Dirac theory of nucleon-nucleus collective excitation

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We obtain the relativistic distorted wave impulse approximation amplitude for nucleon-nucleus collective transitions using an eikonal formalism. The effect of the Darwin term is to suppress the upper-to-upper contribution and enhance the lower-to-lower compared with a nonrelativistic treatment. In collective transitions these combine in a way that results in a transition density proportional to the derivative of the total Dirac distortion including quadratic and spin-orbit pieces. This leads to the same phenomenologically successful relations between elastic and inelastic amplitudes as in the Schrödinger description for collective transitions and promises a better treatment of noncollective transitions. We comment on the vanishing of $P-A$ in this treatment.

I. INTRODUCTION

Extensive phenomenological studies have shown that a relativistic approach to nucleon-nucleus elastic scattering gives an equally good description of the cross section as a nonrelativistic approach, and a far superior description of the spin dependent observables.[1] Recently it was shown that the phenomenological interaction strengths are close to what would be expected from the impulse approximation in the context of the Dirac equation,[2] and further, that good parameter-free descriptions of cross sections, analyzing powers, and spin rotation parameters are obtainable from a relativistic impulse approximation description.[3]

In this paper we undertake to examine the implications of the Dirac equation for inelastic scattering. For simplicity, we consider the excitation of collective transitions which only connect upper-to-upper and lower-to-lower components of the projectile wave function. Using an eikonal formalism we show that the effect of the Darwin term is to suppress the upper-to-upper and enhance the lower-to-lower contributions to the transition. For a collective transition in which the transition strength is the same as the two-body interaction responsible for elastic scattering and in which the transition density is of the Tassie type (powers of r times the derivative of the density), these enhancement and suppression factors combine in an intriguing way. One finds that in the two-component reduction, the transition density is proportional to the derivative of the full equivalent distortion potential, including the spin-orbit and quadratic density terms. Thus, just as the reduction of the elastic scattering from a four component to an equivalent two component equation generates these additional nonlinear potential terms, so too, the reduction of the collective transition amplitude introduces the corresponding terms in the transition density.

These results depend critically on assuming that the transition interaction is the same as the distorting interaction in elastic scattering. In noncollective transitions, this may not be the case. Upper-to-upper and lower-to-lower transitions may enter with different strengths than in elastic scattering. In that case the Darwin suppression or enhancement factors will become important. For other classes of transitions, mixed terms (e.g., upper-to-lower) will enter further, thus enriching the situation. It is well known that the treatment of such transitions based on the Schrödinger equation encounters difficulties which we anticipate the relativistic treatment will overcome.

For collective transitions, the Schrödinger approach with a Tassie or Bohr-Mottleson model transition density is normally very successful so long as a formulation is used that also fits elastic scattering. We see here that the agreement will continue to hold in the relativistic treatment, but again with emphasis on the close connection between the elastic and inelastic interactions. In particular, the data-to-data formulae[5] relating inelastic cross sections, asymmetries, and spin rotations derived using the Schrödinger starting point and employed with such great phenomenological success[6] remain valid in the relativistic treatment. In fact, they gain in credentials since the entire treatment of spin observables is on much better footing in the Dirac approach.

Our results add further to the success of the Dirac equation applied to nucleon-nucleus interactions. They show that for the class of transitions where the Schrödinger based description has been successful, use of the Dirac equation will not alter this success. Rather, the Dirac based approach adds to it by putting the phenomenology on a firmer footing. However, for noncollective transitions difficult to describe with a Schrödinger equation, use of the Dirac equation may well give different, presumably improved, results. In the next section we derive the expression for the transition density in the framework of a relativistic distorted wave impulse approximation (DWIA). We define a relativistic generalization of the collective transition interaction and use relativistic eikonal distorted waves.[4] We show how the Darwin term suppresses the upper-to-upper transition, while enhancing the lower-to-lower, and show how these combine for a collective transition leading to a transition density proportional to the derivative of the full distortion. In the final section we discuss the expected sizes of the suppression factor for upper-to-upper transitions, the enhancement for lower-to-lower transitions, and the correction terms for collective transitions. Our results are then summarized briefly.

