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Theory for structures in the fusion cross sections and an application to $^{12}C+^{28,29,30}Si$

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In this work, we present a theory involving bound states embedded in the continuum in order to explain the observed structures in heavy ion fusion data. As opposed to the continuum-continuum coupling considered in the standard coupled channel calculations, we examine here the effects of interaction of bound states among themselves as well as coupling of the continuum channels with these interacting bound states. By making reasonable approximations within the context of bound states embedded in the continuum, we show that the S matrix for each partial wave can be expressed as a sum of a smooth background term and a resonance term. For the case of a single bound state only, the resonant part of the S matrix is shown to reduce to the Breit-Wigner form. The background part of the S matrix is calculated by using a parabolic barrier in the presence of the Coulomb interaction. The theory is then applied to analyze the fusion excitation functions for the reactions $^{12}C+^{28,29,30}Si$ and is found to account for the structures in the data quite well.

[NUCLEAR REACTIONS Heavy ion fusion; coupled channel calculation; bound states embedded in the continua; resonant structures; application to $^{12}C+^{28,29,30}Si$ systems.]

I. INTRODUCTION

The fusion cross section between two nuclei exhibit in many cases structures superimposed on a smooth background. In recent works,[1,2] we have been able to account for the smooth background for a wide range of data as transmission through a barrier consisting of a parabola joined smoothly to the Coulomb interaction and using a formalism based on the theory of Feshbach, Peaslee, and Weisskopf[3] with a boundary condition which is embedded in a Kapur-Peierls[4] type of R-matrix theory. Whereas other models, such as those of Glas and Mosel,[5] Dethier and Stancu,[6] Avishai,[7] and Bass[8] can also account for the smooth background part of the fusion cross section usually in a limited energy range, our treatment[1,2] is valid at all energies and has been successful in explaining a wide range of data pertinent to the smooth background. Thus, while the smooth energy dependence of the fusion cross section is well understood, attempts to understand the observed structures in the data have just begun.[9–13]

The purpose of this paper is to develop a simple theoretical framework to incorporate resonances in the calculation of fusion cross sections and apply it to the fusion of ^{12}C by $^{28,29,30}Si$. To this end, we adopt the coupled channel approach of Mustafa and Malik[14] which is connected to the theory of Feshbach[15] but avoids the explicit use of projection operators and is based on Trefftz's[16] treatment of dielectronic recombination. From the treatment of Ref. 14, it is evident that the key to resonance in the scattering amplitude is the interaction between continuum and bound states embedded in the continuum. Within the framework of a suitable approximation emphasizing this aspect, we show that the S matrix for each partial wave can be written as a sum of a term depicting scattering from an average potential and terms containing resonances. This work, therefore, also supports the approach of Lee, Wilschut, and Ledoux,[13] and Lee, Chu, and Kuo.[12] We finally apply the theory to the fusion of ^{12}C by $^{28,29,30}Si$, using widths and energy shifts as parameters.

II. THEORY

A. The coupled equations

Let the total Hamiltonian H of two heavy ions be expressed in terms of their relative and intrinsic coordinates as

$$H=-\frac{\hbar^2}{2\mu}\nabla^2_{\vec{r}}+H_0(1,2,\ldots,A)+H_0(1',2',\ldots,A')+H_{\rm int}(\vec{r};1,2,\ldots,A;1',2',\ldots,A')\ , \qquad (1)$$

where $\vec{r}$ represents the relative coordinate between the two ions and the sets $(1,2,\ldots,A)$ and $(1',2',\ldots,A')$ are the intrinsic coordinates of each ion with respect to their individual centers of mass. μ is the reduced mass of the system. The total wave function Ψ of the system in a given state can be expanded in terms of a function $f_n(\vec{r})$ and a product of the individual intrinsic function ϕ's forming a complete set,

$$\Psi=\sum_{n,n'} f_n(\vec{r})\Phi_{n'} \,, \tag{2}$$

with

$$\Phi_{n'}\equiv\phi_\alpha(1,2,\ldots,A)\phi_\beta(1',2',\ldots,A') \,. \tag{3}$$

