Charge-symmetry analysis of elastic $\pi^{\pm}$d scattering below the 33 resonance

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It is shown that elastic πd differential cross sections from 47.5 MeV up to 116 MeV are quite well described by the impulse approximation theory in its form-factor formulation. This theory is used to derive a theoretical expression for the charge asymmetry A, valid for $T_\pi \lesssim 100$ MeV, which includes the effects of Coulomb distortion, Δ- and target-baryon mass differences, and internal Coulomb corrections. The theoretically predicted A is found to be in reasonable agreement with the only existing data for A below 100 MeV at $T_\pi = 65$ MeV.

[NUCLEAR REACTIONS ^{2}H($\pi^{\pm},\pi^{\pm}$), $T_\pi = 65$ MeV; impulse approximation theory; charge-asymmetry A; (Δ^{++},Δ^{-}) mass difference; Coulomb distortion; internal Coulomb correction.]

It is, of course, well known that one of the consequences of the charge symmetry of the π-nucleon interaction is the equality of the π^+d and π^-d elastic cross sections apart from Coulomb effects. With the advent of high intensity pion beams together with the refinement in experimental techniques and accuracy for pion laboratory energies $\lesssim 140$ MeV (we especially allude here to the availability of both π^+d and π^-d data at the same energy), the experimental study of the $\pi^{\pm}$d system has naturally turned to the examination of the consistency of the data with simply Coulomb-corrected charge symmetric theory.[1] This comparison constitutes a very sensitive test of nuclear charge symmetry, particularly in the case of the differential cross sections, and is customarily[2,3] made in terms of the charge asymmetry A defined by

$$A(\theta) = \frac{(d\sigma/d\Omega)^{\pi^- d} - (d\sigma/d\Omega)^{\pi^+ d}}{(d\sigma/d\Omega)^{\pi^- d} + (d\sigma/d\Omega)^{\pi^+ d}} . \tag{1}$$

While it has been pointed out previously[1,4] that the intermediate-energy πd system is very well suited for testing charge symmetry, we want to emphasize here the optimal character of the pionic kinetic energy region < 100 MeV for such analyses from the point of view of *theory*. Because of the success of the three-body approach to πd interactions,[5] including the elastic channel, it may not be so well known that the experimental data from 47.5 MeV (Refs. 6 and 7) up to 116 MeV (Refs. 3 and 8) are still well described by the "old" impulse approximation (IA) theory. To set the stage for the charge symmetry calculations in that older framework which ensue, we first summarize the status and the use which we have made of that earlier (and surprisingly efficacious) theory here. Briefly, the extensive numerical study of the single-scattering contribution [which is expected to dominate the pion-nucleus interaction at low energy ($T_L^\pi \lesssim 100$ MeV), where the πN interaction is rather weak] to the elastic π^+d cross section in the 25–100 MeV region of McMillan and Landau[9] (which examines effects owing to the presence of the D-state component of the deuteron wave function, to the internal motion of the target nucleons, and to uncertainties in the off-energy-shell pion-nucleon scattering amplitudes and the pion-nucleon phase shifts) finds that while the deuteron cross section is sensitive to the presence of the D-state wave function component, it is *not* particularly sensitive to ambiguities (i.e., differing off-energy-shell functions and choice of interaction energy) in the off-energy-shell pion-nucleon amplitudes and indeed, that it is given quite accurately[10] by a form-factor approximation. This last approximation, which may be the most useful formulation of the theory, requires a choice of the momentum $\vec{k}_{av}$ of the struck nucleon at which the πN T matrices are to be evaluated. We have adopted Carlson's choice,[11,12]

$$\vec{k}_{av} = -\tfrac{1}{4}\vec{q} , \tag{2}$$

where $\vec{q}$ is the recoil momentum of the deuteron which McMillan and Landau find very accurate for all pion scattering angles through the 25–100 MeV region. In fact, they conclude that in the above energy and angular domain, the single scattering approximation to the πd scattering amplitude can, with little error, be written in terms of essentially on-shell πN amplitudes and deuteron form-factor integrals.

To make matters simpler, we use the McGee wave function[13] (with a 7% D-state probability), which considerably facilitates numerical (and, indeed, analytical) work, as well as the energy-dependent phase shifts of Rowe, Salomon, and Landau[14] in the present incarnation of the form-factor approximation. These phase shifts, a convenient parametrization of the on-shell πN interaction below 400 MeV, fitting the available S, P, and D wave πN phase shifts of various groups, reproduce most of the experimental cross sections well.