II. THEORY

We wish to calculate the transition amplitude for a collective transition excited by an inelastic proton-nucleus scattering using the Dirac equation. The interaction potential of the proton is given by $V_v+\beta V_s$ in the Dirac equation, where V_v and V_s are the vector and scalar parts of the potential and β is the usual Dirac matrix. We assume that a collective transition is one in which the transition density is proportional to the derivative of this interaction

$$V_t=\frac{1}{r}\frac{d}{dr}(V_v+\beta V_s)\equiv V_v'+\beta V_s' \ . \tag{1}$$

This is the natural and simplest extension of the Tassie form to the relativistic formalism (we suppress factors of $r^L Y_{LM}$ that are essential to the details but irrelevant to our argument here).

The transition amplitude in DWIA is given by the expectation value of this transition density (1) between distorted waves. Since β is block diagonal, this expectation value has a particularly simple form. Consider the transition from a proton initial state of asymptotic momentum $\vec{k}$ and spin s to a final state $\vec{k}'s'$. The amplitude is

$$\begin{aligned}\langle \vec{k}'s' \mid V_t \mid \vec{k}s \rangle &= \int u^{\dagger}_{\vec{k}',s'}(\vec{r})(V_v'+V_s')u_{\vec{k},s}(\vec{r})d^3r \\ &\quad + \int w^{\dagger}_{\vec{k},s'}(\vec{r})(V_v'-V_s')w_{\vec{k},s}(\vec{r})d^3r \\ &\equiv A_1+A_2 \ ,\end{aligned} \tag{2}$$

where u and w are the upper and lower components of the distorted wave. They satisfy

$$w=\frac{1}{E+M+V_s-V_v}(\vec{\sigma}\cdot\vec{P})u \ , \tag{3a}$$

$$\left[\vec{\sigma}\cdot\vec{P}\frac{1}{E+M+V_s-V_v}\vec{\sigma}\cdot\vec{P}-(E-M-V_s-V_v)\right]u=0 \ . \tag{3b}$$

Using (3a), A_2 of (2) can be written in terms of the u's,

$$\begin{aligned}A_2&=\int w^{\dagger}_{\vec{k}',s'}(V_v'-V_s')w_{\vec{k},s}d^3r \\ &=\int u^{\dagger}_{\vec{k}',s'}(\vec{\sigma}\cdot\vec{P})\frac{1}{E+M+V_s-V_v}(V_v'-V_s') \\ &\quad \times\frac{1}{E+M+V_s-V_v}(\vec{\sigma}\cdot\vec{P})u_{\vec{k},s}d^3r \ ,\end{aligned} \tag{4}$$

which can be further simplified using (3b) to yield

$$\begin{aligned}A_2&=\int u^{\dagger}_{\vec{k},s'}(V_v'-V_s')\frac{E-M-V_s-V_v}{E+M+V_s-V_v}u_{\vec{k},s}d^3r \\ &\quad -\tfrac{1}{2}\int u^{\dagger}_{\vec{k}',s'}\left[\vec{\sigma}\cdot\vec{P},\frac{1}{E+M+V_s-V_v}\left[\vec{\sigma}\cdot\vec{P},\frac{1}{E+M+V_s-V_r}(V_v'-V_s')\right]\right]u_{\vec{k},s}d^3r \\ &\equiv A_2^{(1)}+A_2^{(2)} \ .\end{aligned} \tag{5}$$

To make further progress in simplifying (5) or A_1, we need a form for the distorted wave spinor. We take this from an eikonal treatment, which gives

$$u_{\vec{k},s}(\vec{r})=e^{i\vec{k}\cdot\vec{r}}\exp\left[-i(M/k)\int_{-\infty}^{z}[V_c(r')+V_{so}(r')(\vec{\sigma}\cdot\vec{r}'\times\vec{k}-ikz')]dz'\right] , \tag{6}$$

where z is along $\vec{k}$ and where the central potential V_c is given by

$$V_c(r)=V_s(r)+\frac{E}{M}V_v(r)+\frac{V_s^2-V_v^2}{2M} \ , \tag{7}$$

and the spin orbit V_{so} potential by

$$V_{so}(r)=\frac{1}{2M(E+M+V_s-V_v)}\frac{1}{r}\frac{d}{dr}[V_v(r)-V_s(r)]=\frac{1}{2M(E+M+V_s-V_v)}(V_v'-V_s') \ . \tag{8}$$