We can then obtain a set of equations of the following type using the resonating group approach,[17]

$$\left[-\frac{\hbar^2}{2\mu}\nabla^2_{\vec{r}}-(E-\epsilon_n)+V_{nn}(\vec{r})+K_{nn}\right]f_n(\vec{r}) =-\sum_{n'\neq n}[V_{nn'}(\vec{r})+K_{nn'}]f_{n'}(\vec{r}) \,, \tag{4}$$

where

$$V_{nn'}(\vec{r})=\langle\Phi_n,H_{\rm int}\Phi_{n'}\rangle \,, \tag{5}$$

and $K_{nn'}$ is a nonlocal potential of the type

$$K_{nn'}f_{n'}(\vec{r})\equiv\int d\vec{r}\,'K_{nn'}(\vec{r},\vec{r}\,')f_{n'}(\vec{r}\,') \,, \tag{6}$$

originating from the Pauli exclusion principle. E is the total center of mass energy of the system and ϵ_n is the sum of the two eigenenergies of intrinsic states given by (3).

For further analysis, we shall assume $V(\vec{r})$ to be spherically symmetric and avoid explicit consideration of K, although it can be incorporated in our treatment. Writing the radial part of f_n as $f_n(r)=\chi_n(r)/r$, we obtain the following coupled equations for $\chi_n(r)$:

$$\left[\frac{d^2}{dr^2}-\frac{l_\lambda(l_\lambda+1)}{r^2}+k_\lambda^2-W_{\lambda\lambda}(r)\right]\chi_\lambda(r)=\sum_b W_{\lambda b}(r)\chi_b(r)+\sum_{c\neq\lambda}W_{\lambda c}(r)\chi_c(r) \tag{7}$$

and

$$\left[\frac{d^2}{dr^2}-\frac{l_b(l_b+1)}{r^2}+k_b^2-W_{bb}(r)\right]\chi_b(r)=W_{b\lambda}(r)\chi_\lambda(r)+\sum_{b'\neq b}W_{bb'}(r)\chi_{b'}(r)+\sum_{c\neq\lambda}W_{bc}(r)\chi_c(r) \,. \tag{8}$$

In the above equations, we have explicitly separated the incoming channel λ (which is also the elastic channel), the other continuum channels c, and the bound-state—like states ($b=1,2,\ldots,N$) in order to facilitate subsequent analysis. k_n and $W_{nn'}$ ($n,n'=b,c$) are defined as

$$k_n^2=(2\mu/\hbar^2)(E-\epsilon_n) \,, \tag{9}$$

$$W_{nn'}=2\mu V_{nn'}/\hbar^2 \,. \tag{10}$$

In the usual coupled channel calculations, the first term on the right hand side of Eq. (7) is set equal to zero [and hence Eq. (8) is neglected] and as discussed in many works including Ref. 14, this does not always explain resonancelike structures in the continuum channels. Our aim here is to explore the other alternative, i.e., set $W_{\lambda c}=0$ for all $c\neq\lambda$ and examine the ramifications of coupling the continuum channels, in particular the elastic channel λ, to the bound states.

B. Bound state solutions

We may construct a solution of the bound state wave function, χ_b, of Eq. (8) by expanding χ_b in terms of an orthonormal set,[16]

$$\chi_b=\sum_n a_{nb}\chi_{nb} \,, \tag{11}$$

with

$$\int\chi_{nb}(r)\chi_{n'b'}(r)dr=\delta_{nn'}\delta_{bb'} \,. \tag{12}$$

The wave functions χ_{nb} are solutions of the homogeneous part of Eq. (8), i.e., solutions of Eq. (8) with the right-hand side equal to zero. In principle, the summation includes an integration over the continuum states (q). In that case,

$$\int\chi_{qb}(r)\chi_{q'b}(r)dr=\delta(q-q') \,.$$

However, the presence of such a term does not affect the calculations and we shall not consider this explicitly any further.