The IA expression for the elastic $\pi^{\pm}$d differential cross section is given in the form-factor approximation in the laboratory by[15] ($\omega_{k_L} \to \omega_{k_L'}$, where ω_{k_L} is the initial total pion energy in the laboratory and $\vec{q}=\vec{k}_L-\vec{k}'_L$)

$$\frac{d\sigma^{\pi^{\pm}d}}{d\Omega_L}(T_L^{\pi},\theta_L)=\frac{k_L'^3E_d(q)}{k_L[k_L'^2(M_d+\omega_{k_L})-\vec{k}_L\cdot\vec{k}'_L\omega_{k'_L}]}\times[|F_S^{\pm}|^2+|F_D^{\pm}|^2+\tfrac{2}{3}|G_S+G_D|^2] \quad (3)$$

and

$$F_S^{\pm}=R(T_L^{\pi},\theta_L,\vec{k}_{av})f^{\pm}(\vec{\kappa}'_{av},\vec{\kappa}_{av};E)A_0(q)\ , \quad (4a)$$

$$G_S=R(T_L^{\pi},\theta_L,\vec{k}_{av})g(\vec{\kappa}'_{av},\vec{\kappa}_{av};E)A_0(q)\ , \quad (4b)$$

$$|F_D^{\pm}|^2=[R(T_L^{\pi},\theta_L,\vec{k}_{av})]^2|f^{\pm}(\vec{\kappa}'_{av},\vec{\kappa}_{av};E)|^2\times\{(C_0(q))^2+4[B_2(q)-\tfrac{1}{4}\sqrt{2}C_2(q)]^2\}\ , \quad (4c)$$

$$G_D=R(T_L^{\pi},\theta_L,\vec{k}_{av})g(\vec{\kappa}'_{av},\vec{\kappa}_{av};E)\times\left[-\tfrac{3}{2}C_0(q)+\frac{\sqrt{2}}{2}B_2(q)+\tfrac{1}{2}C_2(q)\right]\ , \quad (4d)$$

where

$$R(T_L^{\pi},\theta_L,k)=\frac{(ss')^{1/4}}{[E_N(k)E_N(\vec{k}+\vec{q})]^{1/2}}\ , \quad (5)$$

$$E_N(k)=(M^2+k^2)^{1/2}\ , \quad (6)$$

with the customary two-body invariants given by

$$s=m_\pi^2+M^2+2[\omega_{k_L}E_N(k)-\vec{k}_L\cdot\vec{k}]\ , \quad (7)$$

$$s'=m_\pi^2+M^2+2[\omega_{k'_L}E_N(\vec{k}+\vec{q})-\vec{k}'_L\cdot(\vec{k}+\vec{q})]\ , \quad (8)$$

$$t=2(m_\pi^2-\omega_{k_L}\omega_{k'_L}+\vec{k}_L\cdot\vec{k}'_L)\ . \quad (9)$$

Note that in the present calculations we have elected to take $E=\sqrt{s}$. The initial and final pion center-of-mass momenta are given by

$$\kappa=\{[s-(M+m_\pi)^2][s-(M-m_\pi)^2]/4s\}^{1/2}\ , \quad (10a)$$

$$\kappa'=[s'-(M+m_\pi)^2][s'-(M-m_\pi)^2]/4s'\}^{1/2}\ , \quad (10b)$$

respectively. It follows trivially from Eq. (8) that

$$\vec{\kappa}\cdot\vec{\kappa}'=\tfrac{1}{2}t+\omega_\kappa\omega_{\kappa'}-m_\pi^2\ . \quad (11)$$

Note that by $\vec{\kappa}_{av}$, $\vec{\kappa}'_{av}$ we mean $\vec{\kappa}[s(\vec{k}=-\frac{1}{4}\vec{q})]$, $\vec{\kappa}'[s'(\vec{k}=-\frac{1}{4}\vec{q})]$. The form factors A_0, B_2, C_0, and C_2 are

$$A_0=\int_0^\infty j_0(\tfrac{1}{2}qr)u^2(r)dr\ , \quad (12a)$$

$$B_2=\int_0^\infty j_2(\tfrac{1}{2}qr)u(r)w(r)dr\ , \quad (12b)$$

$$C_0=\int_0^\infty j_0(\tfrac{1}{2}qr)w^2(r)dr\ , \quad (12c)$$

$$C_2=\int_0^\infty j_2(\tfrac{1}{2}qr)w^2(r)dr\ , \quad (12d)$$

where the S-state radial wave function is given by[13]

