Generator coordinate method for the multicluster system

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We have introduced a new "hole" formulation by using two rules for deriving the generator coordinate method kernels of the multicluster system with any cluster partitions. The uniform formulae for the overlap kernel, kinetic energy kernel, and potential energy kernel given in this paper can be used easily and are available for computing codes without any further treatment. We have used them to calculate the binding energies of $1p$ shell nuclei and $1p$ shell Λ hypernuclei.

[NUCLEAR STRUCTURE Derived formulae of kernels. Calculated binding energies of $1p$ shell nuclei, Λ hypernuclei.]

I. INTRODUCTION

During the last ten years, the formulation of the multicluster model of nuclei has progressed. In the seventies the three-cluster structure of ^{12}C was investigated by Horiuchi,[1] Smirnov *et al.*,[2] and Kato and Bando[3] and that of ^{10}B by Deenen.[4] Recently, Schmid developed the three-cluster "fish-bone" model and a theory for describing the N-cluster system.[5] Meanwhile, in the two cluster formulation, the p orbit of single particle states has been taken into account.[6] In either of the two cases, the derivation of the kernels is an essential procedure, but it is a laborious and tedious one. It is a major problem to ensure the correctness of the kernels derived. This paper is devoted to giving a unified method which can be used to derive any kind of generator coordinate method (GCM) kernel in an easy and compact way. We call this method the "hole" formulation.

In the "hole" formulation, we regard the nuclei as consisting of nucleons and "holes" as Deenen did.[4] But the quotation marks around the word hole, hereafter deleted, mean that it does not have the conventional meaning. Here we introduce the hole to indicate the nucleon missing with respect to the biggest $4mn$ nucleus in the region under consideration, where m means the number of single particle states in the cluster and n the number of clusters. The $4mn$ nucleus is called the standard nucleus hereafter. For example, ^{10}B may be regarded as ^{12}C with two holes or ^{16}O with six holes, while ^{12}C or ^{16}O is taken as a standard nucleus. In contrast to previous hole approaches,[4] we do not take a hole state as the time-reversal of a particle state; neither do we consider it as belonging to any Hilbert space. With the hole formulation, we can put the direct kernel and exchange kernel in an analytic form so that we can treat the nucleus with any partition systematically.

In Sec. II, the hole formulation of the GCM is given. In Sec. III, we use this formulation to calculate the binding energies of 1-p shell nuclei. In Sec. IV, the binding energies of 1-p shell Λ hypernuclei are given.

II. HOLE FORMULATION OF GCM KERNELS

A nucleus of A nucleons is partitioned into n clusters. The ith cluster exhibits a harmonic oscillator shell model configuration with the potential well at $\vec{\mathrm{R}}_i$, and the state of the nucleon in the cluster is denoted by φ with a superscript q ($i=1,2,\ldots,n$ and $q=s,p,d,\ldots,m$), that is,

$$\varphi^q_\mu(\vec{\mathrm{r}}-\vec{\mathrm{R}}_i)\equiv\varphi^q(\vec{\mathrm{r}}-\vec{\mathrm{R}}_i)\chi_\mu\ , \tag{1}$$

where $\varphi^q(\vec{\mathrm{r}}-\vec{\mathrm{R}})$ is the spatial part, χ_μ is the spin-isospin part, and μ is the spin-isospin quantum number ($\mu=\alpha,\beta,\gamma,\delta$). For clarity in our discussion, we take α for a proton with spin upwards, β for a proton with spin downwards, γ for a neutron with spin upwards, and δ for a neutron with spin downwards.

It should be noted that the oscillator parameter b is taken to be the same for all the states of the clusters, and is also taken as one of the generating coordinates.

The generating function for the system is written as

$$|\Psi(\vec{\mathrm{r}},\vec{\mathrm{R}})\rangle=\mathscr{A}\,|\Phi(\vec{\mathrm{r}},\vec{\mathrm{R}})\rangle=\mathscr{A}\left[\prod_{iq\mu}^{A}\varphi^q_\mu(\vec{\mathrm{r}}_{iq\mu}-\vec{\mathrm{R}}_i)\right]$$
$$=\frac{1}{\sqrt{A!}}\det|\{\varphi^q_\mu(\vec{\mathrm{r}}_{i'q'\mu'}-\vec{\mathrm{R}}_i)\}|\ , \tag{2}$$

where $\mathscr{A}$ is the total antisymmetrization operator for the nucleons, and the subscript $iq\mu$ ($i'q'\mu'$) of $\vec{\mathrm{r}}$ denotes that the nucleon concerned belongs to the q (q') state of the ith (i'th) potential well with spin-isospin μ (μ') before antisymmetrization.

The GCM wave function is given by

$$\Psi^{\mathrm{GCM}}(\vec{\mathrm{r}})=\int d\vec{\mathrm{R}}\,f(\vec{\mathrm{R}})\,|\Psi(\vec{\mathrm{r}},\vec{\mathrm{R}})\rangle\ , \tag{3}$$

where the integration is performed over the whole generator coordinate space $[\vec{\mathrm{R}}_i,(i=1,2,\ldots,n),b]$. The amplitude $f(\vec{\mathrm{R}})$ is obtained by solving the Hill-Wheeler equation

$$\int d\vec{R}[H(\vec{R},\vec{R}')-EN(\vec{R},\vec{R}')]f(\vec{R})=0\ , \tag{4}$$

with

$$H(\vec{R},\vec{R}')=\langle\Phi(\vec{r},\vec{R})\,|\,H\,|\,\Psi(\vec{r},\vec{R}')\rangle\ , \tag{5}$$

$$N(\vec{R},\vec{R}')=\langle\Phi(\vec{r},\vec{R})\,|\,\Psi(\vec{r},\vec{R}')\rangle\ , \tag{6}$$

where H is the Hamiltonian of the system and is given by

$$H=\frac{-\hbar^2}{2m}\sum_{iq\mu}^{A}\nabla^2_{iq\mu}+\tfrac{1}{2}\sum_{iq\mu\neq i'q'\mu'}^{A}\sum_{i'q'\mu'}^{A}V(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'})$$
$$+V_{\rm Coul}-T_{\rm c.m.}\ , \tag{7}$$

where

$$V(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'})$$

is the effective nucleon-nucleon potential, in general given by

$$V(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'})=\hat{V}(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'})$$
$$\times\{W-MP^{\sigma}P^{\tau}+BP^{\sigma}-HP^{\tau}\}\ , \tag{8}$$

or by the Volkov interaction[7], and $T_{\rm c.m.}$ is the kinetic energy of the center of mass, and $V_{\rm Coul}$ is the Coulomb potential.

In the Hartree-Fock (HF) GCM variational method, the total binding energy of the nuclear system is given by the variational

$$\delta\left\{\frac{H(\vec{R},b,\vec{R},b)}{N(\vec{R},b,\vec{R},b)}\right\}=0\ , \tag{9}$$

in which the position of the center of the well of the cluster $\vec{R}_i$ and the oscillator parameter b are the variational parameters.

A. Hole state and rule I

For clarity in stating the hole formulation, we shall specify the column and row subscripts of the wave function of the system in the form of a determinant.

Consider a standard nucleus of $4mn$ nucleons. The determinant is one with $4mn\times 4mn$ functions $\varphi^q_\mu(\vec{r}_{jr\nu}-\vec{R}_i)$, where the subscripts $jr\nu$ of $\vec{r}$ label column components and $iq\mu$ label row components. In arranging the determinant, we first consider different values of i (j), then different states q (r), and finally μ (ν).