Coefficients a_{nb} satisfy the equation

$$(k_b^2-k_{nb}^2)a_{nb}=\int\chi_{nb}(r)\sum_{b'\neq b}W_{bb'}(r)\chi_{b'}(r)dr+\int\chi_{nb}(r)W_{b\lambda}(r)\chi_\lambda(r)dr+\int\chi_{nb}(r)\sum_{c\neq\lambda}W_{bc}(r)\chi_c(r)dr \,. \tag{13}$$

From Eq. (13), it is clear that a_{nb} exhibits resonant behavior when k_b^2 is close to k_{nb}^2. Near such resonances it is reasonable to approximate χ_b as

$$\chi_b(r)\simeq a_{nb}\chi_{nb}(r), \quad b=1,2,\ldots,N \,, \tag{14}$$

because the other amplitudes are very small compared to the resonant ones. It is to be noted that this is not a case of an isolated resonance but a series of successive resonances.

C. Continuum channel solutions

Following Mott and Massey,[18] we write the elastic channel solution χ_λ, given by Eq. (7), and other continuum channel solutions χ_c, given by a similar equation and defined by appropriate asymptotic forms, in terms of integral equations. Such integral equations for the continuum wave functions are ($c=\lambda$ is the elastic channel)

$$\chi_c(r)=\chi_c^{(0)}\delta_{\lambda c}+\int G_c(r,r')\sum_b W_{cb}(r')\chi_b(r')dr' , \quad (15)$$

where

$$G_c(r,r')=-\frac{1}{k_c}[\chi_c^{(0)}(r_<)\chi_c^{(1)}(r_>)+i\chi_c^{(0)}(r_<)\chi_c^{(0)}(r_>)] . \quad (16)$$

$\chi_c^{(0)}$ and $\chi_c^{(1)}$ are, respectively, the regular and irregular solutions of the homogeneous part of Eq. (7) having the asymptotic forms

$$\chi_c^{(0)}\sim\sin(k_c r-\pi l_c/2-\eta_c\ln 2k_c r+\sigma_c+\delta_c) , \quad (17)$$

$$\chi_c^{(1)}\sim\cos(k_c r-\pi l_c/2-\eta_c\ln 2k_c r+\sigma_c+\delta_c) . \quad (18)$$

Here,

$$\eta_c=\mu Z_1 Z_2 e^2/(\hbar^2 k_c) ,$$

$Z_1 e$ and $Z_2 e$ being the charges of the two ions. σ_c and δ_c are, respectively, the Coulomb and non-Coulomb phase shifts.

Substituting Eqs. (14)—(16) into Eq. (13), we get

$$\sum_{b'}\Bigg\{(k_{b'}^2-k_{nb'}^2)\delta_{bb'}-\int\chi_{nb}(r)W_{bb'}(r)\chi_{nb'}(r)dr$$
$$-\int\int\chi_{nb}(r)\Big[W_{b\lambda}(r)\mathrm{Re}\,G_\lambda(r,r')W_{\lambda b'}(r')+\sum_{c\neq\lambda}W_{bc}(r)\mathrm{Re}\,G_c(r,r')W_{cb'}(r')\Big]\chi_{nb'}(r')dr\,dr'$$
$$-i\int\int\chi_{nb}(r)\Big[W_{b\lambda}(r)\mathrm{Im}\,G_\lambda(r,r')W_{\lambda b'}(r')+\sum_{c\neq\lambda}W_{bc}(r)\mathrm{Im}\,G_c(r,r')W_{cb'}(r')\Big]\chi_{nb'}(r')dr\,dr'\Bigg\}a_{nb'}$$
$$=\int\chi_{nb}(r)W_{b\lambda}(r)\chi_\lambda^{(0)}(r)dr . \quad (19)$$

