^p is a ground state since the maximum value of $a^2_{T_0+1,T_0}$ among all nuclear ground states is 0.006 (Ref. 9) and the other coefficients are of the same order of magnitude.

This framework for the treatment of the Coulomb mixing is very close both to the standard procedure of estimating isospin impurities[8] and to the "analog-state method"[6] to calculate ΔE. The present approach is more approximate than the "analog-state method" in that here we represent the two-body potentials by their one-particle approximants—but this is a good approximation.[8] Our earlier model[3] was more limited than this one because of its special purpose. It was devised to bridge over the gap between the pure-isospin reaction model of Lane[10] and the isospin-mixing structure model of Brown, Massen, and Hodgson[11] for the target ground state as Φ^p. Since Φ^a is not within the scope of that structure model, it had to be shaped consistently by additional assumptions. It was assumed to be similar to Φ^p, and so no $T=T_0-1$ component was included. Furthermore, the perturbation $\mathscr{U}^0_0$ and excitations of types 1 and 2 were dealt with in an *ad hoc* manner. Here we wish to elaborate the model in full generality and treat all excitations and perturbations properly.

The transition density and the Coulomb displacement energy are obtained by substitution of (9) and (10) into (1) and (6). The terms independent of the coefficients a_{TT_Z} can be related to

$$\rho^{\rm ne}=2\langle T_0T_0 \mid \hat{\rho}^1_0 \mid T_0T_0\rangle$$

with the use of (2) and (5a), respectively, and of the Wigner-Eckart theorem. In the first-order approximation for the coefficients a_{TT_Z} we obtain

$$\Delta\rho=\tfrac{1}{2}T_0^{-1/2}\rho^{\rm ne}-\left[\sum_{T=T_0-1}^{T_0+1}E_T^{-1}[\langle T_0 \mid \mathscr{M}^{1\dagger}]^T_{T_0-1}\hat{\rho}^1_{-1} \mid T_0T_0\rangle\langle T_0,T_0-1 \mid (\mathscr{C}^1_0+\mathscr{U}^0_0)[\mathscr{M}^1 \mid T_0\rangle]^T_{T_0-1}\right.$$
$$\left.+\sum_{T=T_0}^{T_0+1}E_T^{-1}[\langle T_0 \mid \mathscr{M}^{1\dagger}]^T_{T_0}(\mathscr{C}^1_0+\mathscr{U}^0_0) \mid T_0T_0\rangle\langle T_0,T_0-1 \mid \hat{\rho}^1_{-1}[\mathscr{M}^1 \mid T_0\rangle]^T_{T_0}\right], \quad (11)$$

$$\Delta E=(2T_0)^{-1}\int\rho^{\rm ne}(r)V_C(r)d\vec{r}-2\left[\sum_{T=T_0-1}^{T_0+1}E_T^{-1}[\langle T_0 \mid \mathscr{M}^{1\dagger}]^T_{T_0-1}(\mathscr{C}^1_0+\mathscr{U}^0_0) \mid T_0,T_0-1\rangle\langle T_0,T_0-1 \mid \mathscr{C}^1_0[\mathscr{M}^1 \mid T_0\rangle]^T_{T_0-1}\right.$$
$$\left.-\sum_{T=T_0}^{T_0+1}E_T^{-1}[\langle T_0 \mid \mathscr{M}^{1\dagger}]^T_{T_0}(\mathscr{C}^1_0+\mathscr{U}^0_0) \mid T_0T_0\rangle\langle T_0T_0 \mid \mathscr{C}^1_0[\mathscr{M}^1 \mid T_0\rangle]^T_{T_0}\right].$$

The formulae of $\Delta\rho$ and ΔE do apparently show some formal similarity. To elucidate the analogy more explicitly, consider a case in which the states concerned have good isospins and are related by

$$\Phi^a=T_0^{-1/2}\sum_i\hat{t}_{i,-1}\Phi^p\,.$$

Using the Wigner-Eckart theorem, one can prove that

$$\Delta E=T_0^{-1/2}\int V_C(r)\Delta\rho(r)d\vec{r}\,. \quad (12)$$

It is conspicuous that the zero-order terms of (11) satisfy this relationship. Because of the isoscalar nature of $\mathscr{U}^0_0$, however, (12) must hold for the monopole admixture caused by $\mathscr{U}^0_0$ as well. On the other hand, for the contributions $\Delta\rho^{\rm Coul}$ and $\Delta E^{\rm Coul}$ from the isovector Coulomb admixture, the Wigner-Eckart theorem implies

$$\Delta E^{\rm Coul}=2T_0^{-1/2}\int V_C(r)\Delta\rho^{\rm Coul}(r)d\vec{r}\,. \quad (13)$$

Equation (12) is thus not maintained unless $\Delta E^{\rm Coul}=0$. However, as we shall see in Sec. II B, such a situation is not entirely unrealistic.