We substitute (6) and the corresponding form for $u^*_{k's'}$ in A_1 (taking care of the effect of the boundary conditions on the integration limits and noting that $V_{\text{in}}=V^*_{\text{out}}$) to yield (suppressing spin indices)

$$\begin{aligned}A_1=\int d^3r\, e^{-i\vec{k}'\cdot\vec{r}}&\exp\left[-i(M/k)\int_z^{\infty}dz'[V_c+V_{so}(\vec{\sigma}\cdot\vec{r}'\times\vec{k}+ikz')]\right] \\ &\times(V_v'+V_s')e^{i\vec{k}\cdot\vec{r}}\exp\left[-i(M/k)\int_{-\infty}^{z}dz'[V_c+V_{so}(\vec{\sigma}\cdot\vec{r}'\times\vec{k}-ikz')]\right] ,\end{aligned} \tag{9}$$

where we have made the usual eikonal assumption that $\vec{k}$ and $\vec{k}'$ are nearly colinear and have the same magnitude. Using the fact that $V_{\rm so}$ is even in z, (9) can be simplified to give

$$A_1=\int d^3r\, e^{i\vec{q}\cdot\vec{r}+i\chi}\exp\left[2M\int_z^\infty z'dz'V_{\rm so}(V_v'+V_s')\right], \tag{10}$$

where $\vec{k}-\vec{k}'=\vec{q}$,

$$\chi=-\frac{M}{k}\int_{-\infty}^{\infty}dz'[V_c+V_{\rm so}\vec{\sigma}\cdot\vec{r}'\times\vec{k}]\ ,$$

and where the last exponential form comes from the Darwin term in (9). Using (8), the definition of $V_{\rm so}$, this last exponential term simplifies remarkably. It can be written

$$\begin{aligned}\exp\left[2M\int_z^\infty z'V_{\rm so}dz'\right]&=\exp\left[\int_z^\infty dz'\frac{1}{E+M+V_s-V_v}\frac{d}{dz'}(V_v-V_s)\right]\\&=\exp-\left[\int_z^\infty dz'\frac{d}{dz'}\ln(E+M+V_s-V_v)\right]=\exp\left[\ln\left[\frac{E+M+V_s-V_v}{E+M}\right]\right]\\&=\frac{E+M+V_s-V_v}{E+M}\end{aligned} \tag{11}$$

where in the second step we used

$$\frac{1}{r}\frac{d}{dr}=\frac{1}{z}\frac{d}{dz}$$

for functions of r and in the next to the last step used V_s and $V_v\to 0$, $z\to\infty$. Using (11), (10) becomes

$$A_1=\int d^3r\, e^{i\vec{q}\cdot\vec{r}+i\chi}\left[\frac{E+M+V_s-V_v}{E+M}\right](V_s'+V_v')\ . \tag{12}$$

For an attractive potential, $(E+M+V_s-V_v)<(E+M)$, and the factor

$$\frac{E+M+V_s-V_v}{E+M}$$

coming from the Darwin term in the distortion *reduces* A_1 over its naive or nonrelativistic amplitude. If we defined a collective transition as one that only connected upper component spinors, this reduction would be valid. As we shall see for our more natural definition of transition density, the reduction in the upper components is exactly compensated for by the lower components. Alternately, in an example where the dynamics dictate a transition density that is all in the upper components, this Darwin term suppression can be important.

To see what happens to the contribution from the lower components, we return to the first term of (5), $A_2^{(1)}$. Since none of the steps involved in going from (10) to (12) depended on the detailed form of the transition density, other than its being spin independent, we can use the results of these steps to write (for eikonal distortions and again suppressing spin indices)

$$A_2^{(1)}=\int d^3\, e^{i\vec{q}\cdot\vec{r}+i\chi}\left[\frac{E-M-V_v-V_s}{E+M}\right](V_v'-V_s')\ . \tag{13}$$

Combining (12) and (13), we have

$$A_1+A_2^{(1)}=\int d^3r\, e^{i\vec{q}\cdot\vec{r}+i\chi}\frac{2M}{E+M}\left[\frac{1}{r}\frac{d}{dr}V_c(r)\right] \tag{14}$$

with V_c the central potential defined in (7). It should be noted that this remarkable result expresses a major part of the transition amplitude in terms of the derivative of the equivalent Schrödinger central potential with no Darwin suppression or enhancement factors.