The second term on the left-hand side of Eq. (19) will be absent if $b=b'$ (one bound state only). The above equation can be written as

$$\sum_{b'}\left[(E-E_{nb'})\delta_{bb'}+\Delta E_{nbb'}+\frac{i}{2}\Gamma_{bb'}\right]a_{nb'}=\gamma_{b\lambda} , \quad (20)$$

where the total width is defined as

$$\Gamma_{bb'}=\frac{4}{\hbar v_\lambda}\gamma_{b\lambda}\gamma_{b'\lambda}+\sum_{c\neq\lambda}\frac{4}{\hbar v_c}\gamma_{bc}\gamma_{b'c} \quad (21)$$

and

$$\gamma_{bj}=\int\chi_{nb}(r)V_{bj}(r)\chi_j^{(0)}(r)dr \quad (j=\lambda,c) . \quad (22)$$

The energy shift, $\Delta E_{nbb'}$, is given by

$$\Delta E_{nbb'}=-\int\chi_{nb}(r)V_{bb'}(r)\chi_{nb'}(r)dr+\frac{2}{\hbar v_\lambda}\int\int\chi_{nb}(r)V_{b\lambda}(r)\chi_\lambda^{(0)}(r_<)\chi_\lambda^{(1)}(r_>)V_{\lambda b'}(r')\chi_{nb'}(r')dr\,dr'$$
$$+\sum_{c\neq\lambda}\frac{2}{\hbar v_c}\int\int\chi_{nb}(r)V_{bc}(r)\chi_c^{(0)}(r_<)\chi_c^{(1)}(r_>)V_{cb'}(r')\chi_{nb'}(r')dr\,dr' . \quad (23)$$

The first term in Eq. (23) represents the shift due to interaction between bound states only and the last two terms between a bound state and a series of continuum states.

D. S matrix

To obtain the S matrix elements, we rewrite Eq. (7) as

$$\left[\frac{d^2}{dr^2}-\frac{l_\lambda(l_\lambda+1)}{r^2}+k_\lambda^2-W_{\lambda\lambda}^{\mathrm{Coul}}(r)\right]\chi_\lambda(r)$$
$$=W_{\lambda\lambda}^{\mathrm{nucl}}(r)\chi_\lambda(r)+\sum_b W_{\lambda b}(r)\chi_b(r) , \quad (24)$$

where we have separated $W_{\lambda\lambda}(r)$ into an appropriate Coulomb part and a nuclear part, i.e.,

$$W_{\lambda\lambda}(r)=W_{\lambda\lambda}^{\mathrm{Coul}}(r)+W_{\lambda\lambda}^{\mathrm{nucl}}(r) .$$

The solution of this equation, satisfying the asymptotic condition

$$\chi_c(r)\to I_c\delta_{\lambda c}-S_{\lambda c}O_c , \quad (25)$$

with

$$I_c=\exp[-i(k_c r-\pi l_c/2-\eta_c\ln 2k_c r+\sigma_c)] , \quad (26)$$

$$O_c = \exp[i(k_c r - \pi l_c/2 - \eta_c \ln 2k_c r + \sigma_c)] , \tag{27}$$

can be written in the same spirit as that of Eq. (15), and is given by

$$\chi_c(r) = u_c^{(0)}\delta_{\lambda c} + \int \mathcal{G}_c(r,r')F(r')dr' , \tag{28}$$

where

$$F(r') = W_{\lambda\lambda}^{\text{nucl}}(r')\chi_\lambda(r') + \sum_b W_{\lambda b}(r')\chi_b(r') \tag{29}$$

and

$$\mathcal{G}_c(r,r') = -\frac{1}{k_c}[u_c^{(0)}(r_<)u_c^{(1)}(r_>) + iu_c^{(0)}(r_<)u_c^{(0)}(r_>)] . \tag{30}$$

$u_c^{(0)}$ and $u_c^{(1)}$ are the regular and irregular solutions of Eq. (24) with the right-hand side set to zero. They have the same asymptotic forms as $\chi_c^{(0)}$ and $\chi_c^{(1)}$ of Eqs. (17) and (18), respectively, but with $\delta_c = 0$.