B. Simplified model

Having outlined the general framework we shall work with, we temporarily introduce three further approximations:

(i) We neglect the isospin splitting of the monopole state, i.e., take all $E_T\equiv E$. This is expected to be a reasonable approximation for nuclei of small T_0 since the splitting of E is roughly proportional to T_0. (See Sec. III.)

(ii) We assume that $\mathscr{M}^1_\tau$ involves neither excitations from nor to sp states that are occupied by neutrons but are not occupied by protons. The excitations allowed now are those denoted by type 0 in Fig. 1. This approximation again seems allowable for nuclei of small T_0 since the fewer the excess neutrons are the less their contribution counts in the collective monopole excitation.

(iii) We ignore $\mathscr{U}^0_0$. When adopting this approximation, one may look for support in former calculations for ΔE. For ^{41}Ca it was found[6] that the cp adds to ΔE only 0.1 MeV, and its contribution grows only by a factor of 3.5 between ^{41}Ca and ^{208}Pb (Ref. 12).

It follows from approximation (ii) that $\mathscr{M}_1^1 \mid T_0, T_0-1\rangle$ is a pure 2p2h state, and hence is orthogonal to $\mathscr{C}_0^1 \mid T_0 T_0\rangle$, which is a 1p1h state. For similar reasons, $\mathscr{M}_1^1 \mid T_0, T_0-2\rangle$ is orthogonal to $\mathscr{C}_0^1 \mid T_0, T_0-1\rangle$. Using these facts and the Wigner-Eckart theorem, it is straightforward to prove that

$$\begin{aligned}
&\langle T_0 T_Z \mid \mathscr{C}_0^1[\mathscr{M}^1 \mid T_0\rangle]_{T_Z}^T = \langle 10 T_0 T_Z \mid T T_Z\rangle \langle T_0 T_0 \mid \mathscr{C}_0^1 \mathscr{M}_0^1 \mid T_0 T_0\rangle \,, \\
&\langle T_0, T_0-1 \mid \hat{\rho}_{-1}^1[\mathscr{M}^1 \mid T_0\rangle]_{T_0}^T = -\langle 11 T_0, T_0-1 \mid T T_0\rangle \langle T_0 T_0 \mid \hat{\rho}_0^1 \mathscr{M}_0^1 \mid T_0 T_0\rangle \,, \\
&[\langle T_0 \mid \mathscr{M}^{1\dagger}]_{T_0-1}^T \hat{\rho}_{-1}^1 \mid T_0 T_0\rangle = \langle 1, -1, T_0 T_0 \mid T, T_0-1\rangle \langle T_0 T_0 \mid \mathscr{M}_0^1 \hat{\rho}_0^1 \mid T_0 T_0\rangle \,.
\end{aligned} \tag{14}$$

Substituting these formulae into (11) and using (i), (iii), and the orthogonality relation of the Clebsch-Gordan coefficients, we find that the expressions in the large square brackets in (11) vanish, so

$$\Delta\rho = \tfrac{1}{2} T_0^{-1/2} \Delta\rho_0 \quad (\Delta\rho_0 \equiv \rho^{\mathrm{ne}}) \,, \tag{15a}$$

$$\Delta E = (2T_0)^{-1} \Delta E_0 \quad \left[\Delta E_0 \equiv \int \rho^{\mathrm{ne}}(r) V_C(r) d\vec{r}\right] . \tag{15b}$$

Equations (15) satisfy (12), and (15a) is nothing but prescription (4) save that ρ^{ne} here is built up from orbits in the average of the neutron and proton sp potentials. Had we neglected the $T = T_0 - 1$ term of Φ^a as in Ref. 3, there would be a net contribution from the Coulomb terms in the large square brackets, which shows that this neglect is not justifiable. We can also easily prove that approximations (i)—(iii) are necessary to obtain prescription (4). The relationship (12) is, however, satisfied without requiring (iii).

C. Corrections to the simplified model

We shall now restore the original model and present the corrections entailed. The "exact" $\Delta\rho$ and ΔE may be derived by substituting the second-quantized form of $\mathscr{M}_\tau^1$, $\hat{\rho}_\tau^1$, $\mathscr{C}_0^1$, $\mathscr{U}_0^0$, $\mid T_0, T_0-1\rangle$, and $\mid T_0, T_0-2\rangle$ into (11) and evaluating the vacuum expectation values obtained. In doing so, one has to observe the isospin-polarized nature of the vacuum $\mid T_0 T_0\rangle$. Since some of the terms are laborious but all are straightforward to calculate, we omit the derivations.

