Double β-decay nuclear matrix elements for the $A=48$ and $A=58$ systems

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The nuclear matrix elements entering the double β decays of the ^{48}Ca-^{48}Ti and ^{58}Ni-^{58}Fe systems have been calculated using a realistic two nucleon interaction and realistic shell model spaces. Effective transition operators corresponding to a variety of gauge theory models have been considered. The stability of such matrix elements against variations of the nuclear parameters is examined. Appropriate lepton violating parameters are extracted from the $A=48$ data and predictions are made for the lifetimes of the positron decays of the $A=58$ system.

[RADIOACTIVITY Double β decay. Gauge theories. Lepton nonconservation. Neutrino mass. Shell model calculations.]

I. INTRODUCTION

Neutrinoless double β decay[1,2] has recently become the subject of extensive investigations because it may give answers to some of the most interesting questions of modern physics such as whether the neutrino is massive and whether lepton number is not strictly conserved. Such old questions have recently been revived due to the developments in modern gauge theories,[3,4] and in particular those that attempt at grand unification (GUTS). It is therefore generally expected that one may extract some of the interesting parameters of such theories (neutrino masses, neutrino mixing angles, the possibility of right-handed admixtures in weak interactions, Higgs particles, etc.) from such $\beta\beta$-decay data. However, such an extraction crucially depends on the reliable estimation of the nuclear matrix elements involved in such transitions. In this paper we are going to present calculations of such matrix elements.

It is well known that double β decay can be observed only in the case of nuclear systems which cannot undergo single β decay due to energy conservation or angular momentum mismatch. If lepton number (charge) is absolutely conserved, only 2ν decays are possible, e.g.,

$$(A,Z)\rightarrow(A,Z+2)+e^-+e^-+\tilde{\nu}_e+\tilde{\nu}_e \,, \tag{1}$$

$$(A,Z)\rightarrow(A,Z-2)+e^++e^++\nu_e+\nu_e \,, \tag{2}$$

$$e_b^-+(A,Z)\rightarrow(A,Z-2)+e^++\nu_e+\nu_e \,. \tag{3}$$

If, however, lepton number is not conserved, one expects to see 0ν processes like

$$(A,Z)\rightarrow(A,Z+2)+e^-+e^- \,, \tag{4}$$

$$(A,Z)\rightarrow(A,Z-2)+e^++e^+ \,, \tag{5}$$

$$e_b^-+(A,Z)\rightarrow(A,Z-2)^*+e^+ \,, \quad (A,Z-2)^*\rightarrow\gamma,x \,. \tag{6}$$

Clearly, by measuring the energies of all detectable particles (electrons, positrons, photons) one can distinguish between 0ν and 2ν processes and draw conclusions about lepton conservation. Since the 0ν processes are kinematically favored compared to the 2ν processes, from the mere nonobservation of any kind of $\beta\beta$ decay in the laboratory, one expects the lepton violating parameter η (Refs. 2 and 5) to be small ($\eta<10^{-4}$).

The 0ν $\beta\beta$ decay rates depend sensitively on the available energy and the nuclear matrix elements involved. Both of these factors must be large for an observable rate. In the case of negaton emission large available energy is also necessary in order to cope with the problem of background radioactivity. Fortunately one can choose among 30 possibilities.[2]

The effective transition operators depend on the mechanism for lepton violation, i.e., on the gauge models. In almost all gauge models, however, the nuclear matrix element involves the axial current. Thus one expects large matrix elements for systems which are not spin-isospin saturated and involve unfilled shells with large l values. (Then neutrons can be transformed into protons in the same harmonic oscillator shell in $\beta^-\beta^-$, or vice versa for positron emission). Finally, for technical reasons one would like to deal with nuclei with the simplest possible nuclear structure (most possible $\beta\beta$-decaying systems[2]) have fairly complicated structure). With the above criteria we have selected for study the following two nuclear systems:

(i) $^{48}\text{Ca}\rightarrow{}^{48}\text{Ti}+e^-+e^-$ with transition energy $\epsilon_0=8.4$ (in units of m_ec^2).