Writing out explicitly the asymptotic form of solution (28) and comparing it with Eq. (25), we get

$$S_{\lambda c} = \delta_{\lambda c} - \frac{2i}{k_c}\int dr\, u_c^{(0)}(r)W_{cc}^{\text{nucl}}(r)\chi_c(r)dr - \frac{2i}{k_c}\int dr\, u_c^{(0)}(r)\sum_b W_{cb}(r)\chi_b(r)dr . \tag{31}$$

The above equation clearly shows that in the absence of continuum-continuum coupling, the S matrix can be written as the sum of a term containing potential scattering and a term having resonances. The first two terms in Eq. (31) contain the contribution to the S matrix from the diagonal part of the potential in the continuum channel and is termed the background part, $S_{\lambda c}^{(B)}$. It is responsible for the smooth part of the S matrix originating from the scattering potential W_{cc} and exhibits only resonances due to various partial waves. The last term in Eq. (31) is responsible for other types of resonances since the bound state solutions, χ_b, are formed by solving Eq. (20) for a_{nb} which exhibits a resonant behavior. These resonances are of the Breit-Wigner type originating from bound states embedded in the continuum and are denoted by $S_{\lambda c}^{(R)}$.

III. APPLICATION

A. Parametrization

At this stage, we do not intend to calculate explicitly either the widths or the energy shifts from the first principle, but rather use the general structure of $S_{\lambda c}$ to incorporate resonances in our earlier calculation (Ref. 1). As usual, the fusion cross section σ_f is calculated by equating it to the reaction cross section in the incident channel. This is particularly suitable at energies below the barrier where other direct channels are not expected to be dominant. Hence, the fusion cross section is given by

$$\sigma_f = \frac{\pi}{k_\lambda^2}\sum_l (2l+1)(1-|S_{\lambda\lambda}|^2) , \tag{32}$$

where

$$S_{\lambda\lambda} = S_{\lambda\lambda}^{(B)} + S_{\lambda\lambda}^{(R)} ,$$

and is defined by Eq. (31) for each l. Thus, apart from background and resonance contributions, the cross section will also contain contributions from interference between the background and the resonance terms as well as interference between terms involving various bound states.

In principle, $S_{\lambda\lambda}^{(B)}$ can be calculated for a parabolic potential joined smoothly to a Coulomb potential in the exterior region using the method of Ref. 1. All the parameters pertinent to such a model could then be varied as free parameters again because $\chi_\lambda(r)$ in Eq. (31) is now a solution of the coupled equation, whereas in the calculations of Ref. 1 it was not. However, we wish to use the results of Ref. 1 for $S_{\lambda\lambda}^{(B)}$. This implies approximating $\chi_\lambda(r)$ in Eq. (31) by a solution of Eq. (7) having no coupling terms on the right-hand side. Implicitly, it also means replacing $u_c^{(0)}$ by $\chi_c^{(0)}$ in the expressions for both $S_{\lambda\lambda}^{(B)}$ and $S_{\lambda\lambda}^{(R)}$. By making use of Eq. (14), $S_{\lambda\lambda}^{(R)}$ can then be written as

$$S_{\lambda\lambda}^{(R)} \simeq -i\left[\frac{4}{\hbar v_\lambda}\right]\sum_b a_{nb}\gamma_{b\lambda} , \tag{33}$$

where the a_{nb}'s are determined from the system of linear equations (20).

The resonant structure of $S_{\lambda\lambda}^{(R)}$ becomes more explicit for the case of a single bound state, i.e., $b = b'$. In that case

$$a_{nb} = \frac{\gamma_{b\lambda}}{E - (E_{nb} - \Delta E_{nnb}) + i\Gamma_{bb}/2} , \tag{34}$$