If a nucleus has a hole in state $(iq\mu)$ before antisymmetrization with respect to the standard nucleus given beforehand, then the function $\varphi^q_\mu(\vec{r}-\vec{R}_i)$ in the standard nucleus should be replaced by $\xi^q_\mu(\vec{r}-\vec{R}_i)$, which is defined by overlapping

$$\xi^q_\mu(\vec{r}-\vec{R}_i)\equiv\xi^q(\vec{r}-\vec{R}_i)\chi_\mu\ , \tag{10}$$

$$\int d\vec{r}\,\xi^{q'}(\vec{r}-\vec{R}_i)\xi^{q'}(\vec{r}-\vec{R}'_{i'})\equiv\delta_{ii'}\delta_{qq'} \tag{11}$$

$$\int d\vec{r}\,\xi^{q*}(\vec{r}-\vec{R}_i)f(\vec{r}-\vec{R}'_{i'})$$
$$=\int d\vec{r}\,f^*(r-\vec{R}_i)\xi^q(\vec{r}-\vec{R}'_{i'})\equiv 0\ , \tag{12}$$

where $f(\vec{r}-\vec{R}_i)$ is any function other than $\xi^q(\vec{r}-\vec{R}_i)$, and χ_μ is the conventional spin-isospin wave function; strictly speaking, $f(\vec{r}-\vec{R}_i)$ occurring here is a kind of function as follows:

$$f(\vec{r}-\vec{R}_i)=\hat{\mathcal{O}}\varphi^q(\vec{r}-\vec{R}_i)$$
or
$$f(\vec{r}-R_i)=\hat{\mathcal{O}}\xi^q(\vec{r}-\vec{R}_i) \tag{13}$$

with $\hat{\mathcal{O}}$ being any operator.

Here we should point out emphatically that because of Eq. (12) $\xi^q(\vec{r}-\vec{R}_i)$ does not belong to the Hilbert space spanned by $\{\varphi^q(\vec{r}-\vec{R}_i)\}$ and is not the time-reversal of $\varphi^q(\vec{r}-\vec{R}_i)$; therefore it is different from the hole introduced by Deenen.[4]

Because we were only concerned with the overlap of functions in the course of the derivation, the definitions (11) and (12) for $\xi^q(\vec{r}-\vec{R}_i)$ are enough and the particular forms are not of importance.

Therefore, the wave function of a nucleus with A nucleons and h holes in hole formulation is

$$|\Psi_u(\vec{r},\vec{R})\rangle=\mathscr{A}\,|\Phi_u(\vec{r},\vec{R})\rangle=\mathscr{A}\left[\prod_{iq\mu}^{A}\varphi^q_\mu(\vec{r}_{iq\mu}-\vec{R}_i)\prod_{i'q'\mu'}^{h}\xi^{q'}_{\mu'}(\vec{r}_{i'q'\mu'}-\vec{R}'_{i'})\right]\ , \tag{14}$$

and $A+h=4mn$. The subscript u on Ψ indicates the hole formulation.

In the conventional formulation, the overlap kernel is thus easily written as

$$N(\vec{R},\vec{R}')\equiv\langle\Phi(\vec{r},\vec{R})\,|\,\Psi(\vec{r},\vec{R})\rangle$$
$$=\prod_{\mu=\alpha,\beta,\gamma,\delta}\det|B_\mu|\ , \tag{15}$$

where B_μ is a matrix resulting after summing over the spin-isospin coordinates μ, with elements

$$(B_\mu)_{iq,i'q'}=B_{iq,i'q'}$$
$$=\int d\vec{r}\,\varphi^{q*}(\vec{r}-\vec{R}_i)\varphi^{q'}(\vec{r}-\vec{R}'_i)\ . \tag{16}$$

The quasidiagonal form of Eq. (15) is due to the orthogonality of spin-isospin states.

Therefore, for the standard nucleus, the overlap kernel

$$N_s(\vec{R},\vec{R}')\equiv\langle\Phi_s(\vec{r},\vec{R})\,|\,\Psi_s(\vec{r},\vec{R}')\rangle=(\det|B^s|)^4\ . \tag{17}$$

Hereafter we call the B^s matrix of Eq. (17) the standard B matrix. The subscript s on B in Eq. (17) emphasizes that it is for the standard nucleus.

Actually, B^s has the same elements with respect to B_μ if the subscripts on the corresponding elements are the same; the only difference between them is the dimension. B^s has mn dimensions and B_μ only has $(mn-h_\mu)$ dimensions, where h_μ is the number of holes of the μ kind of nucleons with respect to the standard nucleus.

In hole formulation, we can similarly write the overlap kernel

$$N_u(\vec{R},\vec{R}') \equiv \langle \Phi_u(\vec{r},\vec{R}) \mid \Psi_u(\vec{r},\vec{R}') \rangle = \prod_{\mu=\alpha,\beta,\gamma,\delta} \det \mid B_\mu^u \mid , \tag{18}$$

with the elements of matrix B_μ^u

$$(B_\mu^u)_{iq,i'q'} = \begin{cases} B_{iq,i'q'} & \text{if } (iq\mu) \text{ is a nucleon state} \\ \delta_{iq,i'q'} & \text{if } (iq\mu) \text{ is a hole state} . \end{cases} \tag{19}$$

With Eq. (19) we have

$$N_u(\vec{R},\vec{R}') = N(\vec{R},\vec{R}') . \tag{20}$$

In summary, we have the following:

Rule (I). Taking the standard matrix B^s, if the $(iq\mu)$ state is a hole, put the nondiagonal elements in the (iq) column and the (iq) row into zero, and the diagonal element into one. The resultant matrix is nothing but the matrix B_μ^u in Eq. (18). By this rule the overlap kernel $N(\vec{R},\vec{R}')$ can be obtained from $N_s(\vec{R},\vec{R}')$, the one for the standard nucleus.

B. One body operator and rule II

We consider the kinetic energy operator as an example of the one body operator,

$$T = -\frac{\hbar^2}{2m} \left\langle \Phi \left| \sum_{jr\nu}^{A} \nabla_{jr\nu}^2 \right| \Psi \right\rangle . \tag{21}$$

Correspondingly, in hole formulation we have

$$T_u = -\frac{\hbar^2}{2m} \left\langle \Phi_u \left| \sum_{jr\nu}^{A+h} \nabla_{jr\nu}^2 \right| \Psi_u \right\rangle = T_u^A + T_u^h , \tag{22}$$

where the superscript $A(h)$ stands for the part which comes from

$$\sum_{jr\nu}^{A(h)} \nabla_{jr\nu}^2 .$$

Explicitly we have

$$T_u^h = N_u(\vec{R},\vec{R}') \sum_{jr\nu}^{h} \sum_{i,q}^{n,m} (t_\nu^u)_{jr,iq} (B_\nu^{u\,-1})_{iq,jr} = 0 \tag{23}$$

where

$$(t_\nu^u)_{jr,iq} = -\frac{\hbar^2}{2m} \int d\vec{r}\, \xi^{*r}(\vec{r}-\vec{R}_j) \nabla^2 \varphi^q(\vec{r}-\vec{R}_i') = 0$$

or

$$(t_\nu^u)_{jr,jr} = -\frac{\hbar^2}{2m} \int d\vec{r}\, \xi^{*r}(\vec{r}-\vec{R}_j) \nabla^2 \xi^r(\vec{r}-\vec{R}_j') = 0 \tag{24}$$

is due to the definition (12) and the elements of the inverse matrix $B_\nu^{u\,-1}$,

$$(B_\nu^{u\,-1})_{iq,jr} = \delta_{iq,jr} , \tag{25}$$

are from Eq. (19).