(ii) $^{58}Ni \rightarrow {}^{58}Fe + e^{+} + e^{+}$ with transition energy $\epsilon_0 = 0.16$. It is expected that the low counting rates associated with such a small energy may be compensated by experimental advantages of positron detection[6] in conjunction with the possibility of making large targets, e.g., using ^{58}Ni in the wiring of a time projection counter.[6]

For more details relevant to double β decay the reader is referred to previous literature.[1,7,8]

II. NUCLEAR STRUCTURE

Simple calculations of the $A = 48$ system relevant to the $0\nu\ \beta\beta$ decay have previously been reported. The first such calculation was done by Khodel in the context of Migdal's theory for finite nuclear systems.[9] Subsequent shell model calculations[10,7,8] included nucleons only in the $0f_{7/2}$ shell and employed the effective interaction resulting from the bare G-matrix elements of Kuo and Brown.[11] In such a scheme the ^{48}Ti ground state wave function has the structure

$$^{48}\mathrm{Ti(g.s.)} = \sum_i C_i \, | f_{7/2}^6(n)J_i; f_{7/2}^2(p)J_i; 0^+ \rangle \ . \tag{7}$$

In such simple calculations the coefficients C_i were found[7,8] to be $C_i = 0.845$, -0.523, 0.109, and 0.001 for $J_i = 0$, 2, 4, and 6, respectively. Unfortunately, for some of the nuclear matrix elements entering double β decay, there exists a cancellation[8–10] between the two dominant components associated with $J_i = 0$ and $J_i = 2$. Such cancellations are also predicted[12,13] in the simple, i.e., without configuration mixing, Nilsson scheme as a manifestation of the fact that the protons and neutrons occupy Nilsson orbitals which do not satisfy certain selection rules.

It is therefore interesting to investigate the stability of such calculations against variations both in the effective interaction and the model space involved. As a first step we will extend the model space to include active protons in the entire $0f$-$1p$ shell while we restrict the active neutrinos in the $0f$ shell. The above restriction on the neutrons was dictated by practical considerations, i.e., to avoid very large ($10\,000 \times 10\,000$) matrices. Furthermore, we believe it to be good approximation. Calculations along these lines have been recently reported,[14] but they involve only a subset of the operators of interest in $\beta\beta$ decay. Secondly, we will employ the normalized two-body Kuo-Brown[9] matrix elements, which seem more appropriate for limited shell model spaces.

The above space is not adequate to guarantee that the ^{48}Ti ground state wave function has definite isospin. Since, however, the matrix element of $\vec{I}^{\,2}$ has been found to be 6.206 38 instead of 6 we know that this wave function has predominantly isospin $I = 2$ with small spurious $I = 3$ and $I = 4$ components. The effective Hamiltonian has been constructed using the renormalized two-body Kuo and Brown[11] matrix elements together with the following set of empirical single particle energies:

$$\begin{aligned} &\epsilon_{7/2} = 0.0, \ \ \epsilon_{5/2} = 6.0 \ , \\ &\epsilon_{3/2} = 2.07, \ \ \epsilon_{1/2} = 4.13 \ . \end{aligned} \tag{8}$$

Repeating the calculation in the limited $0f_{7/2}$ space with the same effective interaction we found $C_i = 0.908$, -0.418, 0.013, and -0.004 [in the order of Eq. (7)]. In the enlarged space the above four components exhaust 88% of the wave function, which now has 20 components.

The nuclear matrix elements involving the ^{58}Ni-^{58}Fe system have also recently been computed.[8] In these calculations, however, the proton-neutron interaction was put in only phenomenologically, i.e., in the context of the pairing vibration model. In the present calculation we assume a ^{48}Ca closed core and permit the active protons to occupy the $0f_{7/2}$ shell and the active neutrons to be distributed in the $0f_{5/2}$, $1p_{3/2}$, and $1p_{1/2}$ shells. Thus we consider eight protons and two neutrons in the initial ^{58}Ni nucleus and six protons and four neutrons in the ^{58}Fe final system. The intermediate ^{58}Co nucleus consists of seven protons and three neutrons.