$$u(r)=N\sum_{i=1}^{5}S_ie^{-\alpha_im_\pi r/\hbar c}\ , \quad (13)$$

with

$$\begin{aligned}&(S_1,\alpha_1)=(1.0,0.3294)\ ,\\&(S_2,\alpha_2)=(-0.63608,5.733\alpha_1)\ ,\\&(S_3,\alpha_3)=(-6.6150,12.844\alpha_1)\ ,\\&(S_4,\alpha_4)=(15.2162,17.331\alpha_1)\ ,\\&(S_5,\alpha_5)=(-8.9651,19.643\alpha_1)\ ,\\&N=1.05607(m_\pi/\hbar c)^{1/2}\ ,\end{aligned} \quad (14)$$

and the D-state radial wave function is given by[13]

$$w(r)=0.0269N\sum_{i=1}^{6}D_ie^{-\beta_im_\pi r/\hbar c}\left[1+\frac{3}{\beta_i(m_\pi r/\hbar c)}+\frac{3}{\beta_i^2(m_\pi r/\hbar c)^2}\right]\ , \quad (15)$$

with

$$\begin{aligned}&(D_1,\beta_1)=(1.0003,0.3294)\ ,\\&(D_2,\beta_2)=(-20.34,4.833\alpha_1)\ ,\\&(D_3,\beta_3)=(-36.60,10.477\alpha_1)\ ,\\&(D_4,\beta_4)=(-123.02,14.506\alpha_1)\ ,\\&(D_5,\beta_5)=(305.11,16.868\alpha_1)\ ,\\&(D_6,\beta_6)=(-126.16,21.154\alpha_1)\ .\end{aligned} \quad (16)$$

One has for the spin-non-flip ($f^{\pm}$) and spin-flip (g) two-body amplitudes,

$$f^{\pm}=\tfrac{4}{3}f^{(3/2)}+\tfrac{2}{3}f^{(1/2)}\pm f_C\ , \quad (17)$$

$$g=\tfrac{4}{3}g^{(3/2)}+\tfrac{2}{3}g^{(1/2)}\ , \quad (18)$$

where

$$f^{(I)}(\vec{\kappa}',\vec{\kappa};E)=\sum_{l=0}^{1}\left[(l+1)\sigma_{l+}^{(I)}(\kappa',\kappa;\kappa_0)\frac{e^{i\delta_{l+}^{(I)}(\kappa_0)}}{\kappa_0}\sin\delta_{l+}^{(I)}(\kappa_0)+l\sigma_{l-}^{(I)}(\kappa',\kappa;\kappa_0)\frac{e^{i\delta_{l-}^{(I)}(\kappa_0)}}{\kappa_0}\sin\delta_{l-}^{(I)}(\kappa_0)\right]P_l(\cos\theta) \quad (19)$$

and

$$g^{(I)}(\vec{\kappa}',\vec{\kappa};E)=\sum_{l=0}^{1}\left[\sigma_{l+}^{(I)}(\kappa',\kappa;\kappa_0)\frac{e^{i\delta_{l-}^{(I)}(\kappa_0)}}{\kappa_0}\sin\delta_{l+}^{(I)}(\kappa_0)-\sigma_{l-}^{(I)}(\kappa',\kappa;\kappa_0)\frac{e^{i\delta_{l-}^{(I)}(\kappa_0)}}{\kappa_0}\sin\delta_{l-}^{(I)}(\kappa_0)\right]P_l^1(\cos\theta)\ , \quad (20)$$

with

$$\kappa_0 = \kappa(s = s_0 = E^2) \ . \tag{21}$$

We take for the off-shell functions $\sigma_{l\pm}^{(I)}$ (Refs. 9 and 16),

$$\sigma_{0+}^{(I)} = 1 \ , \tag{22a}$$

$$\sigma_{1\pm}^{(I)}(\kappa', \kappa; \kappa_0) = \kappa\kappa'/\kappa_0^2 \ . \tag{22b}$$

After McMillan and Landau,[9] we write the off-shell modified point Coulomb amplitude ($\alpha = \frac{1}{137}$) as

$$f_C = -\frac{\alpha M(T_L^{\pi} + m_{\pi})}{E\kappa_0^2(1-\cos\theta)} \ . \tag{23}$$