All that remains is the spin-orbit contribution. This is in $A_2^{(2)}$. It can be written

$$A_2^{(2)}=-M\int d^3r\, u^\dagger_{\vec{k}',s'}\left[\vec{\sigma}\cdot\vec{P},\frac{1}{E+M+V_s-V_v}[\vec{\sigma}\cdot\vec{P},V_{\rm so}]\right]u_{\vec{k},s}\ , \tag{15}$$

using (5) and (8). Working out the commutators gives

$$\begin{aligned}A_2^{(2)}&=2M\int d^3r\, u^\dagger_{\vec{k}',s'}\frac{1}{E+M+V_s-V_v}\frac{1}{b}\frac{d}{db}[V_{\rm so}(r)\vec{\sigma}\cdot\vec{b}\times\vec{k}]u_{\vec{k},s}\\&\quad+2M\int d^3r\, u^\dagger_{\vec{k}',s'}\left[\frac{1}{E+M+V_s-V_v}\frac{1}{r}\frac{d}{dr}V_{\rm so}+\frac{1}{2}\frac{d}{dr}\left[\frac{1}{E+M+V_s-V_v}\frac{dV_{\rm so}}{dr}\right]\right.\\&\qquad\left.-\frac{1}{E+M+V_s-V_v}V_{\rm so}\frac{1}{b}\vec{\sigma}\cdot\vec{b}\times\vec{k}\right]u_{\vec{k},s}\\&\equiv A_{2,\rm so}^{(2)}+A_{2\rm corr}^{(2)}\ ,\end{aligned} \tag{16}$$

where we have introduced the vector $\vec{b}$ orthogonal to $\vec{k}$, and used

$$\frac{1}{r}\frac{d}{dr}=\frac{1}{b}\frac{d}{db}$$

on functions of r only. We have also used the leading eikonal approximation $\vec{p}u_{\vec{k},s}=\vec{k}u_{\vec{k},s}$. The first term in (16) is just what we need; the second contains a number of correction terms that are small essentially because $V_{so} \ll V_v$ or V_s. The only thing that normally makes V_{so} important is that it is multiplied by $\vec{\sigma}\cdot\vec{L}$ and L is of order 50. Neglecting the second term in (16) and using the eikonal distortion techniques of (10)–(12) on the rest (all spin factors are $\vec{\sigma}\cdot\vec{b}\times\vec{k}$ so they all commute) we obtain

$$A_{2,so}^{(2)}=\frac{2M}{E+M}\int d^3r\, e^{i\vec{q}\cdot\vec{r}+i\chi}\frac{1}{b}\frac{d}{db}[V_{so}(r)\vec{\sigma}\cdot\vec{b}\times\vec{k}] \,. \tag{17}$$

Combining all the pieces we have

$$\begin{aligned}A_1+A_2 &\cong A_1+A_2^{(1)}+A_{2,so}^{(2)}\\ &=\frac{2M}{E+M}\int e^{i\vec{q}\cdot\vec{b}}e^{i\chi}\frac{1}{b}\frac{d}{db}(V_c+V_{so}\vec{\sigma}\cdot\vec{b}\times\vec{k})d^3r \,,\end{aligned} \tag{18}$$

where we used the fact that $\vec{q}\cdot\vec{k}\simeq 0$. We can perform the z integration to write (18) in terms of χ of (10)

$$A_1+A_2\simeq\frac{-2k}{E+M}\int e^{i\vec{q}\cdot\vec{b}}e^{i\chi}\frac{1}{b}\frac{d\chi}{db}d^2b \,, \tag{19}$$

or equivalently,

$$A_1+A_2\simeq\frac{2ik}{E+M}\int e^{i\vec{q}\cdot\vec{b}}\left[\frac{1}{b}\frac{d}{db}e^{i\chi}\right]d^2b \,. \tag{20}$$

Thus, the effect of the simple collective transition density (1) in the relativistic picture is to give an equivalent Schrödinger transition amplitude in which the transition is driven by the derivative of the *full* distortion, including quadratic density pieces and spin-orbit coupling, and all without any Darwin term suppression or enhancement. This remarkable result revalidates the data-to-data methods and indicates that their phenomenological success is not diminished as one switches from a Schrödinger- to a Dirac-based approach. The relativistic treatment seems to change things for the better when required but does not spoil the previous nonrelativistic successes.