The energy matrices have been constructed using the effective Hamiltonian of Benson and Johnstone[15] which has been found to satisfactorily describe the low-lying states of several nuclei in the $A = 51-57$ region. The single particle energies of Ref. 10 have been adapted to a ^{48}Ca core and are given by

$$\epsilon_{3/2} = 0.0, \ \ \epsilon_{1/2} = 1.56, \ \ \epsilon_{5/2} = 4.01 \ . \tag{9}$$

III. NUCLEAR OPERATORS

As has been shown in Ref. 8, which hereafter will be denoted as I, the nuclear matrix elements entering the $0\nu\ \beta\beta$ decay involve the following types of operators:

(i) *Mechanisms involving intermediate neutrinos:*

(1) *Light neutrinos.* In this case the effective two nucleon operator is

$$\Omega_\nu = \sum_{i \neq j} \tau_+(i)\tau_+(j) \frac{R_0}{r_{ij}} \left[\frac{f_V^2}{f_A^2} - \vec{\sigma}_i \cdot \vec{\sigma}_j \right] \tag{10}$$

with $R_0 = r_0 A^{1/3}$ ($r_0 = 1.1$ fm) the nuclear radius, $f_V = 1$, and $f_A = 1.24$.

(2) *Heavy neutrinos.* In this case one deals with two types of operators Ω_N and Ω_π. The operator Ω_N arises in nuclear models involving only nucleons and takes the form of Eq. (10) with an additional radial dependence, which arises using a dipole shape[7,8] form factor

$$F(\vec{q}^{\,2}) = 1/(1 + \vec{q}^{\,2}/m_A^2)^2$$

for nucleon, given by

$$\begin{aligned} &F_1(x) = \frac{m_A}{m_e} x(x^2 + 3x + 3)e^{-x} \ , \\ &x = m_A r_{ij}, \ \ m_A = 0.85 \text{ GeV} \ . \end{aligned} \tag{11}$$

The operator Ω_π arises by considering the double β decay of pions in flight between two nucleons,[16] i.e.,

$$\pi^- \rightarrow \pi^+ + e^- + e^- \ ,$$

and takes the form

$$\Omega_\pi=0.06\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0}{r_{ij}}\left[F_2(x_\pi)\vec{\sigma}_i\cdot\vec{\sigma}_j+F_3(x_\pi)\left[\frac{24\pi}{5}\right]^{1/2}Y^2(\hat{r}_{ij})(\vec{\sigma}_i\otimes\vec{\sigma}_j)^2\right] \tag{12}$$

with

$$F_2(x)=\frac{m_A}{m_e}(x-2)e^{-x},\quad F_3(x)=\frac{m_A}{m_e}(x+1)e^{-x},\quad x_\pi=m_\pi r_{ij}\ . \tag{13}$$

(ii) *Mechanisms involving heavy Higgs particles.* In this case in addition to the operator Ω_N mentioned above one may encounter the operator[17]

$$\Omega_\Delta=\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0}{r_{ij}}\left[\alpha_s F_1(x_A)+\alpha_p\vec{\sigma}_i\cdot\vec{\sigma}_j F_4(x_A,x_\pi)+\alpha_p\left[\frac{24\pi}{5}\right]^{1/2}Y^2(\hat{r}_{ij})(\vec{\sigma}_i\otimes\vec{\sigma}_j)^2F_5(x_A,x_\pi)\right] \tag{14}$$

with

$$F_4(x_A,x_\pi)=\frac{m_A}{m_e}\left\{\tfrac{1}{2}[(x_\pi-2)e^{-x_\pi}+(x_A-2)e^{-x_A}]-2\left[\left[\frac{m_\pi}{m_A}\right]^2e^{-x_\pi}-e^{-x_A}\right]\right\},$$

$$F_5(x_A,x_\pi)=\frac{m_A}{m_e}\left\{\tfrac{1}{2}[(x_\pi+1)e^{-x_\pi}+(x_A+1)e^{-x_A}]+2\left[\left[\frac{m_\pi}{m_A}\right]^2Z(x_\pi)-Z(x_A)\right]\right\}, \tag{15}$$

$$Z(x)=\left[1+\frac{1}{x}+\frac{1}{x^2}\right]e^{-x},\quad \alpha_s=0.25,\quad \alpha_p=0.40\ .$$