We shall present the "exact" $\Delta\rho$ and ΔE step by step, in terms of additive corrections obtained by lifting approximations (i), (ii), and (iii) successively. The result is thus expressible as

$$\begin{aligned}
\Delta\rho &= \tfrac{1}{2} T_0^{-1/2} (\Delta\rho_0 + \Delta\rho_1 + \Delta\rho_2 + \Delta\rho_3) \,, \\
\Delta E &= (2T_0)^{-1} (\Delta E_0 + \Delta E_1 + \Delta E_2 + \Delta E_3) \,.
\end{aligned} \tag{16}$$

In presenting these terms it is useful to define the following quantities:

$$R_\alpha(r) = \frac{1}{\sqrt{2}} \sum_{nn'ljm}^{(\alpha)} \frac{c_{nn'lj}}{\sqrt{2j+1}} \phi_{nljm}^*(\vec{r}) \phi_{n'ljm}(\vec{r}) \,, \tag{17a}$$

$$C_\alpha = \int V_C(r) R_\alpha(r) d\vec{r} \,, \tag{17b}$$

$$U_\alpha = \int U(r) R_\alpha(r) d\vec{r} \quad (\alpha = 0, \ldots, 3) \,, \tag{17c}$$

$$R_{12} = \tfrac{1}{2}(R_1 + R_2), \quad C_{12} = \tfrac{1}{2}(C_1 + C_2) \,. \tag{17d}$$

Once approximation (i) is abandoned, E_T^{-1} does not factor out in (11), and the orthogonality of the Clebsch-Gordan coefficients no longer causes a full cancellation. We obtain

$$\Delta\rho_1 = \theta_0(-4T_0 C_0 R_0 / E_{T_0+1}) \,, \tag{18a}$$

$$\Delta E_1 = 2\theta_0(-4T_0 C_0^2 / E_{T_0+1}) \,, \tag{18b}$$

where

$$\theta_0 = \frac{2T_0 - 1}{2T_0(T_0+1)(2T_0+1)} \times \left[\frac{T_0+1}{E_{T_0-1}} - \frac{2T_0+1}{E_{T_0}} + \frac{T_0}{E_{T_0+1}}\right] E_{T_0+1} \,. \tag{19}$$

Correction (ii) amounts to allowing for excitations of types 1 and 2. The number of possible excitations 1 and 2 increases with T_0, and for heavy nuclei their weight is comparable with those of type 0. Therefore, it is not enough to keep the linear terms in the quantities of index 1,2. The result is

$$\begin{aligned}
\Delta\rho_2 &= \theta_1[-2T_0(C_0 R_{12} + C_{12} R_0)/E_{T_0+1}] \\
&\quad + \theta_2(-4T_0 C_{12} R_{12} / E_{T_0+1}) \,,
\end{aligned} \tag{20a}$$

$$\begin{aligned}
\Delta E_2 &= 2\theta_1(-4T_0 C_0 C_{12} / E_{T_0+1}) \\
&\quad + 2\theta_2(-4T_0 C_{12}^2 / E_{T_0+1}) \,,
\end{aligned} \tag{20b}$$

where

$$\begin{aligned}
\theta_1 &= \frac{2T_0 - 1}{T_0^2} \left[\frac{1}{E_{T_0-1}} - \frac{1}{E_{T_0}}\right] E_{T_0+1} \,, \\
\theta_2 &= \frac{2T_0 - 1}{2T_0^3} \left[\frac{2T_0+1}{E_{T_0-1}} - \frac{T_0+1}{E_{T_0}}\right] E_{T_0+1} \,.
\end{aligned} \tag{21}$$

The third correction is obtained as

$$\begin{aligned}
\Delta\rho_3 &= -4\Omega[T_0 R_0 + (T_0+1) R_{12}] \,, \\
\Delta E_3 &= -4\Omega[T_0 C_0 + (T_0+1) C_{12}]
\end{aligned} \tag{22}$$

with

$$\begin{aligned}
\Omega = \{ &U_0 + [(2T_0 - 1)/(4T_0)] U_1 \\
&+ [(2T_0+5)/(4T_0)] U_2 \} / E_{T_0} \,.
\end{aligned} \tag{23}$$

We see that, as envisaged in Sec. II A, correction (iii) satisfies Eq. (12), while corrections (i) and (ii) obey Eq. (13).

III. ESTIMATE FOR THE CORRECTIONS

The adequacy of Eq. (4) was primarily inferred[4] from a case of $T_0=\frac{1}{2}$ (Ref. 6). Looking at Eqs. (19) and (21), we see that for $T_0=\frac{1}{2}$ the first two corrections happen to vanish. This fact shows that the case of $T_0=\frac{1}{2}$ is very special, and underlines the importance of a closer examination of the behavior of the corrections. This could be done by carrying through the Tamm-Dancoff calculation and the evaluation of the corrections for many cases. However, sacrificing accuracy, we can gain more insight by estimating the ingredients of $\Delta\rho$ by general considerations.