where

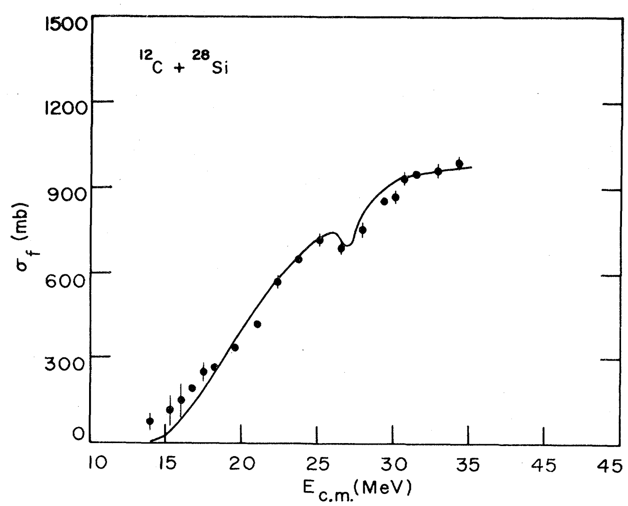

FIG. 1. A comparison between the calculated (solid line) and experimentally measured values (Ref. 20) of fusion cross sections for the reaction $^{12}C + ^{28}Si$. The experimental values are denoted by solid circles. The theoretical values are calculated by using only one bound state ($b=1$) with $E_r = 27.0$ MeV, $\Gamma_{b\lambda}^{(0)}(E_r) = 0.95$ MeV, and $\Gamma_{b0}^{(0)}(E_r) = 1.80$ MeV.

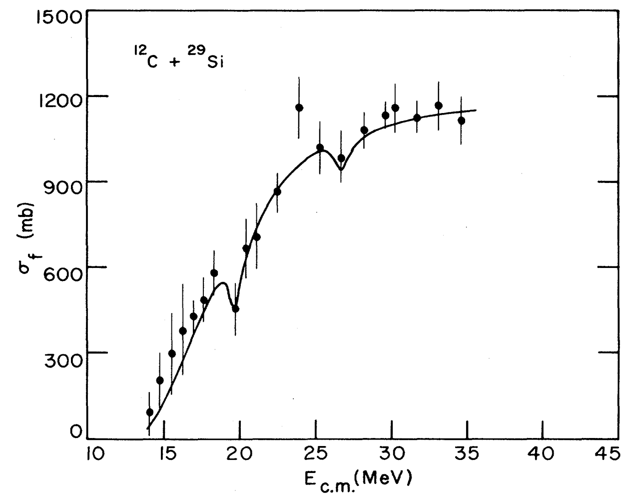

FIG. 2. The same as that of Fig. 1 but for $^{12}C+{}^{29}Si$. Two bound states ($b=1,2$) with $E_r=19.6$ and 26.8 MeV, $\Gamma_{b\lambda}^{(0)}(E_r)=0.75$ and 0.70 MeV, and $\Gamma_{b0}^{(0)}(E_r)=1.10$ and 1.55 MeV are considered here.

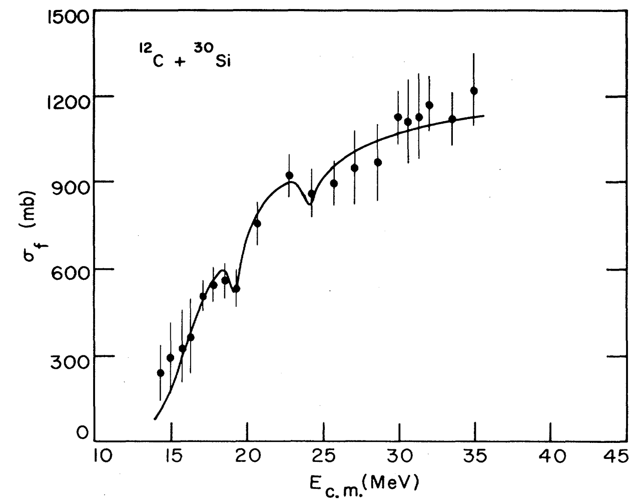

FIG. 3. The same as that of Fig. 1 but for $^{12}C+{}^{30}Si$. Two bound states ($b=1,2$) with $E_r=19.0$ and 24.2 MeV, $\Gamma_{b\lambda}^{(0)}(E_r)=0.80$ and 0.85 MeV, and $\Gamma_{b0}^{(0)}(E_r)=1.40$ and 1.80 MeV are considered here.