Theory, which has been so compactly summarized in Eqs. (2)—(23), and experiment, at 47.5,[6,7] 65,[3] 82, 116, and 142 MeV,[8] (π^+), and at 47.5 (preliminary data),[7] 65,[3] and 142 MeV (π^-),[2] are compared in Figs. 1—3. One sees that the quality of the fits provided by the form-factor approximation ranges from marginally acceptable (47.5 MeV) (Ref. 17) to very good (82 MeV) beyond 60° in the laboratory. In this connection it seems appropriate to point out once again that the inclusion of *P*-wave pionic absorption leads to a further improvement in the backward fit[18] to data provided by the phase shift input of Rowe *et al.*[14] This is illustrated in Figs. 4 and 5 in the case of the 65 MeV ($\pi^{\pm}$) Saclay data; for π exchange only (using the McGee wave function[13]), the theoretical differential cross section enhanced by the *P*-wave dispersive contribution[18] is seen to agree remarkably well with the experimental data for backward angles for both π^+ and π^- on d. We conclude that *the IA theory in the form factor appproximation offers a viable alternative (to the three-body theory) description of the data in the pion kinetic energy range* $\lesssim$ *100 MeV.*

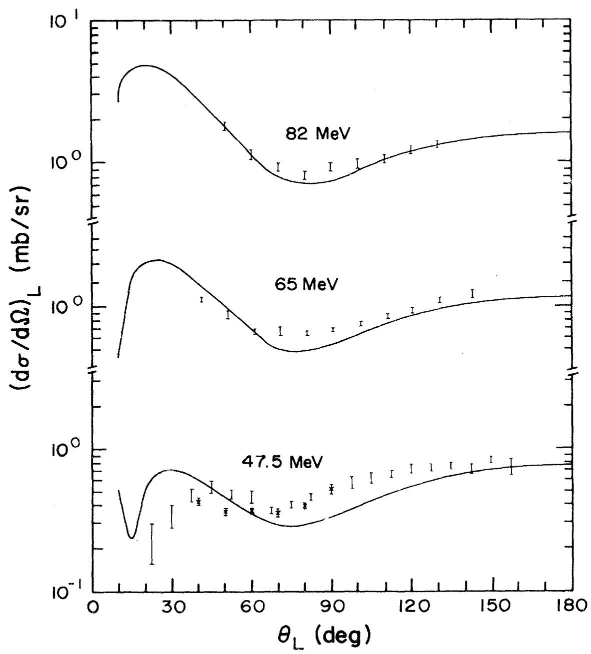

FIG. 1. Comparison of elastic π^+d differential cross sections in the form factor version of IA theory with data at 47.5 (Refs. 6 and 7), 65 (Ref. 3), and 82 (Ref. 8) MeV.

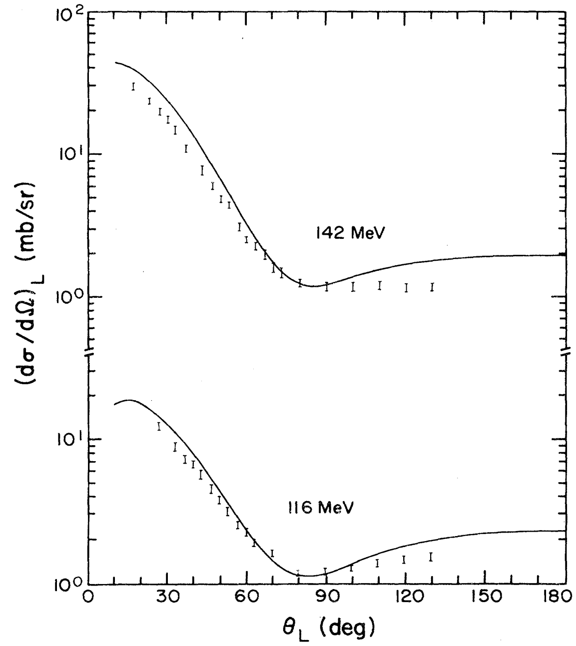

FIG. 2. Same as Fig. 1 at 116 and 142 MeV (Ref. 8).

We turn now to the consequences of this theory in its form-factor approximation for charge symmetry in terms of the predicted $A(\theta_L)$ [Eq. (1)] and to the introduction of the simple modifications of IA input which enable one to treat the charge-symmetry-breaking effects on A from Δ-isobar and spectator-mass differences and from external and internal Coulomb corrections.