The behavior of the corrections is largely determined by their implicit dependence on T_0 through C_0R_0, $C_0R_{12}+C_{12}R_0$, $C_{12}R_{12}$, Ω, and E_T. Therefore, to explore $\Delta\rho_i$, we need to know something of these quantities. It provides a starting point that $\rho_{\rm np}$ is also expressible in terms of them:

$$\rho_{\rm np}=2\langle\Phi^p|\hat{\rho}_0^1|\Phi^p\rangle=\rho^{\rm ne}+4\left[\left[\frac{1}{E_{T_0}}\frac{T_0}{T_0+1}+\frac{1}{E_{T_0+1}}\frac{1}{T_0+1}\right]C_0R_0+\frac{1}{E_{T_0}}(C_0R_{12}+C_{12}R_0)+\frac{1}{E_{T_0}}\frac{T_0+1}{T_0}C_{12}R_{12}\right]$$
$$-4\Omega[T_0R_0+(T_0+1)R_{12}]\,. \tag{24}$$

Moreover, owing to the normalization condition (8), for a given Z, the quantities $R\equiv R_0+R_{12}$ and $C\equiv C_0+C_{12}$ are more or less independent of T_0. In particular, $T_0=0$ implies $R=R_0$, $C=C_0$, and $R_{12}=C_{12}=0$. In a nucleus of $T_0=0$, $\rho_{\rm np}$ ($\equiv\rho_{\rm np}^0$) comes solely from the Coulomb force, thus (24) reduces to

$$\rho_{\rm np}^0=4CR/E_{T_0+1}\,. \tag{25}$$

To estimate $\Delta\rho_1$ and $\Delta\rho_2$, we proceed in two steps: we first relate $\rho_{\rm np}^0$ to $\Delta\rho_0=\rho^{\rm ne}$, and then relate $\Delta\rho_1$ and $\Delta\rho_2$ to $\rho_{\rm np}^0$ through decomposing C and R into C_0+C_{12} and R_0+R_{12}, respectively.

For ^{42}Ca we find (Fig. 2) that, beyond the outer (negative) peak of $\rho_{\rm np}^0$, $\rho_{\rm np}-\rho_{\rm np}^0\approx-\rho_{\rm np}^0$. Indeed, the relationship

$$\rho_{\rm np}-\rho_{\rm np}^0\approx-2T_0\rho_{\rm np}^0$$

is found to hold in the $4.5<r<6$ fm region for all Ca isotopes.[3] If the cp is neglected, this takes the form

$$\rho^{\rm ne}\approx-2T_0\rho_{\rm np}^0\ \text{(for Ca, } 4.5<r<6\text{ fm)}\,. \tag{26}$$

To extract something general from this observation, we have to know the behavior of $\rho_{\rm np}^0$ as a function of A. Owing to the uniformity of the Coulomb repulsion for all nuclei and to the vanishing of the volume integral, $\rho_{\rm np}^0$ has much the same shape, as in Fig. 2, for all A: it has a large positive and a smaller negative peak and a flat tail. From the results for ^{16}O (Ref. 13), ^{40}Ca, and ^{56}Ni (Ref. 3) we can infer that at equivalent points (e.g., at corresponding peaks and along the tail) $\rho_{\rm np}^0\propto A$. On the other hand, the proportionality of the nuclear volume to A implies $\rho^{\rm ne}/T_0\propto A^{-1}$. With these trends taken into account, Eqs. (25) and (26) yield

$$\tfrac{1}{2}(A/40)^2\rho^{\rm ne}\approx-4T_0CR/E_{T_0+1}$$
$$(1.3A^{1/3}<r<1.75A^{1/3}\text{ fm})\,. \tag{27}$$

We decompose R into $R_0+\frac{1}{2}(R_1+R_2)$ by assuming that each term R_i is proportional to the number N_i of possible nucleon excitations $nljm\to n+1,ljm$ of type i (Fig. 1). This is in accord with the spirit of a first-order Tamm-Dancoff calculation. In this picture we have

$$C_0R_0=\frac{N_0^2}{N^2}CR,\quad C_0R_{12}+C_{12}R_0=\frac{2N_0N_{12}}{N^2}CR\,,$$
$$C_{12}R_{12}=\frac{N_{12}^2}{N^2}CR\,, \tag{28}$$

where $N_{12}=\frac{1}{2}(N_1+N_2)$ and $N=N_0+N_{12}$. Now, substituting (28) and then (27) into (18a) and (20a), we find that, in the tail region ($1.3A^{1/3}<r<1.75A^{1/3}$ fm), the magnitude of $\Delta\rho_1$ and $\Delta\rho_2$ relative to $\rho^{\rm ne}$ is given by the coefficients

$$\Lambda_1=\frac{1}{2}\left[\frac{A}{40}\right]^2\frac{N_0^2}{N^2}\theta_0\,,$$
$$\Lambda_2=\frac{1}{2}\left[\frac{A}{40}\right]^2\frac{N_{12}}{N^2}(N_0\theta_1+N_{12}\theta_2)\,, \tag{29}$$

respectively.