For the part T_u^A, we have

$$T_u^A = N_u(\vec{R},\vec{R}') \sum_{jr\nu}^{A} \sum_{i,q}^{n,m} (t_\nu^u)_{jr,iq} (B_\nu^{u\,-1})_{iq,jr} , \tag{23'}$$

where

$$(t_\nu^u)_{jr,iq} = -\frac{\hbar^2}{2m} \int d\vec{r}\, \varphi^{*r}(\vec{r}-\vec{R}_j) \nabla^2 \varphi^q(\vec{r}-\vec{R}_i') \equiv t_{jr,iq}$$

or (26a)

$$(t_\nu^u)_{jr,iq} = -\frac{\hbar^2}{2m} \int d\vec{r}\, \varphi^{*r}(\vec{r}-\vec{R}_j) \nabla^2 \xi^q(\vec{r}-\vec{R}_i') = 0 \tag{26b}$$

and

$$(B_\nu^{u\,-1})_{iq,jr} = (B_\nu^{-1})_{iq,jr}$$

or (27)

$$(B_\nu^{u\,-1})_{iq,jr} = 0$$

if the $(iq\nu)$ state is a nucleon state or a hole state, respectively. Equation (27) is obtained from Eq. (19) also.

From Eqs. (24)–(27), we have

$$T_u = T . \tag{28}$$

In order to make use of advantages of the hole formulation, we introduce a standard matrix t^s of mn dimensions with elements (26a) (in other words, it is nothing but one for the standard nucleus), and a matrix C_ν, which is defined by

$$(C_\nu)_{iq,jr} \equiv \begin{cases} 0 & \text{if } iq = jr \text{ and } (iq\mu) \text{ is a hole} \\ (B_\nu^{u\,-1})_{iq,jr} & \text{otherwise} , \end{cases} \tag{29}$$

with mn columns and rows.

In terms of t^s and C_ν, we finally get

$$T = N_u(\vec{R},\vec{R}') \sum_{jr,iq}^{nm} (t^s)_{jr,iq} \left[\sum_\nu (C_\nu)_{iq,jr} \right] . \tag{30}$$

In summary, we have the following:

Rule (II). Taking the matrix B_ν^u from B^s and carrying out the inverse $B_\nu^{u\,-1}$, if the $(iq\nu)$ state is a hole before antisymmetrization, put the diagonal element of the (iq) row of the inverse matrix $B_\nu^{u\,-1}$ into zero; the resultant matrix is nothing but the matrix C_ν. By this rule the kinetic energy kernel T can be obtained easily. We should point out that Eq. (29) is the backbone of our hole formulation, with which uniform expressions for T and V can be obtained.

C. Two body operator

The typical two body operator is the potential energy operator. Using the same arguments as in subsection B, we have

$$\left\langle \Phi \left| \sum_{iq\mu \neq jr\nu}^{A} V(\vec{r}_{iq\mu}-\vec{r}_{jr\nu}) \right| \Psi \right\rangle = \left\langle \Phi_u \left| \sum_{ir\mu \neq jr\nu}^{A+h} V(\vec{r}_{iq\mu}-\vec{r}_{jr\nu}) \right| \Psi_u \right\rangle . \tag{31}$$

We first consider the Wigner part of the nuclear potential; the kernel of this part is

$$V_u^W = \frac{W}{2} \left\langle \Phi_u \left| \sum_{iq\mu \neq i'q'\mu'}^{A+h} \hat{V}(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'}) \right| \Psi_u \right\rangle . \tag{32}$$

Since $\hat{V}$ operates on two different columns of $|\Psi_u\rangle$ only and the terms in the expression are all determinant, similar to the work of Horiuchi[8] we can formally set

$$\hat{V}(\vec{r}_{iq\mu}-\vec{r}_{i'q'\mu'}) \equiv \hat{\mathcal{O}}_{iq\mu} \hat{\mathcal{O}}_{i'q'\mu'} . \tag{33}$$

Then we can find

$$V_u^W = \frac{W}{2} N_u(\vec{R},\vec{R}') \left[\sum_{\substack{iq,i'q' \\ jr,j'r'}}^{nm} \left[\sum_{\mu \neq \nu} (U_{\mu\nu}^u)_{iq,i'q';jr,j'r'} (B_\mu^{u\,-1})_{i'q',iq} (B_\nu^{u\,-1})_{j'r',jr} + \sum_{\mu} (U_{\mu\mu}^u)_{iq,i'q',jr,j'r'} [(B_\mu^{u\,-1})_{i'q',iq} (B_\mu^{u\,-1})_{j'r',jr} - (B_\mu^{u\,-1})_{i'q',jr} (B_\mu^{u\,-1})_{j'r',iq}] \right] \right] , \tag{34}$$

with

$$(U_{\mu\nu}^u)_{iq,i'q';jr,j'r'} = \begin{cases} U_{iq,i'q';jr,j'r'} & \text{if } (iq\mu),\ (i'q'\mu),\ (jr\nu), \text{ and } (j'r'\nu) \text{ are all nucleons} \\ 0 & \text{otherwise} \end{cases} \tag{35}$$

and

$$U_{iq,i'q';jr,j'r'} = \int d\vec{r}_1 d\vec{r}_2 \varphi^{q*}(\vec{r}_1-\vec{R}_i) \varphi^{r*}(\vec{r}_2-\vec{R}_j) \hat{V}(\vec{r}_1-\vec{r}_2) \varphi^{q'}(\vec{r}_1-\vec{R}'_{i'}) \varphi^{r'}(\vec{r}_2-\vec{R}'_{j'}) . \tag{36}$$

Obviously, if the subscripts of U run over $(1,s)$ to (n,m), we have the standard matrix U^s with the elements

$$U^s_{iq,i'q';jr,j'r'} = U_{iq,i'q';jr,j'r'} .$$

In terms of U^s and C_ν, we finally have

$$V^W = V_u^W = \frac{W}{2} N_u(\vec{R},\vec{R}') \sum_{\substack{iq,i'q' \\ jr,j'r'}}^{nm} U^s_{iq,i'q';jr,j'r'} \left[\sum_{\mu \neq \nu} (C_\mu)_{i'q',iq} (C_\nu)_{j'r',jr} + \sum_{\mu} [(C_\mu)_{i'q',iq} (C_\mu)_{j'r',jr} - (C_\mu)_{i'q',jr} (C_\mu)_{j'r',iq}] \right] . \tag{37}$$

For the Bartlett term

$$V^B = V_u^B = \frac{B}{2} \left\langle \Phi_u \left| \sum_{iq\mu \neq jr\nu}^{A+h} \hat{V}(\vec{r}_{iq\mu}-\vec{r}_{jr\nu}) P^\sigma \right| \Psi_u \right\rangle , \tag{38}$$

we should note the following conditions:

(a) If $(\mu,\nu)=(\alpha,\gamma)$, $(\mu,\nu)=(\beta,\delta)$, or $\mu=\nu$, the operator P^σ does not result in any new contribution to the wave function $|\Psi_u\rangle$; in other words, P^σ is just a unit operator in this case.

(b) If $(\mu,\nu)=(\alpha,\beta)$ or $(\mu,\nu)=(\gamma,\delta)$, the operator P^σ will interchange α and β, and γ and δ in $|\Psi_u\rangle$.