We take up the external Coulomb correction first. Here, we follow Pedroni *et al.*[1] in adopting a potential

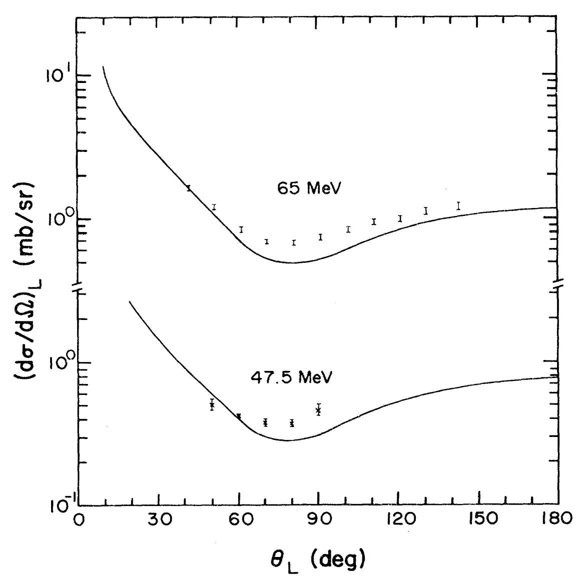

FIG. 3. Same as Fig. 1 for π^-d at 47.5 (Ref. 7) and 65 (Ref. 3) MeV.

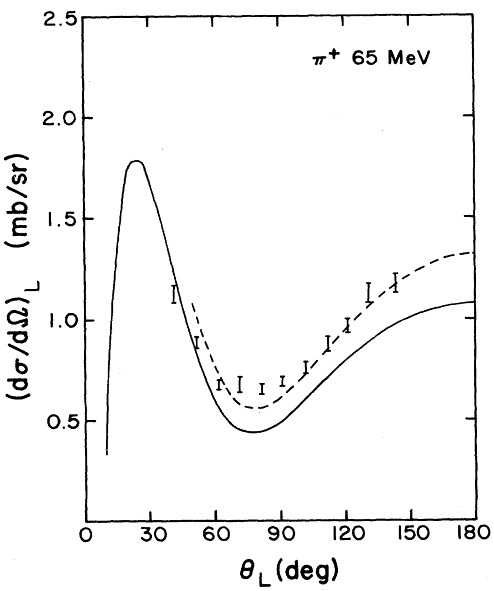

FIG. 4. Comparison of 65 MeV π^+ Saclay data (Ref. 3) with IA prediction [with Coulomb distortion (solid curve)] and with P-wave dispersive enhancement (Ref. 18) with π exchange only (dashed curve).

model approach suitable for the low-energy regime. The correction for Coulomb distortion in the two-body channels ($\pi^{\pm}$p) entails the substitutions[1]

$$a^{(3/2)}_{3/2^+}(\kappa_0)=[e^{2i\delta^{(3/2)}_{3/2}}-1]/2i\kappa_0$$
$$\rightarrow[e^{2i(\delta^{(3/2)}_{3/2^+}+C^{+}_{33})}-1]/2i\kappa_0 \quad (24)$$

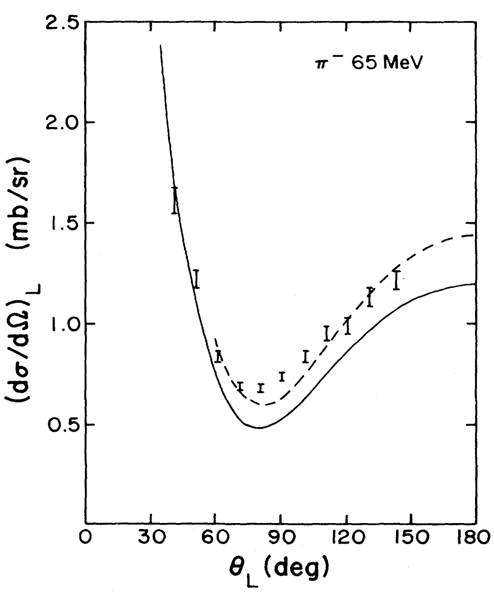

FIG. 5. Comparsion of 65 MeV π^- Saclay data (Ref. 3) with IA prediction [with Coulomb distortion (solid curve)] and with P-wave dispersive enhancement (Ref. 18) with π exchange only (dashed curve).