(iii) *Processes involving right-handed currents.* In the case of heavy intermediate neutrinos one encounters the operator Ω_N mentioned above. For light intermediate neutrinos one encounters the following three operators:

$$\Omega_1=\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0}{r_{ij}}\left[3\frac{f_V^2}{f_A^2}-\vec{\sigma}_i\cdot\vec{\sigma}_j+2\left[\frac{24\pi}{5}\right]^{1/2}Y^2(\hat{r}_{ij})(\vec{\sigma}_i\otimes\vec{\sigma}_j)^2\right], \tag{16}$$

$$\Omega_2=6\frac{f_V^2}{f_A^2}\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0}{r_{ij}}\ , \tag{17}$$

$$\Omega_3=3\frac{f_V}{f_A}\sum_{i\neq j}\tau_+(i)\tau_+(j)\frac{R_0R_{ij}}{r_{ij}^2}(\vec{\sigma}_i\cdot\vec{\sigma}_j)(i\hat{r}_{ij}\times\hat{R}_{ij})\ . \tag{18}$$

In all the above operators we have used the definitions

$$\vec{r}_{ij}=\vec{r}_i-\vec{r}_j,\quad \vec{R}_{ij}=\tfrac{1}{2}(\vec{r}_i+\vec{r}_j)\ . \tag{19}$$

For positron emission one should make the substitution $\tau_+\to\tau_-$.

The above operators do not depend on the structure of the intermediate nuclear states since, in their derivation, closure over such states was invoked. Here this seems to be a good approximation since the intermediate states are dominated by the energy of the intermediate particles (neutrinos or Higgs particles). The same is not true for the 2ν process, since in the latter the intermediate states contain physical neutrinos. This 2ν process is dominated by the axial hadronic current. Thus the nuclear matrix element for $0^+\to0^+$ decays takes the form

$$(\tilde{M})^A_{\rm nuc}=\sum_n\frac{1}{\mu_n}\langle f\,|\,\vec{Y}\,|\,n\rangle\langle n\,|\,\vec{Y}\,|\,i\rangle\ , \tag{20a}$$

where

$$\vec{Y}=\sum_i\tau_+(i)\vec{\sigma}(i)$$

is the Gamow-Teller operator and

$$\mu_n=\frac{1}{m_e}(E_{xn}+\Delta m)+\epsilon_0/2\ .$$

Δm is the mass difference $m(z\pm1)-m(z)$ and E_{xn} is the excitation energy of the intermediate nuclear states. It has, however, been found convenient[1,8,10] to absorb into the kinematics a suitable[8] quantity μ_0 [see Eq. (3) of I], an average of the intermediate denominators μ_n, and cast the nuclear matrix element in the form

$$(M)^A_{\rm nuc}=\mu_0(\tilde{M})^A_{\rm nuc}=\sum_n\frac{\mu_0}{\mu_n}\langle f\,|\,\vec{Y}\,|\,n\rangle\langle n\,|\,\vec{Y}\,|\,i\rangle\ . \tag{20b}$$

If one defines the above quantity μ_0 so that

$$\langle f\,|\,\Omega_4\,|\,i\rangle\equiv-\sum_n\frac{\mu_0}{\mu_n}\langle f\,|\,\vec{Y}\,|\,n\rangle\langle n\,|\,\vec{Y}\,|\,i\rangle\ , \tag{21a}$$

where

$$\Omega_4=\sum_{i\neq j}\tau_+(i)\tau_+(j)\vec{\sigma}_i\cdot\vec{\sigma}_j=\vec{Y}\cdot\vec{Y}\ , \tag{21b}$$

then one can compute the relevant matrix element employing the operator Ω_4. In this case no approximation has been made. In most calculations performed thus far, however, the operators Ω_4 was employed in computing the 2ν nuclear matrix element while μ_0 was estimated from the energy spectrum. This, of course, is only approximately true. In the present calculation our model space is not prohibitively large and the summation over the intermediate states can be explicitly performed. Thus in our model μ_0 is computed exactly employing Eq. (21a). We note that in the limit $r_{ij} \simeq R_0$ (the Rosen-Primakoff approximation[1]) the operator Ω_ν coincides with Ω_4, something that was taken for granted in earlier[10] work, but it was found inadequate[7] in subsequent work.