We take ^{12}C as an example and consider the interaction $\hat{V}(\vec{r}_{2\alpha}-\vec{r}_{3\beta})P^\sigma$ only. Since ^{12}C is a $4n$ nucleus,

$$V^B(2\alpha,3\beta) = \frac{B}{2} \langle \Phi(^{12}\mathrm{C}) | \hat{V}(\vec{r}_{2\alpha}-\vec{r}_{3\beta}) P^\sigma | \Psi(^{12}\mathrm{C}) \rangle$$

$$= \frac{B}{2} \begin{vmatrix} B_{11} & 0 & B_{31} & 0 & 0 & \theta_{31} & & \\ B_{12} & 0 & B_{32} & 0 & 0 & \theta_{32} & 0 & 0 \\ B_{13} & 0 & B_{33} & 0 & 0 & \theta_{33} & & \\ 0 & \theta_{21} & 0 & B_{11} & B_{21} & 0 & & \\ 0 & \theta_{22} & 0 & B_{12} & B_{22} & 0 & 0 & 0 \\ 0 & \theta_{23} & 0 & B_{13} & B_{23} & 0 & & \\ & 0 & & & 0 & & B_\gamma & 0 \\ & 0 & & & 0 & & 0 & B_\delta \end{vmatrix} , \tag{39}$$

where

$$B_{ij}=\int d\vec{r}\,\varphi(\vec{r}-\vec{R}_i)\varphi(\vec{r}-\vec{R}'_j)\,, \tag{40}$$

$$\theta_{ij}\equiv\int d\vec{r}\,\varphi(\vec{r}-\vec{R}_i)\hat{\mathcal{O}}(\vec{r})\varphi(\vec{r}-\vec{R}'_j)\,, \tag{41}$$

and the symbol $\equiv$ indicates that Eq. (41) is only as formal as Eq. (33).

Permutating the 2α (second) column with the 3β (sixth) column, we obtain

$$V^B(2\alpha,3\beta)=-\det|B_\gamma|\det|B_\delta|\begin{vmatrix}B_{11}&\theta_{31}&B_{31}\\B_{12}&\theta_{32}&B_{32}\\B_{13}&\theta_{33}&B_{33}\end{vmatrix}\begin{vmatrix}B_{11}&B_{21}&\theta_{21}\\B_{12}&B_{22}&\theta_{22}\\B_{13}&B_{23}&\theta_{23}\end{vmatrix}\frac{B}{2}$$

$$=-\frac{B}{2}N(\vec{R},\vec{R}')\sum_{i,j}U_{2i,3j}(B^{-1})_{i3}(B^{-1})_{j2}\,, \tag{42}$$

$$\theta_{3j}\theta_{2i}\equiv U_{2i,3j}\,. \tag{33'}$$

In general,

$$V^B(iq\mu,jr\nu)=-\frac{B}{2}N(\vec{R},\vec{R}')\sum_{i'q',j'r'}(U_{\mu\nu})_{iq,i'q';jr,j'r'}(B_\mu^{-1})_{j'r',iq}(B_\nu^{-1})_{i'q',jr}\,. \tag{43}$$

It is found that the operator P^σ results in the exchange of state subscripts (i',q') and $(j'r')$ with an additional minus sign.

(c) If $(\mu,\nu)=(\alpha,\delta)$ or $(\mu,\nu)=(\beta,\gamma)$, then the result after the operation of P^σ is zero. We take ^{12}C as an example again:

$$V^B(2\alpha,3\delta)=\frac{B}{2}\langle\Phi(^{12}C)|\hat{V}(\vec{r}_{2\alpha}-\vec{r}_{3\delta})P^\sigma|\Psi(^{12}C)\rangle$$

$$=0\,. \tag{44}$$

For the Heisenberg operator P^τ, we have the following three conditions:

(a) If $(\mu,\nu)=(\alpha,\beta)$, $(\mu,\nu)=(\gamma,\delta)$, or $\mu=\nu$, the operator P^τ does not change the wave function $|\Psi_u\rangle$, just like a unit operator.

(b) If $(\mu,\nu)=(\alpha,\gamma)$ or $(\mu,\nu)=(\beta,\delta)$, the operator P^τ will interchange spin-isospin states α and γ, or β and δ in $|\Psi_u\rangle$, resulting in the exchange of the state subscripts $(i'q')$ and $(j'r')$ in the final results with an alternation of sign, just like in (b) of the Bartlett term.

(c) If $(\mu,\nu)=(\alpha,\delta)$ or $(\mu,\nu)=(\beta,\gamma)$, then the result after the operation of P^τ is zero.

For the Majorana operator $P^r=-P^\tau P^\sigma$, we have the following:

(a) For the case $(\mu,\nu)=(\alpha,\beta)$ or $(\mu,\nu)=(\gamma,\delta)$, the operation of $P^\tau P^\sigma$ gives the same effect as the operation of P^σ in case (b) above.

(b) For the case $(\mu,\nu)=(\alpha,\gamma)$ or $(\mu,\nu)=(\beta,\delta)$, the operation of $P^\tau P^\sigma$ gives the same effect as the operation of P^τ in case (b) above.

(c) For the case $(\mu,\nu)=(\alpha,\delta)$ or $(\mu,\nu)=(\beta,\gamma)$, the operation of $P^\tau P^\sigma$ behaves the same as case (b) of both P^τ and P^σ.

In short, for the cases $\mu\neq\nu$, the operation of $P^\tau P^\sigma$ results in the exchange of the state subscripts $(i'q')$ and $(j'r')$ with an alternation of sign, just like (b) of both P^τ and P^σ.

(d) For the case $\mu=\nu$, the operator $P^\tau P^\sigma$ does not change $|\Psi_u\rangle$, just like a unit operator.

Finally, for potential (8) in terms of U^s and C_μ, we obtain the following general expression:

$$V(\vec{R},\vec{R}')=\langle\Phi|V|\Psi\rangle$$

$$=\tfrac{1}{2}N_u(\vec{R},\vec{R}')\sum_{\substack{iq,i'q'\\jr,j'r'}}^{mn}U^s_{iq,i'q';jr,j'r'}((W+B-H-M)\sum_\mu[(C_\mu)_{i'q',iq}(C_\mu)_{j'r',jr}-(C_\mu)_{i'q',jr}(C_\mu)_{j'r',iq}]$$

$$+\sum_{\mu\neq\nu}[W(C_\mu)_{i'q',iq}(C_\nu)_{j'r',jr}+M(C_\mu)_{j'r',iq}(C_\nu)_{i'q',jr}]$$

$$+2\{B[(C_\alpha)_{i'q',iq}(C_\gamma)_{j'r',jr}+(C_\beta)_{i'q',iq}(C_\delta)_{j'r',jr}$$

$$-(C_\alpha)_{j'r',iq}(C_\beta)_{i'q',jr}-(C_\gamma)_{j'r',iq}(C_\delta)_{i'q',jr}]$$

$$-H[(C_\alpha)_{i'q',iq}(C_\beta)_{j'r',jr}+(C_\gamma)_{i'q',iq}(C_\delta)_{j'r',jr}$$

$$-(C_\alpha)_{j'r',iq}(C_\gamma)_{i'q',jr}-(C_\beta)_{j'r',iq}(C_\delta)_{i'q',jr}]\}) \ . \tag{45}$$

III. CLUSTER STRUCTURE IN LIGHT NUCLEI

As the first example of application of the formulae for the kernels, we investigate the 1-p shell light nuclei. As mentioned in Sec. II, for this region we can take ^{16}O as the standard nucleus to construct the standard matrix B^s,t^s,U^s. Then, according to the nucleus we want to investigate, we assume a special configuration, that is, specify some states being occupied by protons or neutrons, some being empty or holes before antisymmetrization. Next, taking rules (I) and (II) into account, we obtain the matrix B^u_μ and C_μ.

In this paper, we assume the 1-s shell state is the only state that can be available in every cluster, so that at least four clusters are needed for a nucleus heavier than ^{12}C. It is worth pointing out that if a nucleus under consideration is assumed to be in a configuration with some clusters completely empty, that is, with the cluster number n less than four, we still treat it as a four cluster structure with some hole clusters. Therefore in the hole formulation we can systematically treat 1-p and 1-s shell nuclei with one, two, three, and four cluster structures in a single computing code. The configurations of the particular nucleus are treated as input data of the code.

