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Intermediate-energy-proton scattering, the Dirac equation, and nuclear structure

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The Dirac equation employing an impulse approximation optical potential based on *NN* phase shifts has been shown to provide a description of the intermediate energy proton-nucleus spin-dependent elastic scattering data not yet found in the context of a Schrödinger approach. Using analytic methods generalized to the Dirac equation, we trace this success to the nonlinear density contributions which arise as a natural consequence of the Dirac approach. This new understanding resurrects the hope of using intermediate energy protons as a probe of nuclear structure details.

[NUCLEAR REACTIONS Intermediate energy. Dirac impulse approximation, Dirac-eikonal amplitude, analytic methods, the Dirac approach and nuclear structure.]

A principal purpose of medium energy hadron-nucleus scattering is to exploit the short wavelength of the probe to provide detailed knowledge of the nuclear geometry. It is clear that in proton-nucleus scattering, for example, the oscillations in the cross section are diffractive and their connection to the nuclear shape was already clear in the work of Kerman, McManus, and Thaler[1] (KMT), who showed in 1959 that a "$t\rho$" optical potential was qualitatively similar to phenomenological potentials in use at the time. Since that time energy and precision have increased and spin dependent observables have been obtained. The observed oscillations in the spin observables are clearly related to the diffraction structure[2,3] but quantitative, detailed calculations aimed at fitting the full range of cross-section and spin observables data in impulse approximation have been forced to introduce parameters and assumptions in the nuclear geometry that are not easily justified.[4] These have included unusually large values for nuclear force ranges,[5] or for neutron-proton radius differences,[6,7] and energy dependent or probe dependent geometry.[8] Workers in the field finally concluded that nothing less than the breakdown of the impulse approximation was underlying the difficulties in describing the spin dependence using credible parameters.[9,10]

By contrast, the Dirac equation approach to proton-nucleus scattering solves these problems in a simple and remarkably straightforward way in the spirit of the KMT impulse approximation $t\rho$ approach. In this paper we explain how the three dynamical geometries needed[4] to account for the cross-section and spin observables arise naturally in the Dirac formalism from a single underlying nuclear geometry. We do this by exploiting insights that arise in an analytic approach to the scattering amplitudes, which approach starts most easily from an eikonal formalism. The qualitative success of the picture makes it clear that if realistic effects of finite range or neutron-proton differences are to be extracted, the Dirac starting point is both natural and necessary. We show, in particular, that although the Dirac scattering approach with but one geometry is qualitatively correct, there remain interesting details that require just such features and revitalize the hope of extracting them with the precise short range probes of medium energy.

The major clue to the need for a Dirac equation approach is that it is the spin-observables that are particularly difficult to describe in a parameter free Schrödinger approach. It has been shown by many[11-13] that in the limit of a spin orbit potential given by $(1/r)(d/dr)$ operating on the central potential the spin asymmetry is structureless to first order in the spin orbit strength, and nearly structureless to all orders if the strengths and ranges are taken from the observed nucleon-nucleon behavior. Some structure may be introduced by folding with the nucleon-nucleon force ranges or by using different proton and neutron densities. For phenomenological success, however, these approaches require either forces inconsistent with the free forces, or energy dependent neutron densities. It is important to stress that parameter-free attempts to describe the asymmetry data result in *qualitative* failure in reproducing the spin structure, as is well documented and will be illustrated again below.

Phenomenological treatment of the scattering based on analytic approximations to the eikonalized Schrödinger amplitude reveals that the cross section and asymmetry require

a central and spin-orbit geometry that differ by a single complex length.[2,3] This length not only completely characterizes the asymmetry but also provides the often sought for connection between the oscillatory structures in the cross section and in the asymmetry. Extending these notions to the spin-rotation parameter Q has revealed that yet a third geometry, or another complex length is required to account for the observations.[4] Thus this purely phenomenological approach showed the need for three distinct geometries and the size of this difference again has no justifiable theoretical origin.

In contrast to this, Clark *et al.*[14] showed that a phenomenological approach based on the Dirac equation allowed one to describe the spin rotation parameter Q with no further adjustment of parameters over those obtained by fitting to the cross section and asymmetry. Very recently it has been shown that an impulse approximation tp approach based on the Dirac equation allows for a parameter-free description of the 500-MeV p-^{40}Ca cross-section polarization and spin rotation parameter.[15–17] This suggests that the geometry differences known to be required phenomenologically, and very difficult, if not impossible to include via a Schrödinger theory, are automatically included in a Dirac theory.