in the non-spin-flip (f) and spin-flip (g) π^+p elastic amplitudes,

$$f^{\pi^+p}=f^{(3/2)}+f_c\ ,$$
$$g^{\pi^+p}=g^{(3/2)}\ , \quad (25)$$

and[1]

$$a^{(3/2)}_{3/2^+}(\kappa_0)\rightarrow[e^{2i[\delta^{(3/2)}_{3/2^+}+(1/3)C^{-}_{33}]}-1]/2i\kappa_0 \quad (26)$$

in the non-spin-flip (f) and spin-flip (g) π^-p elastic amplitudes,

$$f^{\pi^-p}=\tfrac{1}{3}f^{(3/2)}+\tfrac{2}{3}f^{(1/2)}-f_c\ ,$$
$$g^{\pi^-p}=\tfrac{1}{3}g^{(3/2)}+\tfrac{2}{3}g^{(1/2)}\ . \quad (27)$$

Following Zimmermann,[19] we make a two-parameter (A,B) ansatz for the $\frac{3}{2}^+$ wave functions[1] and, neglecting particle size effects, find the approximate results,[20]

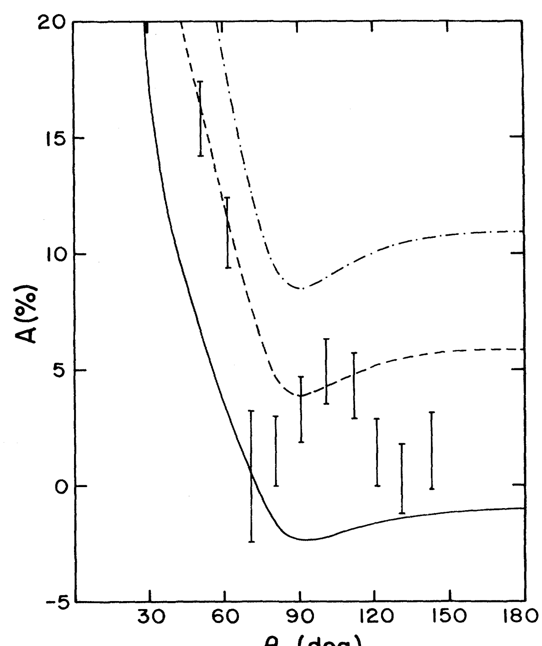

FIG. 6. Comparison of theoretical predictions of $A(\theta_L)$ at 65 MeV with Saclay data (Ref. 3): (i) No Coulomb distortion effects (solid curve); (ii) Coulomb distortion effects in the two-body channels ($\pi^{\pm}$p) only according to Ref. 1 (dashed curve); (iii) Coulomb distortion effects in *all* two-body channels (both $\pi^{\pm}$p and $\pi^{\pm}$n) according to Ref. 1 (dot-dashed curve). Particle size is neglected.

$$C_{33}^{\pm}=\mp\frac{2\alpha}{\beta}\left\{(kr_{\rm N})^4[\tfrac{1}{4}A^2+\tfrac{2}{5}ABkr_{\rm N}+\tfrac{1}{6}B^2(kr_{\rm N})^2]+\frac{1}{(2kr_{\rm N})^2}[1-\cos2(\delta_{33}+kr_{\rm N})]\right.$$
$$-\frac{1}{2kr_{\rm N}}\sin2(\delta_{33}+kr_{\rm N})+\tfrac{1}{2}[\gamma+\ln2kr_{\rm N}-(kr_{\rm N})^2]\cos2\delta_{33}$$
$$\left.-\left[-\frac{\pi}{4}+kr_{\rm N}\right]\sin2\delta_{33}-(-1+\tfrac{1}{2}\gamma+\tfrac{1}{2}\ln2kr_{\rm N})\right\}, \tag{28}$$

adequate for our purposes. The effect of the correction for Coulomb distortion in the case of pionic scattering on the proton in the deuteron is shown in Fig. 6 (dashed curve). Note that already at this stage of correction one finds theory and experiment in good agreement for $\theta_L \lesssim 50°$. In the case of pionic scattering on the *neutron* in the deuteron ($\pi^{\pm}$n channels), the approach of Ref. 1 furnishes us with an *effective* two-body correction [Eqs. (5.2a), (5.6), and (5.11) of Ref. 1] in what is essentially a three-body context. [Here we take[1] for the effective charge radius of the neutron $r_c \simeq \frac{8}{3}R_{\rm d}^2$, with $R_{\rm d}\simeq 2.0$ fm, where $R_{\rm d}$ is the deuteron root-mean-square (rms) radius.] This additional distortion correction calculated numerically here leads to quite sizeable effects (see the dot-dashed curve of Fig. 6).