In the Hartree-Fock GCM formulation, Eq. (9) has seven variational parameters for four cluster configurations. The seven variational parameters consist of one intrinsic parameter b, the harmonic oscillator, and six geometrical parameters, S_1, S_2, θ, S_3, ψ, and φ, which are the Jacobi coordinates of the centers of the potential well as shown in Fig. 1,

$$\vec{S}_1=\vec{R}_2-\vec{R}_1 \ ,$$

$$\vec{S}_2=\vec{R}_3-\frac{n_1\vec{R}_1+n_2\vec{R}_2}{n_1+n_2} \ ,$$

$$\vec{S}_3=\vec{R}_4-\frac{n_1\vec{R}_1+n_2\vec{R}_2+n_3\vec{R}_3}{n_1+n_2+n_3} \ ,$$

$$0=\sum_{i=1}^{4} n_i\vec{R}_i \ ,$$

where n_i and $\vec{R}_i$ are the numbers of nucleons and coordinates of the well in the ith cluster, respectively. $\vec{S}_1$ is taken parallel to the axis Z, $\vec{S}_2$ parallel to the x-z plane so that θ is the angle of $\vec{S}_2$ with respect to the z axis, and $\vec{S}_3$ has the polar angles ψ and φ with modS_3.

We take the Volkov No. 1 force as the effective two-body nuclear force[7]:

$$\hat{V}(\vec{r}_1-\vec{r}_2)=\{-83.34\exp[-\beta_1(\vec{r}_1-\vec{r}_2)^2]+144.86\exp[-\beta_2(\vec{r}_1-\vec{r}_2)^2]\}(0.44-0.56P^\sigma P^\tau) \tag{46}$$

in MeV, with $\beta_1=0.39062$ fm^{-2} and $\beta_2=1.48721$ fm^{-2}.

The resulting binding energies and configuration parameters of the wave function for the nuclei with assumed cluster partitions are shown in the following subsections.

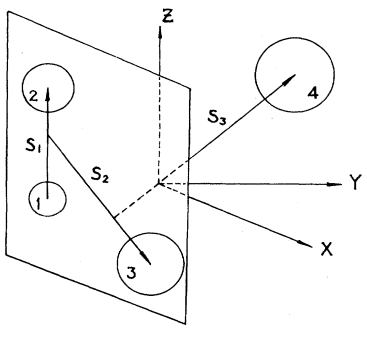

FIG. 1. The Jacobi coordinates for a four cluster configuration.

A. ^{6}Li

As shown in Table I, three cases are treated, $\alpha+d$, ^{3}He + T, and $\alpha+p\uparrow+n\uparrow$, where each term means a cluster ordered in a series of $i=1,2,3$ from left to right, and the arrow $\uparrow$ ($\downarrow$) means spin upwards (downwards). It is interesting to notice that $\alpha+d$ and $\alpha+p\uparrow+n\uparrow$ both manifest themselves as being the same thing. In the $\alpha+p\uparrow+n\uparrow$ configuration, in spite of the fact that the proton and neutron have been artificially separated into two clusters, the spacial structure of this configuration with minimum energy still exhibits a spacial configuration in which the proton cluster and neutron cluster are close to each other to recombine to a deuteron subsystem. A crude estimate of the rms radius of the cluster $\bar{R}=\langle r^2\rangle^{1/2}\sim 1.5^{1/2}b$. Here $b\sim 1.6$ fm as shown in Table I; then $\bar{R}\sim 1.9$ fm for each cluster. The distance between the centers of the proton and neutron clusters is about 0.3 fm. Therefore in the deuteron subsystem, the distributions of the proton and neutron are almost overlapping. The radius of the deuteron subsystem is 1.9 fm + 0.15 fm$\simeq$2 fm,

TABLE I. ^{6}Li. The resulting binding energy and configuration from the HF GCM calculation. The experimental binding energy of the ground state is -31.994 MeV (Ref. 9).

Configuration of clusters		$\alpha+d$	$^3\mathrm{He}+T$	$\alpha+p\uparrow+n\uparrow$
S_1 (fm)		1.53	0.937	1.85
S_2 (fm)				0.927
θ (rad)				-0.5×10^{-5}
b (fm)		1.587	1.620	1.591
$\left.\frac{\langle\Psi\mid H\mid\Psi\rangle}{\langle\Psi\mid\Psi\rangle}\right\vert_0$	(MeV)	-16.099	-13.45	-16.20
Projected	$L=0$	-19.99	-16.76	-19.84
	1	-2.72	-8.02	-2.34
	2	-17.93	-14.72	-17.80
	3	-1.59	-11.38	-1.85
	4	-8.59	-16.44	-9.09

which is somewhat less than the one of the free deuteron. This means that the deuteron cluster is more tightly bound than the free deuteron.

This example gives us an indication that the partitions of clusters may be chosen somewhat artificially, but some of them are equivalent. Later ^{8}Be will be given as another example. It brings us some benefits. First, it shows that the nucleus in the cluster model has its own cluster structure which is somewhat independent of the cluster partition given beforehand. Second, as it is known, Volkov force is well determined from ^{4}He data, but it is not appropriate for describing the deuteron, so now we may find an alternative or a comparison for the deuteron cluster.

In order to get more information, we have carried out the orbital momentum projection

$$|\Psi_L\rangle=[(2L+1)/8\pi^2]\int d\omega\,\mathscr{D}^L_{00}(\omega)\hat{R}(\omega)\,|\Psi\rangle\ .$$

The excitation energies of odd parity $L=1,3$ states are far from those of even parity $L=0,2,4$ states, which is in qualitative agreement with experiments. The quantitative discrepancy is expectable. For higher excitation, the simple projection could not describe the properties of these states well enough. Other effects must be taken into account, such as the variations of geometry shape (here the distances between the clusters) or of the well parameter with respect to the low lying states; that is, we must solve the Hill-Wheeler equation for these states.

From Table I, we find that the lowest state of the $^3\mathrm{He}+T$ cluster structure in ^{6}Li is at -16.76 MeV with orbital momentum $L=0$, which is 3.2 MeV above the corresponding one of $\alpha+d$ structure. Since we did not introduce spin-orbit coupling (the introduction of which is straightforward) in these calculations, naturally we can assume that the experimental 3.563 MeV (Ref. 9) (with $J^\pi=0^+$) state of ^{6}Li is identified as this state and the 5.37 MeV (2^+) is identified as the $L=2$ state of $^3\mathrm{He}+T$ [three states located in this region, the 5.65 MeV (1^+), the 4.31 MeV (2^+), and the 2.185 MeV (3^+), are identified as the $L=2$ state of $\alpha+d$, because of the consideration of isospin].

But we should point out that because these cluster structures have close energies, the interference between them is not negligible. Therefore the above identification only reflects the main character of these states.

TABLE II. ^{7}Li. The resulting binding energy and configuration from the HF GCM calculation. The experimental binding energy at the ground state is -39.2465 MeV (Ref. 9).

Configuration of clusters		$\alpha+T$	$\alpha+d\uparrow+n\uparrow$	$\alpha+d\uparrow+n\downarrow$
S_1 (fm)		2.246	1.382	2.296
S_2 (fm)			0.4635	1.793
θ (rad)			3.025	1.744
b (fm)		1.520	1.693	1.637
$\left.\frac{\langle\Psi\mid H\mid\Psi\rangle}{\langle\Psi\mid\Psi\rangle}\right\vert_0$	(MeV)	-25.27	-12.10	-11.49
Projected	$L=0$	-13.86	-4.37	-6.01
	1	-30.89	-14.795	-18.168
	2	-11.79	-0.11	-5.22
	3	-9.70	-9.69	-16.67
	4	-6.48	-0.02	-3.74

B. ^{7}Li

Three cluster structures, $\alpha+T$, $\alpha+d\uparrow+n\uparrow$, and $\alpha+d\uparrow+n\downarrow$, have been assumed for low lying states. But from Table II, we find that there is a big gap in energy which separates the spectrum of $\alpha+T$ far from the others. Therefore we think that at least below 12 MeV excitation energy the $\alpha+T$ structure is dominant.