In this paper we try to understand this success from an analytic perspective. We present an eikonalized solution to the Dirac equation, which has not to our knowledge been previously presented. From this, following the work of others, we exhibit the Schrödinger equivalent potential, which indeed has three terms with different geometries as required by the data. These three terms arise as different functions of a single assumed underlying density. We show that with no force range differences and no differences in the neutron and proton densities, structure in P and Q is obtained which is qualitatively in agreement with the data. The Schrödinger or single geometry approach fails utterly in this regard. The Dirac equation leads to these sizeable differences among the three interactions geometries in a natural way. It is clear from these results that while a Schrödinger based analysis of a single cross section can yield qualitatively correct information about a single density, Schrödinger based analyses of density differences, neutron densities, spin observables, or energy dependence must be questioned.

To more easily understand the physics of the Dirac equation approach we have derived an eikonal form for the elastic scattering amplitude for a Dirac particle of mass m, energy E interacting with both scalar (S) and vector (V) potentials:

$$f(q)=\frac{-k_a}{2\pi}\int d^2b e^{i\vec{q}\cdot\vec{b}}(e^{-\chi(b)}-1)\ ,\quad -\chi(b)=-im/k_a\int_{-\infty}^{\infty}dz\left[S+E/mV+\frac{S^2-V^2}{2m}-\vec{\sigma}\cdot(\hat{r}\times\hat{k}_a)\frac{k_a}{2m}\frac{S'-V'}{E+m+S-V}\right] \tag{1}$$

where $\vec{q}=\vec{k}-\vec{k}'$ and $\vec{k}_a=(\vec{k}+\vec{k}')/2$. The z axis is chosen along $\vec{k}_a$, $\vec{\sigma}$ is the Pauli spin operator of the projectile, and prime denotes differentiation with respect to r. The derivation of this equation will be presented in a separate paper. Here we only note that the result is the same as one would obtain by eikonalizing à la Glauber[18] the "Schrödinger equivalent" potential discussed in Ref. 19 and elsewhere.

In Figs. 1–3 we compare this eikonal approximation to a partial wave solution of the Dirac equation[20] for the case of p-^{40}Ca elastic scattering at 500 MeV. In the calculation V and S are both taken to be of the form $V_0^{(i)}\rho$ where $V_0^{(i)}$ is a complex strength, different for V and S and ρ is a Woods-Saxon form $1/(1+e^{(r-c)/\beta})$ with radius $c=3.55$ fm and diffusivity parameter $\beta=0.64$ fm. The important point is that the same density is used for V and S. Also shown in Fig. 1 is the data of Refs. 9 and 21 for p-^{40}Ca elastic scattering at 500 MeV. The strength parameters are taken from the NN amplitudes following Ref. 16: $S_0=(-303+i73)$ MeV and $V_0=(191-i86)$ MeV. The eikonal and partial wave cross sections are essentially identical and both are close to the data. We see in Fig. 1 that the equivalent single density Schrödinger calculation also gives a reasonable description of the cross section.

In Figs. 2 and 3 the polarization and spin rotation data are plotted along with the single density Schrödinger, partial-wave-Dirac and eikonal-Dirac calculations. The single density Scrhödinger calculation is a standard nonrelativistic zero range one using the same Woods-Saxon density parameters for both the central and spin orbit optical potentials with the strengths extracted from the same NN amplitudes used in the relativistic case. We observe in both cases relatively structureless Schrödinger curves compared to the data. The exact and eikonal Dirac are again very close and both qualitatively duplicate the observed structures in P and Q.

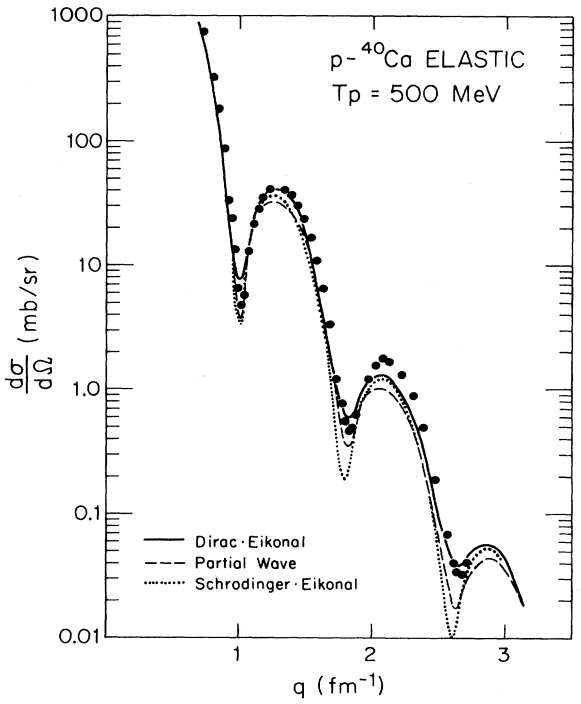

FIG. 1. The 500-MeV p-^{40}Ca elastic scattering cross section data (Ref. 8) is compared to three equal geometry model calculations. The solid curve is the Dirac-eikonal result [Eq. (1)], the dashed curve is the corresponding partial wave Dirac equation calculation, and the dotted curve is the "equivalent" Schrödinger-eikonal result. The potential strengths were taken from Ref. 16. The Woods-Saxon parameters used were $c=3.55$ fm and $\beta=0.64$ fm.