Since in our model space the ground state of the final nucleus is not characterized by good isospin one will have a contribution arising from the vector current. The relevant nuclear matrix element is

$$(M)^V_{\rm nuc} = \frac{f_V^2}{f_A^2}\left\{\sum_n \frac{\mu_0 C_{n0}^2}{\tilde{\mu}_n}\right\}\langle f\,|\,T_+\,|A_0\rangle\langle A_0\,|\,T_+\,|\,i\rangle\ , \tag{22}$$

where T_+ is the isospin raising operator and A_0 is the isobaric analog of the initial state and C_{n0} its amplitude in the intermediate $|0^+n\rangle$ state with energy $\tilde{\mu}_n$. The vector contribution is expected to be small, since, as we have seen, the isospin impurity of the final state is small. In the case of ^{48}Ca in our model there is only one intermediate state, A_0 itself, i.e., $C_{n0}=1$, with excitation energy $E_{xA}=4.8$ MeV, i.e., $\tilde{\mu}_0 \simeq 17$. We thus get

$$(f_V^2/f_A^2)\sum_n (\mu_0 C_{n0}^2/\tilde{\mu}_n) \approx 0.3\ ,$$

i.e., the vector contribution is also kinematically suppressed. In our model we find $(M)^V_{\rm nuc}=0.029$, which is an order of magnitude smaller than the matrix element of the axial current.

As was explained in I we found it necessary, in particular for the somewhat short-ranged operators, to consider the effects of the two nucleon short range correlation function. Our matrix elements were computed with a correlated two-body density which is related to the uncorrelated shell model density as follows[8]:

$$\psi_{\rm cor}(\vec{r}_i,\vec{r}_j) = [1 - C(\,|\,\vec{r}_i - \vec{r}_j\,|\,)]\Psi_{\rm uncor}(\vec{r}_i,\vec{r}_j) \tag{23}$$

with

$$C(r) = e^{-ar^2}(1-br^2)\ ,$$
$$a = 1.1\ {\rm fm}^{-2},\ \ b = 0.68\ {\rm fm}^{-2}\ . \tag{24}$$

The matrix elements of the various transition operators defined by Eqs. (11)–(24) are presented in Table I. The operators Ω_N, Ω_π, and Ω_Δ, being short ranged, receive most of their contribution from the $J=0$ nucleon pairs and are quite stable against the nuclear model variations considered here. On the other hand, these operators depend on the short range correlation function employed. The operators Ω_ν and Ω_4 suffer from the cancellations between the $J=0$ and $J=2$ pairs mentioned earlier. This is the reason why in the case of $A=48$ the 2ν mode is characterized by a small nuclear matrix element, in agreement with what has been known for a long time.[7] The matrix element of operator Ω_ν, however, which is crucial in extracting the neutrino mass[8] from the data, is not small, since the cancellation is not complete. It also seems to be stable. Finally, we notice that the calculated value of μ_0 is also relatively stable. This may be attributed to the fact that in our model there are at most two 1^+ intermediate nuclear states, the lowest of which almost saturates the sum [see Eq. (20)]. In the case of the $A=58$ system the situation is somewhat different. When one compares the results of the present calculation with those of I one finds appreciable differences in the matrix elements of the operators Ω_1, Ω_2, Ω_ν, and Ω_4. As expected, these matrix elements are sensitive to the $0f_{5/2}$ configuration in the ground states. Naturally the quantity μ_0 is model dependent since there are now 21 intermediate 1^+ nuclear states all with appreciable Gamow-Teller strength. The value $\mu_0=6.5$ was obtained using Eq. (21). On the other hand, the value $\mu_0=10$ of our previous calculation[8] was determined from the energy spectrum. This may serve as a warning that the use of the closure relation may lead to unreliable results. Thus disagreement worse than in the present case may occur if some of the terms summed in the closure relation produce opposite contributions. Such effects may very well explain the discrepancy between the calculated[18] 2ν rates and the total $(0\nu+2\nu)$ rates determined by geochemical methods[19] in the cases of ^{82}Se-^{82}Kr and Te-Xe isotopes, even though in our example the tendency is in the opposite direction.

IV. RESULTS

Using the nuclear matrix elements designated a in Table I and the available experimental[20] limit $T^{\rm exp}_{1/2}(0\nu) > 2\times 10^{21}$ yr for the ^{48}Ca→^{48}Ti decay, and proceeding exactly as in I we obtain the following limits on the lepton violating parameters:

TABLE I. Nuclear matrix elements of the various operators entering double β decay (for definitions see text). b refers to the results of Ref. 8 and a to the results of the present calculation. In the case of the $A=48$ system we give our new results for the small space (indicated as c).

