Pauli blocking correction in pion-^{4}He scattering

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We discuss the pion nucleon partial wave mixing due to the Pauli projection operator in π-^{4}He scattering. In a previous numerical investigation we found that this partial wave mixing generates only small corrections in the cross sections. In the present study we demonstrate that the coupling between partial waves vanishes in the limit $m_\pi/m_{\rm N}\to 0$, thereby explaining these numerical findings.

[NUCLEAR REACTIONS π-^{4}He scattering, optical potential, Pauli principle corrections.]

The simplest nontrivial way to obtain a unitary optical potential is to truncate the Hilbert space of nuclear states in the intermediate system to the subspace of 1p-1h states.[1] This is realized for the single scattering process in which the pion scatters from one nucleon at a time if, in addition, the nucleus is described by means of an independent particle shell model. At this level of approximation two medium effects enter the description, viz., the binding correction and the Pauli blocking correction. To incorporate the binding effect one has to solve a three-body problem (pion, nucleon, core) with interactions between the pion nucleon and between the nucleon core two-body subsystems, giving a T operator t for pion bound-nucleon scattering.[2] Subsequently, one has to account for the ground-state exclusion and the Pauli blocking effect to obtain the required π-N G operator τ and the optical potential[3]

$$U_{\rm opt}=A\,\langle\psi_0|\tau|\psi_0\rangle\ . \tag{1}$$

In Ref. 4 we have shown in a discussion of π-^{4}He scattering that the Pauli blocking correction introduces a mixing between different π-N partial waves. There we have found that this partial wave mixing is not very important numerically, although no explanation for this finding was presented. The purpose of this Brief Report is to show that the coupling between partial wave vanishes completely in the limit $m_\pi/m_{\rm N}\to 0$, clarifying the numerical results in view of the smallness of the ratio $m_\pi/m_{\rm N}$. The π-N G operator τ is related to the T operator t by

$$\tau=t-t\,1/eR\,\tau\ . \tag{2}$$

Here the π-A propagator is denoted by $1/e$ and R is the Pauli projection operator projecting on the space of single-nucleon states, which are occupied in the ground-state nucleus. A straightforward but very laborious way to solve Eq. (2) is to consider its matrix representation in the space of occupied single-nucleon states yielding a set of A coupled equations.[2] For the case of π-^{4}He scattering this would involve an expansion of t and τ into the (iso)spin-(non)flip parts. To avoid such coupled equations, here we expand t and τ into partial waves:

$$t=\sum_{Tjl}t_{Tjl}P^lP_l^jP^T \tag{3}$$

and

$$\tau=\sum_{Tjl}\tau_{Tjl}P^lP_l^jP^T\ , \tag{4}$$

where P^l, P^T projects on a π-N partial wave with orbital angular momentum l and total isospin T, respectively. Furthermore, P_l^j projects for a given l on the total angular momentum state j. Substitution of Eqs. (3) and (4) into Eq. (2) gives

$$\tau_{Tjl}P^lP^jP^T=\left(t_{Tjl}P^l-t_{Tjl}P^l1/eR\,\tau_{Tjl}P^l-\sum_{l',l'\neq l}\sum_{j}t_{Tj'l'}P^{l'}P_{l'}^{j'}\,1/eR\,\tau_{Tjl}P^l-t_{Tj'l}P^l[P_l^{j'},1/eR\,]\tau_{Tjl}P^l(1-\delta_{jj'})\right)P_l^jP^T\ . \tag{5}$$

Notice that in Ref. 4 the last term between the brackets, which represents a j-state mixing, resulting from intermediate nucleon spin flips, has not been considered. This neglect can be justified as follows. For $T_\pi\lesssim 300$ MeV contributions from $l\geq 2$ can be ignored. Since a j-state mixing does not arise for the π-N S waves, we only have such a mixing between the P_{33} and P_{31}, and between the P_{13} and P_{11} partial waves. The contributions from both the S waves and the P_{33} partial wave are much larger than the contributions from the other P waves. Therefore one expects the l-state mixing to be more important than the j-state mixing.