Neglecting shifts in minima due to powers of r in the transition density, which are treated in detail by Amado *et al.*,[5] we recover the following specific results for the large q limit:

1. The spin dependent and spin independent amplitudes are each increased by a power of q.

2. The cross section will go like $(qb)^2$ multiplied by the elastic cross section.

3. For the asymmetry polarization and spin rotation parameters, the powers of q will cancel—these will be the same as for elastic scattering.

4. As a consequence, the asymmetry and polarization will be equal as in elastic scattering, or $P-A$ will be zero. The analogous combination involving the spin rotation parameters $D_{LS}+D_{SL}$ will also be zero. It can be shown that $P-A=D_{LS}+D_{SL}=0$ does not depend on any large q assumption, but follows simply from the fact that there are not terms in the distortion (i.e., in elastic scattering) which would allow $P-A\neq 0$.

III. DISCUSSION

We have seen that in a relativistic treatment of the distorted wave impulse approximation for collective transitions the distortion factors combine to yield a transition density proportional to the derivative of the full Dirac distortion, including quadratic density and spin orbit terms. This arises from a subtle interplay of the upper-to-upper and lower-to-lower contributions. Separately, the effect of the Darwin term in the distortion is to suppress the former and enhance the latter. In a noncollective transition which may not combine so simply, these suppressions and enhancements may be significant. To examine their effect we rewrite the transition in Eqs. (12) and (13), as, for A_1,

$$\frac{E+M+V_s-V_v}{E+M}(V_v'+V_s')=\left[1+\frac{V_s-V_v}{E+M}\right](V_v'+V_s') \,, \tag{21}$$

and for A_2,

$$\begin{aligned}\frac{E-M-V_s-V_v}{E+M}(V_v'-V_s') &=\left[1-\frac{V_v+V_s}{E-M}\right]\\ &\quad\times\left[\frac{E-M}{E+M}\right](V_v'-V_s') \,.\end{aligned} \tag{22}$$

The factors immediately to the right of the equal sign in (21) and (22) are the Darwin distortion factors over what one would "naively" expect. Because $V_s-V_v<0$ the factor is a suppression in A_1, but because $V_v+V_s<0$ the factor is an enhancement in A_2. For typical potential strengths and on E corresponding to 500 MeV kinetic energy, one finds

$$\frac{V_s-V_v}{E+M}\sim[-0.22+i(0.07)];$$

$$\frac{V_v+V_s}{E-M}\sim[-0.24-i(0.02)] \,.$$

Both ratios increase at lower energy, but remain nearly equal. The fact that these numbers are roughly equal reflects the well-known fact that V_v and V_s nearly cancel. This makes the remaining factors in (21) and (22) also nearly equal and warns against the view that at 500 MeV the upper-to-upper contribution should be much larger than the lower-to-lower. It is not the large size of V_s and V_v separately, but their near cancellation that makes the relativistic approach both subtle and necessary.

In obtaining our results, Eq. (20), we have neglected the correction terms in (16). As is clear, these are proportional to the spin-orbit force but *without* the factor of L, the angular momentum. The spin orbit potential above is some 100 times smaller than the central potential and is only important because L values get so large. Hence, terms in V_{so} without L are negligible.

In our discussion we also have not addressed the $r^L Y_{LM}$ part of the Tassie form; such factors will just be carried into the final answer Eq. (20) and will only matter in the double commutation of (15). Its presence then will lead to terms in the derivative of r^2 rather than of the potentials; such terms are down by qr. Terms of this order have been discussed previously and are seen to be small.

In conclusion, we have seen that a relativistic treatment of collective excitation recovers the phenomenologically successful relationship among Schrödinger amplitudes—data-to-data formulae—but with better credentials and added emphasis on the need to consider elastic and inelastic processes, particularly spin dependence, together.

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