The lowest state is -30.89 MeV with orbital momen-

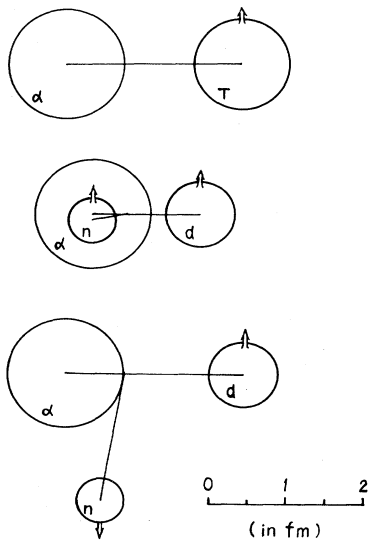

FIG. 2. The spacial structures for ^{7}Li in $\alpha+T$, $\alpha+d\uparrow+n\uparrow$, and $\alpha+d\uparrow+n\downarrow$ configurations. The sizes of the circles do not indicate the real sizes of the clusters and are only used to reflect the number of nucleons included schematically.

TABLE III. ^{8}Be. The resulting binding energy and configuration from the HF GCM calculation. The experimental binding energy of the ground state is -54.499 MeV (Ref. 9).

Configuration of clusters		$\alpha+\alpha$	$\alpha+d\uparrow+d\downarrow$	$\alpha+d\uparrow+d\uparrow$
S_1 (fm)		2.854	3.047	1.385
S_2 (fm)			1.556	1.438
θ (rad)			-0.5×10^{-3}	1.847
b (fm)		1.413	1.430	1.715
$\left.\frac{\langle\Psi\mid H\mid\Psi\rangle}{\langle\Psi\mid\Psi\rangle}\right\vert_0$	(MeV)	-44.97	-41.12	-10.55
Projected	$L=0$	-50.765	-47.465	-16.106
	1		-15.013	-0.058
	2	-48.086	-44.843	-13.478
	3		-11.962	-0.030
	4	-40.361	-37.318	-12.492

tum $L=1$ of $\alpha+T$ structure. Since the triton has intrinsic spin and parity $\frac{1}{2}^+$, the calculated lowest state should be the two states of $J^\pi=\frac{3}{2}^-$ and $\frac{1}{2}^-$, which are the ground state ($\frac{3}{2}^-$) and the first excited state [0.4776 MeV ($\frac{1}{2}^-$)] from experiment.[9]

In the following energy region we can reasonably assume that the ^{5}He + d structure is the predominant one, because in $\alpha+d\uparrow+n\uparrow$ structure α and neutron clusters automatically recombine to a ^{5}He subsystem, shown in Fig. 2.

Although the $\alpha+d\uparrow+n\uparrow$ and $\alpha+d\uparrow+n\downarrow$ structures appear to be different in these calculations, if we consider channel spin or total spin as a good quantum number and take spin-orbit coupling into account, we will have both structures mixed.

C. ^{8}Be

Like in the ^{6}Li case, the structures of $\alpha+\alpha$ and $\alpha+d\uparrow+d\downarrow$ give similar results. Two deuteron clusters

TABLE IV. 1-p shell nuclei. The resulting energies and configurations from the HF GCM calculation.

Nucleus	Configuration	b (fm)	S_1 (fm)	S_2 (fm)	θ (rad)	S_3 (fm)	ψ (rad)	φ (rad)	$\left.\frac{\langle H\rangle}{\langle N\rangle}\right\vert_0$ (MeV)
^{16}O	$\alpha+\alpha+\alpha+\alpha$	1.363	1.473	1.276	1.5708	1.203	1.5708	1.5708	-137.4
^{15}O	$\alpha+\alpha+\alpha+^3$He	1.400	1.563	1.354	1.5695	1.157	1.5671	1.5694	-104.2
^{15}N	$\alpha+\alpha+\alpha+T$	1.403	1.463	1.266	1.5710	0.970	1.5716	1.5714	-111.2
^{14}O	$\alpha+\alpha+^3$He$\uparrow+^3$He$\downarrow$	1.443	1.768	1.422	1.5708	1.153	1.5708	0.9203	-76.8
^{14}C	$\alpha+\alpha+T\uparrow+T\downarrow$	1.439	1.553	1.144	1.5650	0.878	1.5650	0.6226	-91.5
^{14}N	$\alpha+\alpha+^3$He$\uparrow+T\downarrow$	1.439	1.672	1.318	1.5708	0.979	1.5708	0.7632	-84.6
^{14}O	$\alpha+\alpha+\alpha+(p\uparrow p\downarrow)$	1.426	1.722	1.492	1.5708	1.200	1.5708	1.5708	-81.5
^{14}C	$\alpha+\alpha+\alpha+(n\uparrow n\downarrow)$	1.433	1.410	1.221	1.5708	0.697	1.5708	1.5708	-96.1
^{14}N	$\alpha+\alpha+\alpha+d$	1.429	1.611	1.395	1.5708	0.978	1.5708	1.5708	-88.2
^{13}C	$\alpha+\alpha+\alpha+n$	1.437	1.655	1.433	1.5708	0.733	1.5708	1.5708	-81.17
^{13}N	$\alpha+\alpha+\alpha+p$	1.431	1.854	1.606	1.5708	1.094	1.5708	1.5708	-74.84
^{12}B	$\alpha+\alpha+T\uparrow+n\uparrow$	1.502	1.556	1.133	1.377	0.778	0.4216	2.7856	-59.01
^{12}C	$\alpha+\alpha+T\uparrow+p\uparrow$	1.499	1.778	1.236	1.5708	1.114	1.5708	1.5935	-53.11
^{11}C	$\alpha+\alpha+d\uparrow+p\downarrow$	1.476	2.091	1.372	1.5708	1.740	1.5708	-0.2×10^{-4}	-49.58
^{11}B	$\alpha+\alpha+d\uparrow+n\downarrow$	1.471	1.980	1.415	1.5708	1.049	1.5708	0.7×10^{-5}	-56.05
^{11}C	$\alpha+\alpha+d\uparrow+p\uparrow$	1.543	1.980	1.211	1.5708	1.241	1.5708	1.702	-36.49
^{11}B	$\alpha+\alpha+d\uparrow+n\uparrow$	1.543	1.779	1.142	1.353	0.869	0.2854	2.6969	-41.97
^{10}Be	$\alpha+\alpha+n\uparrow+n\uparrow$	1.552	1.882	0.943	2.9906	1.046	-0.1265	1.4489	-38.2
^{10}B	$\alpha+\alpha+p\uparrow+n\uparrow$	1.499	2.187	1.381	1.5708	1.047	1.5708	0.1×10^{-4}	-41.76
^{10}Be	$\alpha+\alpha+n\uparrow+n\downarrow$	1.512	1.967	0.897	2.9681	0.876	2.9879	0.4×10^{-2}	-45.80
^{10}B	$\alpha+\alpha+p\uparrow+n\downarrow$	1.499	2.1870	1.381	1.5708	1.047	1.5708	-0.9×10^{-6}	-41.76
^{8}B	$\alpha+d\uparrow+p\uparrow+p\uparrow$	1.844	1.495	1.807	1.8704	1.827	1.8238	1.8125	-6.57
^{8}B	$\alpha+d\uparrow+p\uparrow+p\downarrow$	1.635	2.431	1.521	1.9844	0.990	-0.1213	0.4×10^{-4}	-13.07
^{8}Be	$\alpha+d\uparrow+n\downarrow+p\uparrow$	1.620	1.810	1.035	0.0666	1.379	1.8831	0.0758	-18.29
^{8}Li	$\alpha+d\uparrow+n\uparrow+n\downarrow$	1.616	1.626	0.886	-0.3×10^{-2}	0.669	3.1066	-0.200	-22.36
^{8}Be	$\alpha+d\uparrow+p\uparrow+n\uparrow$	1.718	1.368	1.368	1.8404	0.779	2.0142	0.5×10^{-5}	-10.03
^{8}B	$\alpha+d\uparrow+p\downarrow+p\downarrow$	1.620	1.979	1.550	0.8267	1.641	0.8447	3.1410	-13.29
^{14}O	$\alpha+\alpha+^3$He$\uparrow+^3$He$\uparrow$	1.457	1.636	1.301	1.5708	1.286	1.5708	1.6682	-71.22
^{14}C	$\alpha+\alpha+T\uparrow+T\uparrow$	1.450	1.422	1.009	1.5708	0.976	1.5708	1.5915	-83.95
^{9}Be	$\alpha+\alpha+n$	1.484	2.410	1.040	1.5680				-40.14
^{9}B	$\alpha+\alpha+p$	1.482	2.590	1.310	1.5708				-36.78
^{10}B	$\alpha+d+\alpha$	1.499	1.703	1.889	1.0976				-41.69
^{12}C	$\alpha+\alpha+\alpha$	1.403	2.101	1.820	1.5708				-77.08