$$\Gamma_{bb}=\Gamma_{b\lambda}+\sum_{c\neq\lambda}\Gamma_{bc} \tag{35}$$

and

$$\Gamma_{bj}=\frac{4}{\hbar v_j}(\gamma_{bj})^2 \quad (j=c,\lambda)\ . \tag{36}$$

Equation (33) for $S_{\lambda\lambda}^{(R)}$ then becomes

$$S_{\lambda\lambda}^{(R)}\simeq\frac{-i\Gamma_{b\lambda}}{E-E_r+i\Gamma_{bb}/2}\ , \tag{37}$$

where

$$E_r=E_{nb}-\Delta E_{nbb}\ .$$

This is the Breit-Wigner resonance for a single bound state.

As mentioned earlier, instead of using definitions (36) and (23) for the widths and energy shifts, we will use them as parameters. We prescribe a simple energy and l dependence for the width $\Gamma_{b\lambda}$, in analogy to the α-decay problem.[19] Thus, we take

$$\Gamma_{b\lambda}^{(l)}(E)=\Gamma_{b\lambda}^{(l)}(E_r)\exp[(E-E_r)/E_r] \tag{38}$$

and

$$\Gamma_{b\lambda}^{(l)}(E)=C_\lambda\theta(l_r-l)\Gamma_{b\lambda}^{(l-1)}(E) \quad (l\neq 0)\ , \tag{39}$$

where

$$\theta(l_r-l)=\begin{cases}1 \text{ for } l<l_r\\ 0 \text{ for } l>l_r\ ,\end{cases} \tag{40}$$

and C_λ is a constant always less than one. We denote the width in all other channels ($\sum_{c\neq\lambda}\Gamma_{bc}$) by $\Gamma_{b0}^{(l)}$ and assume the same energy and l dependence as $\Gamma_{b\lambda}^{(l)}$. However, from Eq. (21) it is evident that $\Gamma_{bb'}$ is the total width and we shall treat it as such. We have also set $\Delta E_{nbb'}=0$ for $b\neq b'$ and in order to minimize the number of parameters, we have used $C_\lambda=C_0=0.9$ and $l_r=9$ for all the reactions considered here. The last one is not an approximation because contributions to $S_{\lambda\lambda}^{(R)}$ from higher l's become increasingly smaller.

TABLE I. Contributions of the background (second column), resonance (third column), and the interference between background and resonance (fourth column) parts of the S matrix to the fusion of ^{12}C by ^{30}Si for the first few partial waves at center of mass energy of 14.0 MeV. The fifth column shows the full contribution to the S matrix. The sixth and seventh are, respectively, the full and background contributions to the cross section.

l	$\lvert S_{\lambda\lambda}^{(B)}\rvert^2$	$\lvert S_{\lambda\lambda}^{(R)}\rvert^2$	$2\,\mathrm{Re}(S_{\lambda\lambda}^{(B)}S_{\lambda\lambda}^{(R)*})$	$\lvert S_{\lambda\lambda}\rvert^2$	$1-\lvert S_{\lambda\lambda}\rvert^2$	$1-\lvert S_{\lambda\lambda}^{(B)}\rvert^2$
0	0.4281	0.0296	0.0189	0.4766	0.5234	0.5719
1	0.4453	0.0243	0.0367	0.5063	0.4973	0.5547
2	0.4804	0.0199	0.0537	0.5540	0.4460	0.5196
3	0.5337	0.0162	0.0694	0.6193	0.3807	0.4663
4	0.6043	0.0132	0.0827	0.7002	0.2998	0.3957

TABLE II. The same as that of Table I except that the center of mass energy is 18.0 MeV.