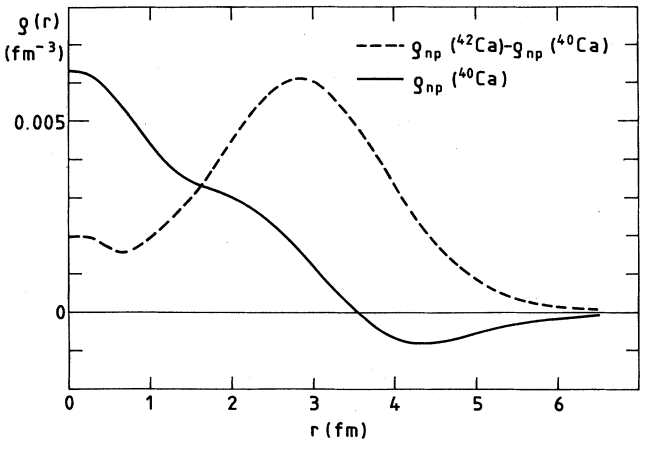

FIG. 2. Comparison of the neutron-proton density difference of ^{40}Ca with that caused by the presence of the ne in ^{42}Ca. It is seen that, beyond 4.5 fm, $\rho_{\rm np}(^{42}{\rm Ca})-\rho_{\rm np}(^{40}{\rm Ca})\approx-\rho_{\rm np}(^{40}{\rm Ca})$. (Calculations described in detail in Refs. 3 and 11.)

The quantities θ_i, N_i, and Λ_i are tabulated for a couple of nuclei in Table I. In calculating E_T, the formula[8]

$$E_T \approx (2\hbar\omega + 10 \text{ MeV}) + \tfrac{1}{4}[T(T+1) - T_0(T_0+1) - 2]U/A \quad (30)$$

was used with $\hbar\omega = 41/A^{1/3}$ MeV and $U = 100$ MeV. In counting the possible nucleon excitations the standard extreme sp model has been adopted.

The results show that correction (i) is insignificant, and correction (ii) is likely to be large only for heavy nuclei. The reliability of these estimates depends on the radial region actually responsible for the transition and on the size of ρ^{ne} in this region relative to the ph states involved in the monopole. The corrections are bound to be underrated in the extreme tail region ($r > 1.75A^{1/3}$ fm) as the ph states contain highly excited particle orbits, whose extention is larger than that of the sp states occupied by the ne. Since all density terms are very small in this region, this effect may only be observable where the cross section comes solely from this region, namely, at extreme forward angles. In the more significant part of the tail ($1.3A^{1/3} < r < 1.75A^{1/3}$ fm), however, the individual shape of ρ^{ne} is not likely to cause fluctuations in ρ^{ne}/T_0 relative to ρ^0_{np} larger than, say, a factor of 2. Because of the very different shapes of $\rho^{ne}(r)$ and $\rho^0_{np}(r)$, however, the more the inner regions contribute the less clear the picture becomes. But even in the extreme case when the whole volume contributes, our estimates are likely to work as order-of-magnitude estimates.

To weigh up the size of $\Delta\rho_3$, we use the fact that it is equal to the last term of ρ_{np} in (24). It is now convenient to apply approximations (i) and (ii). Inserting (23) in (24) and (22), we obtain

$$\rho_{np} = \rho^{ne} + 4(C_0 - T_0U_0)R_0/E \ , \quad (31)$$

$$\Delta\rho_3 = -4T_0U_0R_0/E \ , \quad (32a)$$

$$\Delta E_3 = -4T_0U_0C_0/E \ . \quad (32b)$$

In these equations all R_α, C_α, and U_α appear with subscript 0 only, thus in this approximation correction (iii) represents just the cp. Substituting (17a) and (17c) into (31) and (32), and then (32) into (16), we obtain

$$\rho_{np} = \rho^{ne} - 4E^{-1}R_0 \sum_{nn'ljm}^{(0)} \frac{c_{nn'lj}}{\sqrt{2(2j+1)}} \langle \phi_{nljm} | -V_C + T_0U | \phi_{n'ljm} \rangle \ , \quad (33)$$

$$\Delta\rho = \tfrac{1}{2}T_0^{-1/2}\left[\rho^{ne} - 4E^{-1}R_0 \sum_{nn'ljm}^{(0)} \frac{c_{nn'lj}}{\sqrt{2(2j+1)}} \langle \phi_{nljm} | T_0U | \phi_{n'ljm} \rangle\right] , \quad (34)$$

$$\Delta E = (2T_0)^{-1}\left[\Delta E_0 - 4E^{-1}C_0 \sum_{nn'ljm}^{(0)} \frac{c_{nn'lj}}{\sqrt{2(2j+1)}} \langle \phi_{nljm} | T_0U | \phi_{n'ljm} \rangle\right] .$$

Equation (33) shows that the shapes of the Coulomb and cp terms of ρ_{np} are identical, their signs are opposite, and their magnitudes compare as V_C does to T_0U. However, while the Coulomb term of the transition density, $\Delta\rho^{Coul} = \Delta\rho_1 + \Delta\rho_2$, given by Eqs. (18a) and (20a), is only a small fraction of the large Coulomb term of ρ_{np} [cf. Eq. (24)], in the case of the cp, the full correction term of ρ_{np} is added to $\Delta\rho$. This result confirms the conjecture put forward in our former work.[3] Moreover, since both ρ^{ne} and $\Delta\rho_3$ are proportional to T_0/A, the magnitude of $\Delta\rho_3$ relative to ρ^{ne} must be more or less the same for all nuclei. Therefore, the cp effect in the transition density must be, for most nuclei, far more important than the Coulomb effect.