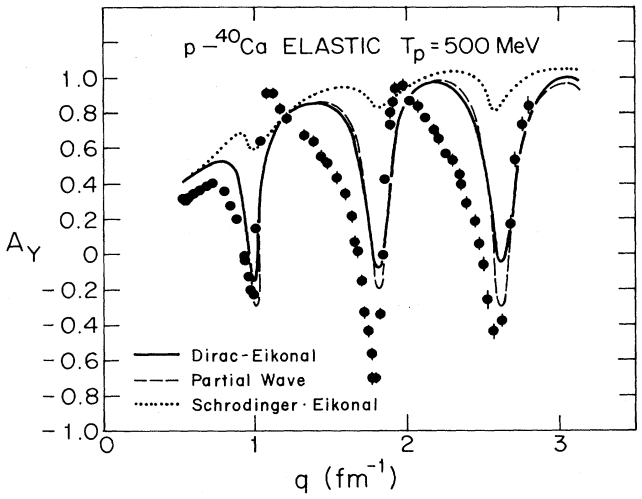

FIG. 2. Same as Fig. 1 except for analyzing power.

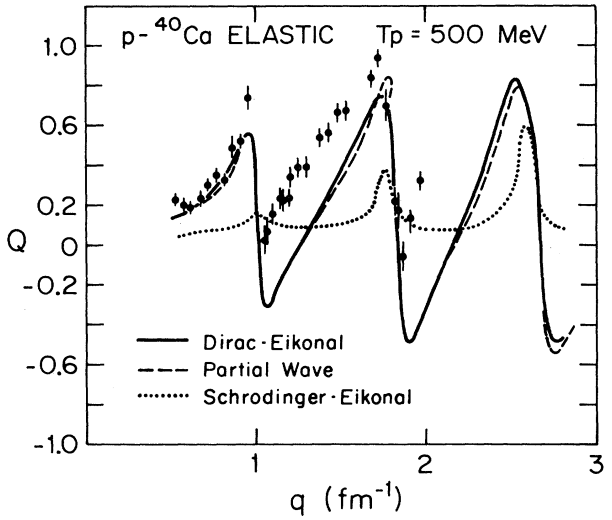

FIG. 3. Same as Fig. 1 except for spin rotation parameter (data from Ref. 21).

The three geometries that enter the Dirac approach are clear in Eq. (1). They are the density ρ in the first term $(S_0+V_0)\rho(r)$, the quadratic density term of the second term $(S_0^2-V_0^2)\rho(r)^2$, and the nonlinear density (the spin orbit term $(S_0-V_0)\rho'(r)/[E+M+S(r)-V(r)]$. That these produce just the geometries and phases required by the physics is most easily seen by using an analytic approach to the evaluation of Eq. (1) in which the rapid oscillation of the integrand is exploited to evaluate the integral by the method of stationary phase. It is the complex singular point of the profile integrals that then dominate the integrals and these can be obtained from the combined effects of the three geometries by a generalization of the methods of Refs. 22 and 23. It is then easily seen that the three "geometries" in Eq. (1) lead to just the three dynamic geometries that are phenomenologically required by P and Q.[4] This inspires the hope that the data-to-data approach for spin observables and inelastic scattering will be as successful in a parameter-free Dirac approach as it was in a phenomenological Schrödinger approach.[23,24] Details of these analytic analyses will be presented later.

In conclusion, we have elucidated how a Dirac equation, impulse approximation approach to medium energy proton-nucleus scattering is able to account for the features of the cross sections and spin observables using only a single underlying nuclear density, although this is impossible in the Schrödinger equation. In the Dirac equation this single underlying nuclear density manifests itself in the equivalent Schrödinger equation as three interaction geometries. An analytic approach based on a Dirac-eikonal scattering formalism allows a connection to be made between these three geometries, and those of previous *ad hoc* phenomenology. For a quantitatively detailed description of the proton-nucleus scattering data, force ranges, neutron-proton density differences, and small components of the nuclear wave function should be included. However, the hope is raised anew that the theory is now adequate for exploiting the amazing precision of present data to determine the detailed structure and possible new features of nuclear densities.

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