System		μ_0	$-\vec{Y}\cdot\vec{Y}$	Ω_ν	$\Omega_N+\Omega_\pi$	Ω_Δ	Ω_1	Ω_2	Ω_3
^{48}Ti(g.s.)	a	9	0.25	1.05	142	284	3.67	1.33	−4.03
	b	12	0.13	0.63	122	260	3.90	0.56	−6.70
	c	11	0.40	1.08	137	296	4.52	0.80	−4.25
^{58}Fe(g.s.)	a	6.5	0.30	1.17	160	146	−1.77	0.79	−1.17
	b	11	1.36	2.22	112	213	0.46	1.08	0.50

TABLE II. The predicted lifetimes for the double β decay of the $A=58$ system based on the lepton violating parameters extracted from the data on ^{48}Ca-^{48}Ti.

	$T_{1/2}(2\nu)$ (yr)	$T_{1/2}(0\nu)$ (yr) $\|\eta_\nu\|=8.4\times10^{-5}$ $n_{RL}=0$	$\|\eta_\nu\|=7.0\times10^{-5}$ $\|\eta_{RL}\|$ 7.0×10^{-5}	$\|\eta_i\|=8.6\times10^{-7}$ $i=N,H,R$	$\eta_\Delta=2.7\times10^{-7}$	$g=2.3\times10^{-3}$
$\beta^+\beta^+$	1.1×10^{37}	1.4×10^{26}	2.6×10^{26}	7.3×10^{27}	1.2×10^{29}	5.4×10^{29}
(e^-,e^+)	2.0×10^{26}	1.0×10^{26}	1.7×10^{26}	5.5×10^{25}	8.9×10^{26}	3.1×10^{24}

$$\eta_\nu| \le 7.0\times10^{-5}, \quad |\eta_{RL}| \le 7.0\times10^{-5},$$
$$|\langle m_\nu\rangle| \le 36 \text{ eV}, \quad |\eta_N| \le 8.6\times10^{-7},$$
$$|\eta_H| \le 8.6\times10^{-7}, \quad |\eta_\Delta| \le 2.7\times10^{-7}.$$

The value of $\langle m_\nu\rangle$ extracted is in agreement with that of Ref. 14. [For precise definitions of these parameters the reader is referred to Eqs. (12), (24), (13), (19a), and (21a) of I. η_R is obtained from η_N by replacing β with $\kappa=(m_{w_L}/m_{w_R})^2$.] Also using $\langle f\,|\,\Omega_4\,|\,i\rangle=0.25$ (see Table I) we obtain $T^{\rm th}_{1/2}(2\nu)=4.1\times10^{19}$ yr, which is very close to the experimental lower limit[20] $T^{\rm exp}_{1/2}(2\nu)>3.6\times10^{19}$ yr. From these we obtain $|g|\le1.7\times10^{-3}$. We remark that this value is about a factor of 3 smaller than the one given in I and it may be somewhat uncertain because the predicted lifetime is close to the experimental limit.

Using the upper limits of the above deduced lepton violating parameters we can predict the corresponding half-lives associated with processes (5) and (6) of the ^{58}Ni-^{58}Fe system which are presented in Table II.

In conclusion, we can say the following for the ^{48}Ca-^{48}Ti decays:

(1) Our improved wave functions have increased the Gamow-Teller matrix element by approximately a factor of 2 compared to the old value. Thus the predicted half-life is now only a factor of 2 longer than the present experimental limit. Thus even a modest improvement of the existing experiments may allow the observation of the lepton allowed 2ν $\beta\beta$ decay in the laboratory. Finally we note that the use of an appropriate effective interaction is more important than the expansion of the model space (compare a, b, and c of Table I).

(2) The expected improvement[6] of the 0ν lifetime by two orders of magnitude will set the stringent limit $|\langle m_\nu\rangle|<4$ eV on the light neutrino mass. The nuclear matrix element characterizing this process is about 1, i.e., bigger than that entering the 2ν process.

For the ^{58}Ni-^{58}Fe decays we notice both the $\beta^+\beta^+$ and electron capture processes proceed with the same rate, which is 10^5 slower than the corresponding one for the $A=48$ system. Thus such experiments appear hopeless without some experimental ingenuity allowing for the construction of a large target. We note that in the $\beta^+\beta^+$ emission no confusion can arise from the 2ν background since the latter is ten orders of magnitude slower than that of the 0ν process.

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