To assess the strength of the cp contribution to the densities, let us consider the Ca isotopes again. For these nuclei $V_C(r) \approx 3U(r)$ (within the nuclear volume), and we may assume that, similarly, $C_0 \approx 3U_0$. Substituting $3U_0$ for C in (25) and inserting ρ^0_{np} of (25) in (26), we find that in the tail region

$$\rho^{ne}(\text{Ca}) \approx -24T_0U_0R_0/E \ .$$

A comparison of this result with (32a) reveals that

$$\Delta\rho_3 \approx \tfrac{1}{6}\rho^{ne} \quad (1.3A^{1/3} < r < 1.75A^{1/3} \text{ fm}) \ , \quad (35)$$

and, as was mentioned above, this estimate should hold for all nuclei.

Just as for the items in Table I, Eq. (35) may be expected to hold within a factor of 2, but in the cp transition amplitude drastic departures are possible if the inner regions contribute to it or the extreme surface dominates the

TABLE I. Coefficients in the correction terms to the transition density. The quantities θ_i are defined in (19) and (21); N_i is the number of possible sp excitations of type i (Fig. 1) and Λ_i are given in (29).

Nucleus	θ_0	θ_1	θ_2	N_0	N_1	N_2	Λ_1	Λ_2
^{42}Ca	7.0×10^{-4}	4.1×10^{-2}	6.0×10^{-1}	18	2	0	3.5×10^{-4}	2.0×10^{-3}
^{48}Ca	4.4×10^{-3}	7.8×10^{-2}	3.4×10^{-1}	18	8	0	2.1×10^{-3}	1.6×10^{-2}
^{90}Zr	2.0×10^{-3}	4.5×10^{-2}	2.5×10^{-1}	32	10	0	3.8×10^{-3}	2.5×10^{-2}
^{116}Sn	2.3×10^{-3}	4.0×10^{-2}	1.8×10^{-1}	34	16	8	5.3×10^{-3}	8.4×10^{-2}
^{208}Pb	2.9×10^{-3}	3.2×10^{-2}	8.7×10^{-2}	42	44	20	1.3×10^{-2}	3.3×10^{-1}

transition. In any case, this contribution is always non-negligible. Since the cp is governed by the distribution of the ne, $\Delta\rho_3$ should follow the fluctuations of $\Delta\rho_0=\rho^{ne}$ between different nuclei. Because of this property, Λ_1 and Λ_2 can be reinterpreted as the magnitudes of $\Delta\rho_1$ and $\Delta\rho_2$ relative to $\Delta\rho_0+\Delta\rho_3$.

The treatment of the cp effect on the (p,n) cross section in Ref. 3 can be considered as an approximation to the model outlined in the present paper. The main additional approximations in it are that the density terms coming from a nonperturbative nuclear model are treated as if they had been calculated perturbatively, and no provision is made for excluding spurious isosopin mixing. The term $\Delta\rho_3$ is calculated by subtracting the Coulomb term from the core contribution to ρ_{np}.

For all the cases calculated[3] (for Ca isotopes at 25 MeV and for ^{58}Ni at 22.8 MeV bombarding energy) in the interval 20°—180° the cp appears as a solid 5—20% effect in accord with Eq. (35). At extreme forward angles, however, the cp effect is much larger. At 0° it enhances the cross section by factors of 4, 2, 1.5, and 1.4 for ^{42}Ca, ^{44}Ca, ^{48}Ca, and ^{58}Ni, respectively. This is obviously the cp version of the surface effect mentioned in conjunction with the Coulomb contribution. Indeed, for the Ca isotopes $\Delta\rho_3$ dominates over $\Delta\rho_0$ in the outermost region.

To estimate the corrections to the Coulomb displacement energy along the lines followed so far would be difficult for ΔE_i gain contributions from $\Delta\rho_i(r)$ *over all r*. Nevertheless, since there is a one-to-one correspondence between the terms of $\Delta\rho$ and ΔE, some qualitative features of ΔE_i can be deduced.

The Coulomb terms of ρ_{np} contribute to ΔE_i multiplied by the same diminutive factors θ that appear in $\Delta\rho_i$. Moreover, when Eqs. (28) are substituted in (18a) and (20a), $\Delta\rho_1$ and $\Delta\rho_2$ appear to be proportional to ρ^0_{np} given in (25). Therefore, if $V_C(r)$ were constant throughout the nuclear volume, Eq. (13) and the vanishing of the volume integral of $\rho^0_{np}(r)$ would imply that $\Delta E_1=\Delta E_2=0$. Of course, Eqs. (28) are not exact, and $V_C(r)$, though smooth and flat, is not constant, so ΔE_1 and ΔE_2 do not in fact vanish. Nonetheless, this speculation indicates that the Coulomb corrections to ΔE must be less important than to $\Delta\rho$.