l	$\|S^{(B)}_{\lambda\lambda}\|^2$	$\|S^{(R)}_{\lambda\lambda}\|^2$	$2\,\mathrm{Re}(S^{(B)}_{\lambda\lambda}S^{(R)*}_{\lambda\lambda})$	$\|S_{\lambda\lambda}\|^2$	$1-\|S_{\lambda\lambda}\|^2$	$1-\|S^{(B)}_{\lambda\lambda}\|^2$
0	0.0304	0.3041	0.0009	0.3354	0.6646	0.9696
1	0.0318	0.2776	−0.0344	0.2750	0.7250	0.9682
2	0.0347	0.2506	−0.0786	0.2068	0.7932	0.9653
3	0.0396	0.2239	−0.1288	0.1347	0.8653	0.9604
4	0.0473	0.1978	−0.1739	0.0712	0.9288	0.9527

B. Results and discussion

In the following, we apply the above formalism to calculate the fusion cross section for the reactions $^{12}C+^{28,29,30}Si$. The data[20] for these reactions show dips at around 27.0 MeV for $^{12}C+^{28}Si$, at 19.6 and 26.8 MeV for $^{12}C+^{29}Si$, and at 19.0 and 24.2 MeV for $^{12}C+^{30}Si$. We, therefore, use these values of energies as E_r for the respective reactions. This implies that we are considering only one bound state for $^{12}C+^{28}Si$ and two bound states for each of the reactions $^{12}C+^{29}Si$ and $^{12}C+^{30}Si$. At the location of each resonance one is to specify the width due to coupling of the elastic channel with bound states $\Gamma^{(0)}_{b\lambda}$ and $\Gamma^{(0)}_{b0}$ which is basically total width less $\Gamma^{(0)}_{b\lambda}$ for the s wave. Thus, for the $^{12}C+^{28}Si$ system, there are two free parameters once E_r is taken to be 27. 0 MeV from the data. For the $^{12}C+^{29}Si$ and the $^{12}C+^{30}Si$ systems, there are four free parameters because of the two resonances considered for each case.

The results of our calculations together with the experimental data[20] are displayed in Figs. 1—3. The values of the width parameters used in the calculation are given in the figures. In order to obtain reasonably good fits to the data, the values of $\Gamma^{(0)}_{b0}(E_r)$ had to be made larger than those of $\Gamma^{(0)}_{b\lambda}(E_r)$. This is reasonable since $\Gamma^{(0)}_{b0}$ represents total width less $\Gamma^{(0)}_{b\lambda}$. In calculating the cross sections, we have used the results of Ref. 1 for the background contribution to the S-matrix elements.

As expected, away from a resonance energy, the contribution of the resonant part of the S matrix to the cross section is very small. However, as opposed to pure background calculation,[1] a reduction of about 10—20 mb in the cross section is observed in the low energy region. This slight reduction in the magnitude of the cross section is caused by interference between the background and the resonance terms. To illustrate this point, we have listed in Table I the various contributions for the first few l values to the cross section at 14 MeV c.m. energy for the reaction $^{12}C+^{30}Si$. In Table I, the sixth column shows the full contribution to the cross section while the seventh column shows only the background contribution. Near a resonance energy, the main contribution to $S_{\lambda\lambda}$ comes from $S^{(R)}_{\lambda\lambda}$ even though the interference term is also significant. The interference term competes with the resonance term as l increases, as can be seen in the third and fourth columns of Table II for c.m. energy of 18.0 MeV. The net result of all these contributions is a reduction in the magnitude of the cross section.

If we consider only one bound state ($E_r=19.0$ MeV) for the reaction $^{12}C+^{30}Si$, no appreciable difference can be noticed in the magnitude of the cross section except for the disappearance of the dip around 24.0 MeV. This is because the two states considered here are far apart (about 5.0 MeV) and do not interfere significantly with each other. However, if the states are spaced closely, the interference effect might become appreciable.

In conclusion, we have provided here a theoretical framework to incorporate resonances in fusion cross section in a systematic way. We have, however, applied a simplified or schematic version of the theory to three systems and have shown that the theory can reasonably reproduce the data.

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