recombine to form an α subsystem only having a distance of separation of 0.5 fm in spacial structure. Furthermore, the energy spectrums of the two structures are the same, and the energy difference between $L=2$ and $L=0$ of $\alpha+\alpha$ is 2.679 MeV and that of $\alpha+d\uparrow+d\downarrow$ is 2.66 MeV. The energy difference between $L=4$ and $L=0$ of $\alpha+\alpha$ is 10.404 MeV and that of $\alpha+d\uparrow+d\downarrow$ is 10.088 MeV (refer to Table III). Unlike the ^{6}Li case, the total binding energies of the two structures have a discrepancy of 3.3 MeV. We attribute this to the Volkov force which we have adopted. In spite of this, we still assume these two structures are equivalent just as in ^{6}Li.

Because the Pauli principle makes the $\alpha+d\uparrow+d\uparrow$ structure move to a higher energy region and have a different spacial structure from $\alpha+\alpha$ and $\alpha+d\uparrow+d\downarrow$, it is reasonable to regard it as the dominant configuration in the higher energy region. According to this, a new cluster structure state may be observed in the background of the continuum of near 30 MeV excitation energy.[10]

D. Nuclei heavier than ^{8}Be

In Table IV, we list the results of nuclei from ^{9}Be to ^{16}O (19 nuclei with 33 cluster structures). The binding energies are in agreement with experiment,[9] especially in tendency. We should point out that if only the relative values are concerned, the agreement is more satisfying. For example, the difference in binding energies between ^{9}Be and ^{9}B has the theoretical value of 3.35 MeV in comparison with the experimental value of 1.85 MeV, and that between ^{9}Be and ^{12}C is 36.94 MeV (theory) in comparison with 34.01 MeV (experimental), and so on.

The above calculations are rudimentary, because we only take them as illustrative examples for the hole formulation. These calculations can be regarded as references for further investigation.

IV. Λ HYPERNUCLEI IN THE CLUSTER MODEL

We have carried out the GCM calculation of Λ hypernuclei. The procedures are the same as described above except for a new ingredient, the Λ particle. This new ingredient brings in two things to do, first, to choose the proper Λ-N interaction and second, to derive the new terms in kernels related to the Λ particle.

We still adopt a Gaussian-type form factor as the form factor of the Λ-N interaction. There have been several kinds of Gaussian-type Λ-N interactions[11] in usual shell model calculations. But none of these gave rise to satisfying results for Λ-particle binding energies, $_\Lambda$BE. Taking $^{13}_\Lambda$C as an example, it is 28.32 MeV in the paper by Bassichis and Gal and 21.48 (17.76) with a zero (0.5) Majorana term of Λ-N interaction by Ho and Volkov, which are all far from the experimental value of 11.69±0.12 MeV. For $^{17}_\Lambda$O, a result of 42 MeV is given in Ref. 11, which is much larger than 22.7 MeV, the upper limit of the $60<A<100$ region of nuclei.[12] Considering that $_\Lambda$BE increases with mass number A, these results are hardly right.

Considering that these Λ-N interactions come from fitting Λ-p low energy scattering data or $_\Lambda$BE of $^5_\Lambda$He as the authors of Ref. 11 declared, we think these Λ-N interactions are dominated by the s wave component and that the p component is underestimated. But for those nuclei mentioned above, the p component of the interaction plays an important role. So the large deviations from experiment occurring in the papers of Ref. 11 are not surprising. Therefore we must find some ways to emphasize the p wave component in the Λ-N interaction if we want to describe the 1-p shell nucleus region with it.

First, considering that there is broken SU_3 symmetry between the N-N and Λ-N forces and that the p component of the N-N potential is weaker than the s one, we suggest that the Λ-N force has similar behavior so that the Λ-N potential has a shortened force range or a weakened strength. Therefore the first way is to shorten the attractive force range from certain kinds of N-N or Λ-N forces which are available. Here we have taken Volkov force No. 1 of the N-N interaction as a starting point from which, through fitting the Λ-particle binding energies of ^{9}Be and ^{13}C, the first set of Λ-N force for the 1-p shell region is obtained.

Second, we take into account the fact that the Λ-N interaction arises from 2π and K meson exchange as the leading term, which gives us some information on the force ranges. Directly, we take these ranges as the attractive and repulsive force range, respectively, then adjust the attractive strength to fit the binding energies of $^9_\Lambda$Be and $^{13}_\Lambda$C. In this way the second set of Λ-N force for the 1-p shell is fixed.

These two obtained sets of Λ-N force of Gaussian type are shown as follows:

$$V^{(\mathrm{I})}_{\Lambda N}(r)=\{-85.8\exp[-(r/1.12\ \mathrm{fm})^2]+145\exp[-(r/0.85\ \mathrm{fm})^2]\}(0.44+0.56P^r)\ , \tag{47}$$

$$V^{(\mathrm{II})}_{\Lambda N}(r)=\{-63\exp[-(r/1.05\ \mathrm{fm})^2]+144.86\exp[-(r/0.60\ \mathrm{fm})^2]\}(0.8+0.2P^r) \tag{48}$$

(in MeV), where P^r is the Majorana operator.