Similar reasoning is applicable to correction (iii). If we restrict correction (iii) to the cp as we did in estimating $\Delta\rho_3$, the volume integral of $\Delta\rho_3$ vanishes. Thus the cp term in ΔE is nonzero due only to the departure of V_C from $V_C=$const, which implies that the relative importance of ΔE_3 is much smaller than that of $\Delta\rho_3$.

These considerations reveal that it was not justified to conclude[4] from the smallness of the corrections to ΔE that they are small also in $\Delta\rho$. On the other hand, they show that our evidence for the cp effect being sizable in $\Delta\rho$ does not contradict the former finding[6] that it is small in ΔE.

IV. SUMMARY AND DISCUSSION

An idealized charge-exchange transition assumes parent and analog states in which the nucleon distributions are the same. In real nuclei, however, both the self-conjugate cores and the valence nucleon orbits are distorted in the two states relative to each other. In an sp model the interactions responsible for this are the isovector Coulomb term (5a) and the asymmetry potential (5b). Their effect consists in mixing the states concerned with the isovector monopole state. The perturbative model used has made it possible to study, term by term, the Coulomb and asymmetry contributions to the transition density $\Delta\rho$ along with the contributions to the Coulomb displacement energy ΔE and the neutron-proton density difference ρ_{np}.

By this means we could simply formulate the correspondence, between $\Delta\rho$ and ΔE, referred to in Sec. I as premise (b). It was found that $\Delta\rho$ and ΔE are built up from terms which strictly correspond to each other. However, the formula for this correspondence contains a coefficient, which depends on whether the terms concerned originate from an isovector mixing, like the Coulomb terms [Eq. (13)], or not, like the ne terms and the asymmetry terms [Eq. (12)]. As was expected, the ne contributions $\Delta\rho_0=\rho^{ne}$ and ΔE_0 are the largest. As to the two corrections to $\Delta\rho$, we can sum up the results as follows.

We know empirically that each Coulomb potential term V_C is somewhat larger than the asymmetry term T_0U. Therefore, if they contribute with equal weights, as in the case of ρ_{np} [Eq. (24)], the Coulomb contribution is larger than the asymmetry contribution. In $\Delta\rho$ (and ΔE), however, the two effects appear very differently. While the asymmetry-potential contribution to $\Delta\rho$ equals that to ρ_{np}, the Coulomb contributions are not related simply. Although the two Coulomb contributions are built up from the same blocks, these add up constructively in ρ_{np}, but largely cancel each other in $\Delta\rho$.

The Coulomb admixture may be split into two parts: the first containing all purely core effects and the second the corrections due to the ne. It is the contribution of the first part to $\Delta\rho$ (as well as to ΔE) that cancels out: it has been shown to vanish exactly in our model. The ne gives two distinct corrections: $\Delta\rho_1$ due purely to the isospin splitting of the monopole state [Eq. (30)], and $\Delta\rho_2$ coming from ph excitations involving the ne (Fig. 1). The estimates show $\Delta\rho_1$ not to exceed 1.5% of ρ^{ne} in the surface region ($1.3A^{1/3}<r<1.75A^{1/3}$ fm), and even $\Delta\rho_2$ appears smaller than what was envisaged for the Coulomb term of $\Delta\rho$ in Ref. 4. It is about 2% for ^{48}Ca and ^{90}Zr, but as large as 33% for ^{208}Pb. (See Λ_1 and Λ_2 in Table I.) This last fact may well be the cause of the hitherto unexplained systematic breakdown[14] of the Lane model[10] found just for the heavy elements in a comprehensive survey.[14,15]

Because of the large spatial extension of the monopole state, further away from the nuclear center, $|\Delta\rho^{Coul}(r)|$ tends to exceed the density of the ne, $\rho^{ne}(r)$; thus $\Delta\rho$ changes sign. This may have a drastic effect on the T_0 dependence of the 0° charge-exchange cross section. Such an anomaly has been observed in (^{3}He,t) scattering from rare earth nuclei.[16] To explain it, the Coulomb effect should be invoked, along with multistep processes.

Not being associated with isospin mixing, the asymmetry contribution $\Delta\rho_3$ does not disturb the validity of the Lane model. It also consists of two parts: the cp correction and a minor correction to the ne. We have shown that the cp correction to $\Delta\rho$ is uniform for all nuclei, and in the tail region ($1.3A^{1/3}<r<1.75A^{1/3}$ fm) it is about

one-sixth of ρ^{ne} [Eq. (35)]. This is a modest but non-negligible departure from prescription (4). The significance of the cp may, however, be greatly enhanced at forward angles in the charge-exchange cross section since, beyond $1.75A^{1/3}$ fm, $\Delta\rho_3(r)$ damps down more slowly than $\rho^{ne}(r)$. Model calculations[3] show that this effect depends on T_0 strongly. In the 0° cross section of pion single charge exchange on Ca isotopes T_0 anomalies have been observed.[17] Since for Ca isotopes both the Coulomb effect and multistep contributions must be small, we suggest that the cp is the likeliest to explain this anomaly. Of course, the cp must also contribute to the similar anomaly[16] in (^{3}He,t). We thus argue that, negligible as it appears[6] in ΔE, the cp effect is appreciable in $\Delta\rho$, and hence premise (a) is not well founded.