Let us consider the commutator $[P_l^j,1/eR\,]$ in more detail. Using

$$P_l^j=\begin{cases}\dfrac{l-\vec\sigma\cdot\vec{1}}{2l+1} & \text{for } j=l-\frac{1}{2}\ ,\\[2mm] \dfrac{l+1+\vec\sigma\cdot\vec{1}}{2l+1} & \text{for } j=1+\frac{1}{2}\ ,\end{cases} \tag{6}$$

we can write

$$\begin{aligned}[P_l^j,1/eR\,]&=\mp\frac{1}{2l+1}[\,\vec\sigma\cdot\vec{1},1/eR\,]\\&=\mp\frac{1}{2l+1}\{1/e[\,\vec\sigma\cdot\vec{1},R\,]+[\,\vec\sigma\cdot\vec{1},1/e\,]R\}\ .\end{aligned} \tag{7}$$

We take nonrelativistic kinematics for simplicity. Then the inverse propagator is given by

$$e = E_0^+ + \epsilon_{1s} - T_{\pi A} - T_{\rm NC} - V_{\rm NC} \ , \tag{8}$$

where ϵ_{1s} is the average single-nucleon separation energy. Furthermore, $T_{\pi A}$ and $T_{\rm NC}$ are the kinetic energy operators for the relative πA and nucleon core (NC) motions, respectively, and $V_{\rm NC}$ is the NC potential. We rewrite Eq. (8) in obvious notation as

$$\begin{aligned} e &= E_0^+ + \epsilon_{1s} - T_{(\pi {\rm N}),C} - T_{\pi {\rm N}} - V_{\rm NC} \\ &= E_0^+ + \epsilon_{1s} - T_{(\pi {\rm N}),C} - \left(\frac{P_r^2}{2\mu} + \frac{l^2}{2\mu r_{\pi {\rm N}}^2} \right) - V_{\rm NC} \ . \end{aligned} \tag{9}$$

The potential $V_{\rm NC}$ is a function of the relative NC coordinate $r_{\rm NC}$. Because

$$\vec{r}_{\rm NC} = \vec{r}_{(\pi {\rm N}),C} - \frac{m_\pi}{m_{\rm N} + m_\pi} \vec{r}_{\pi {\rm N}} \ , \tag{10}$$

an error is involved of the order $m_\pi/m_{\rm N}$ if we approximate $r_{\rm NC} \simeq r_{(\pi {\rm N}),C}$. In this approximation $V_{\rm NC}$ acts on the π-N c.m. degree of freedom, relative to the core C. Therefore, from Eq. (9), one sees that

$$\lim_{m_\pi/m_{\rm N} \to 0} [\vec{\sigma} \cdot \vec{1}, 1/e] = 0 \ . \tag{11}$$

For ^{4}He we simply have $R = |1s\rangle\langle 1s|$, which is a unity operator in spin (and isospin) space. Therefore

$$\begin{aligned} \lim_{m_\pi/m_{\rm N} \to 0} [P_l^j, 1/eR] &= \mp \frac{1}{2l+1} 1/e [\vec{\sigma} \cdot \vec{1}, |1s\rangle\langle 1s|] \\ &= \mp \frac{1}{2l+1} 1/e\, \vec{\sigma} \cdot [\vec{1}, |1s\rangle\langle 1s|] \ . \end{aligned} \tag{12}$$

The $|1s\rangle$ state is a function of $r_{\rm NC}$. From Eq. (10) we see therefore that

$$\lim_{m_\pi/m_{\rm N} \to 0} [P_l^j, 1/eR] = 0 \ . \tag{13}$$

This shows that the j-state mixing vanishes in the limit $m_\pi/m_{\rm N} \to 0$. In a similar way one can easily show that, also,

$$\lim_{m_\pi/m_{\rm N} \to 0} [P^l, 1/eR] = 0 \ ,$$

resulting in a vanishing l-state mixing.

Summing up, we have shown that the π-N partial waves in the evaluation of the Pauli blocking correction completely decouple in the limit of a vanishing $m_\pi/m_{\rm N}$ ratio. This explains the previously found small effects obtained in a numerical evaluation of this mixing.

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