According to the general definition of the Λ-particle binding energy, $_\Lambda$BE,

$$-{}_\Lambda\mathrm{BE}=\mathrm{BE}(^{A+1}_\Lambda\mathrm{X}_g)-\mathrm{BE}(^A\mathrm{X}_g)\ , \tag{49}$$

where $\mathrm{BE}(^A\mathrm{X}_g)$ stands for the total binding energy of nucleus ^{A}X in its ground state, the subscript Λ indicates the hypernucleus, and

$$\mathrm{BE}(^{A+1}_\Lambda\mathrm{X}_g)=\left.\frac{\langle\Psi_\Lambda|H_\Lambda|\Psi_\Lambda\rangle}{\langle\Psi_\Lambda|\Psi_\Lambda\rangle}\right|_0\ , \tag{50}$$

where the subscript zero denotes the HF GCM minimum. The Hamiltonian is given by

$$H_\Lambda=H-\frac{\hbar^2}{2m_\Lambda}\nabla^2_\Lambda+\sum_{iq\mu}V_{\Lambda N}(\vec{r}_{iq\mu}-\vec{r}_\Lambda)\ , \tag{51}$$

and the wave function

$$|\Psi_\Lambda\rangle = |\Psi\rangle\varphi(\vec{r}_\Lambda - \vec{R}_l) ,$$

with H and $|\Psi\rangle$ given in Eq. (7) and Eq. (2), respectively, and $\varphi(\vec{r}_\Lambda - \vec{R}_l)$ being the Λ-particle wave function with the same harmonic oscillator parameter b as nucleons in the hypernucleus (the consistency assumption), $\vec{R}_l$ being the coordinate of the center of the potential well which the Λ particle is engaged in.

In terms of the hole formulation described in the previous section, the kernels which have included the new terms relative to the Λ particle are given by

$$T_\Lambda(\vec{R},\vec{R}') = T(\vec{R},\vec{R}') + \frac{3\hbar^2}{4m_\Lambda b^{1/2}} N(\vec{R},\vec{R}') , \tag{52}$$

$$V_\Lambda(\vec{R},\vec{R}') = V(\vec{R},\vec{R}') + N(\vec{R},\vec{R}') \sum_{\mu,iq,jr}^{A+h} (C_\mu)_{iq,jr} (W_\Lambda U^s_{iq,jr;l,l} + M_\Lambda U^s_{iq,l;l,jr}) , \tag{53}$$

$$N_\Lambda(\vec{R},\vec{R}') = N(\vec{R},\vec{R}') . \tag{54}$$

Still the Volkov force No. 1 is adopted for the N-N interaction, and the results for 1-p shell hypernuclei are shown in Tables V and VI. Good agreement is obtained in the whole region.[12] It is interesting that the agreement is far better than that of the total binding energies. Moreover, it is surprising that the results reproduce not only the general tendency of the Λ-particle binding energy with mass numbers, but also the details.

($^{12}_{\Lambda}$C,$^{12}_{\Lambda}$B) and ($^{8}_{\Lambda}$Be,$^{8}_{\Lambda}$Li) belong to isobaric doublets, there are large splittings in BE. Some ascribe this to the charge symmetry breaking of the Λ-N interaction,[12] because the Coulomb force gives the opposite contribution. In our calculations, we have used the Λ-N force with charge symmetry; still, the results obtained are in good agreement with experiment as shown in Table V. Therefore we suggest that the large splitting in $_{\Lambda}$BE of ($^{12}_{\Lambda}$C,$^{12}_{\Lambda}$B) could be due not only to the charge asymmetry, but could also come from a sort of "consistency effect" in the total system. Because of the repulsive effect of the Coulomb force, the distances between the clusters in $^{12}_{\Lambda}$C should be larger than those in $^{12}_{\Lambda}$B, but the sizes of the clusters in $^{12}_{\Lambda}$C are smaller than in $^{12}_{\Lambda}$B (refer to Table VI); this is due to the N-N and Λ-N interactions to increase overlapping of the clusters and minimize the energy, that is, the "consistency effect" named above. Possibly it could be used to reconcile the charge symmetry of spin-spin splitting in the Λ-N interaction mentioned by some authors.[12]

TABLE V. The Λ-particle binding energies of 1-p shell hypernuclei.

Nuclear core	(^{A}X)	Configuration	BE($^{A+1}_{\Lambda}$X)$^{(I)}_g$	$_{\Lambda}$BE$^{(I)}$	BE($^{A+1}_{\Lambda}$X)$^{(II)}_g$	$_{\Lambda}$BE$^{(II)}$	$_{\Lambda}$BE(exp)
^{16}O		$\alpha+\alpha+\alpha+\alpha$	−151.107	13.741	−158.179	20.813	
^{15}O		$\alpha+\alpha+\alpha+^3$He	−116.928	12.681	−122.359	18.122	
^{15}N		$\alpha+\alpha+\alpha+^3T$	−124.698	13.454	−130.239	18.995	
^{14}O		$\alpha+\alpha+\alpha+(2p)$	−92.925	11.407	−97.022	14.504	
^{14}N		$\alpha+\alpha+\alpha+d$	−100.391	12.187	−104.549	16.345	13.59±0.15
^{13}N		$\alpha+\alpha+\alpha+p$	−85.486	10.644	−88.577	13.735	
^{12}C		$\alpha+\alpha+\alpha$	−86.487	9.407	−88.926	11.846	11.69±0.12
^{11}C		$\alpha+\alpha+^3$He	−61.401	9.287	−62.580	10.466	10.76±0.19
^{11}B		$\alpha+\alpha+T$	−66.314	11.023	−67.183	11.892	11.37±0.06
^{10}B		$\alpha+\alpha+d$	−50.926	10.223	−51.114	10.411	10.24±0.05
^{9}B		$\alpha+\alpha+p$	−44.459	7.679	−44.075	7.295	8.89±0.12
^{8}B		$\alpha+^3$He$+p$	−24.060	8.713	−22.358	7.011	7.88±0.15
^{8}Be		$\alpha+\alpha$	−52.117	7.144	−51.210	6.237	6.71±0.04
^{7}Be		$\alpha+^3$He	−31.802	8.500	−29.743	6.441	6.84±0.05
^{6}Be		$\alpha+(2p)$	−22.493	8.757	−19.679	5.943	5.16±0.08
^{7}Li		$\alpha+T$	−29.888	4.616	−29.947	4.675	6.80±0.03
^{6}Li		$\alpha+d$	−21.182	5.083	−20.405	4.306	5.58±0.03
^{5}He		$\alpha+n$	−24.120	5.499	−22.786	4.165	4.18±0.11
^{3}He		^{3}He	−8.240	2.330	−5.773	−0.137	2.39±0.03
^{4}He		α	−27.347	0.203	−28.444	1.300	3.12±0.02

TABLE VI. The configuration parameters for isobaric doublets $^{12}_{\Lambda}$C and $^{12}_{\Lambda}$B from the HF GCM calculation.

Configuration of $^{12}_{\Lambda}$C	b (fm)	S_1 (fm)	S_2 (fm)	θ (deg)
(I) Λ-N force	1.471	1.648	1.316	100°46′
(II) Λ-N force	1.437	1.651	1.312	81°9′43″
Nuclear core of ^{11}C	1.465	2.140	1.587	89°59′58″
Configuration of $^{12}_{\Lambda}$B	b (fm)	S_1 (fm)	S_2 (fm)	θ (deg)
(I) Λ-N force	1.476	1.504	1.085	107°2′9″
(II) Λ-N force	1.439	1.505	1.084	75°59′12″
Nuclear core of ^{11}B	1.470	1.987	1.392	90°0′22″

V. CONCLUSION

In this paper, a systematic and general approach to finding the kernels of the GCM has been given. As seen from Secs. III and IV, this approach is powerful and compact. With the kernels obtained in the hole formulation, the 1-p shell nuclei, Λ hypernuclei with a one, two, three, or four cluster configuration, and the Λ-N interaction for this nucleus region have been investigated with a single computer code. As already mentioned, this was almost impossible in the cluster model before. Roughly speaking, with the usual procedure it may take several months to find kernels for a single nucleus, say for ^{14}C, and to check whether the kernels obtained are correct. So it is no doubt that our formulation will save a great deal of time.

The course of deriving the kernels of the GCM presented here is almost a conventional routine without any complicated and laborious mathematics, and rather easy to manage with.

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