What we call cp is, in general terms, the effect of the ne on the shape of the orbits in the self-conjugate core. The cp arises because of the $\vec{t}_1\cdot\vec{t}_2$ terms in the nucleon-nucleon force (which may be hidden in the spin-exchange term in the case of zero-range forces) and of the Pauli principle. The latter contributes to the cp by making the interaction matrix elements of like and unlike nucleons different and limiting the phase space available for any one core neutron and proton differently. In our approach the cp is mediated by the asymmetry potential, whose nonlocality was disregarded here. Both empirical evidence and microscopic calculations show that our asymmetry potential is a fairly realistic substitute for the underlying microscopic interaction. In fact, such a potential correctly predicts the observed sp energies[18] as well as charge densities.[7] On the other hand, it agrees qualitatively with the asymmetry potential emerging from the Skyrme interaction in the Hartree-Fock model.[19]

It is nevertheless true that delicate effects such as the Coulomb energy (Nolen-Schiffer) anomaly[20] and the change of the rms charge radii along isotopic chains[21] are not satisfactorily accounted for by a local-potential single-configuration model[11] of the fashion we adopted here. Recently, however, Caurier and Poves demonstrated[9] impressively that these effects may be attributed to a kind of cp. This result casts doubt on premise (c). But the cp that had to be postulated to this end acts in an unexpected fashion. It compresses the neutron distribution and expands the proton distribution. If it could be represented by a local asymmetry potential, this would be negative, which is at variance with all previous findings. Charge-exchange scattering experiments on isotopic chains analyzed in a model treating isospin properly may be the most direct tests on the sign of the cp.

Our model treats the isospin satisfactorily, but in other respects it is inferior to some of the standard nuclear models. For an actual realistic calculation, the reduction of the problem to a local nucleon-nucleus potential, the first-order treatment of the perturbation, the postulation of a discrete isovector monopole state, etc., would be unnecessarily restrictive. Instead of a practical implementation of our model, we therefore recommend the use of an isospin-projected Hartree-Fock model. In such a model the wave functions can be written in the form

$$\Phi^p = a'_{T_0T_0}\Phi_{T_0T_0} + a_{T_0+1,T_0}\Phi_{T_0+1,T_0} + \cdots , \tag{36a}$$

$$\Phi^a = a_{T_0-1,T_0-1}\Phi_{T_0-1,T_0-1} + a'_{T_0,T_0-1}\Phi_{T_0,T_0-1} + a_{T_0+1,T_0-1}\Phi_{T_0+1,T_0-1} + \cdots , \tag{36b}$$

and Eq. (1) can be evaluated with direct numerical methods.

For nuclear ground states the isospin-projected Hartree-Fock model of Caurier, Poves, and Zuker[22,9] is fully appropriate. In some of the integral properties, such as rms radii, ΔE, etc., the isospin projecting may not be crucial, but as far as densities are concerned, it seems to be indispensable.[9] This model, however, needs to be improved by including the $T=T_0-1$ term in Eq. (36b). In fact, $a_{T_0-1,T_0-1}=0$ was assumed even in the calculations[9] intended to explain the Nolen-Schiffer anomaly. However, as had been stressed by Auerbach *et al.*,[6] in calculating ΔE one has either to keep all $T=T_0$, $T_0\pm1$ admixtures or none. In our model the terms of $\Delta\rho$ and ΔE coming from $a_{T_0-1,T_0-1}\neq0$ are

$$\delta\rho = -\tfrac{1}{2}T_0^{-1/2}\{2E^{-1}_{T_0-1}(2T_0-1)[(2T_0+1)^{-1}C_0R_0 + T_0^{-1}(C_0R_{12}+C_{12}R_0) + T_0^{-2}(2T_0+1)C_{12}R_{12}]\} ,$$

$$\delta E = -(2T_0)^{-1}\{4E^{-1}_{T_0-1}(2T_0-1)[(2T_0+1)^{-1}C_0^2 + 2T_0^{-1}C_0C_{12} + T_0^{-2}(2T_0+1)C_{12}^2]\} . \tag{37}$$

Note that in these terms C_0R_0 and C_0^2 appear with coefficients, which do not vanish in the limit of zero monopole-state splitting. Therefore, apart from the case of $T_0=\frac{1}{2}$, for small T_0, $\delta\rho$ and δE are larger than $\Delta\rho_1+\Delta\rho_2+\Delta\rho_3$ and $\Delta E_1+\Delta E_2+\Delta E_3$, respectively. Owing to the cancellations between the positive and negative regions of $R_0(r)$ in C_0 (see Sec. III), this error might not invalidate the conclusions on ΔE, but would certainly be fatal in $\Delta\rho$.

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