That the Δ-isobar mass difference[21] (in particular, $M_{\Delta^-}-M_{\Delta^{++}}$) might be a significant dynamical mechanism for the generation of charge-symmetry-breaking effects was recognized and studied some time ago by Myhrer and Pilkuhn[22] in the case of $\pi^{\pm}$d total cross sections at the Δ resonance. Recently, Rinat and Alexander[23] have taken up this idea again in their study of elastic $\pi^{\pm}$d scattering in the intrinsically off-shell approach of the three-body method with a view to examining the effect of such perturbations on the three-body prediction for A at $T_L^{\pi}=143$ MeV where experimental results for the asymmetry parameter exist.[2] They use (as we do, also) the most recent data[24] for the electromagnetic mass difference of the Δ^- and Δ^{++}, $M_{\Delta^-}-M_{\Delta^{++}}=5.9\pm3.1$ MeV. (With this measurement the accuracy has been improved by a factor of 2 over previous values.) The effect of the Δ-isobar mass difference on $A(\theta_L)$ is easily predicted from the observation that as

$$A(\theta_L)\sim d\sigma^{\pi^-{\rm d}}/d\Omega_L - d\sigma^{\pi^+{\rm d}}/d\Omega_L\ ,$$

the *increased* mass of the Δ^- relative to the Δ^{++} implies a corresponding *weakening* of π^-n scattering in the (dominant) 33 channel vis-à-vis the corresponding two-body scattering of π^+p, thus moving A toward negative values. (There will be some moderation in this effect when one takes into account the mass splitting of the target baryons with $M_{\rm n}-M_{\rm p}\neq 0$.) The various mass perturbations may be handled quantitatively in the simple unitarized model of low-energy 33 scattering made up of the comparable contributions from the Δ resonance and crossed nucleon poles,[25]

$$e^{i\delta_{33}(\kappa_0)}\sin\delta_{33}(\kappa_0)=\frac{4}{3}\left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3}{m_\pi^2}\frac{(M_\Delta-M)}{\omega^*\left[(M_\Delta-M)-\omega^*-i\frac{\Gamma}{2}(\kappa_0)\right]}\ , \tag{29}$$

where

$$\Gamma(\kappa_0)=\frac{8}{3}\left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3(M_\Delta-M)}{m_\pi^2\omega^*(\kappa_0)} \tag{30}$$

and

$$\omega^*=\omega(\kappa_0)+E(\kappa_0)-M\ . \tag{31}$$

After taking $M_\Delta-M=2.1m_\pi$ and "fine-tuning" the coupling to the value, $f^2/4\pi=0.084$, expression (29) is found to yield a phase shift *nearly indistinguishable* from that of Rowe *et al.*[14] up to 20°. (Even at $\delta_{33}\simeq 60°$, the divergence between the two fits is only about 10%.) (See Fig. 7.)

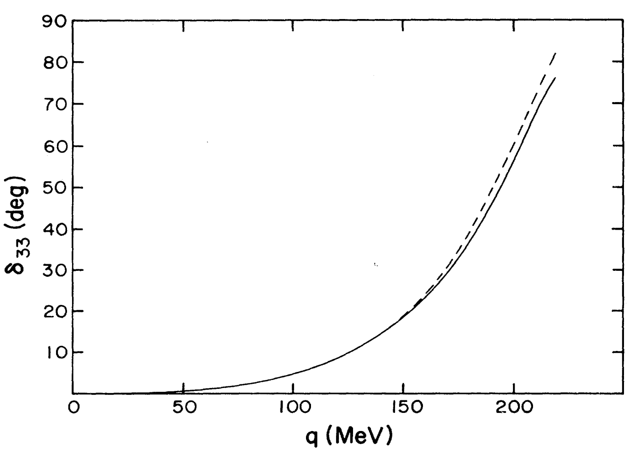

FIG. 7. Comparison of $P(33)$ phase shift (in degrees) in the analysis of Rowe *et al.* (Ref. 14) (solid curve) with the simple unitarized model including nucleon recoil (see text). ($f^2/4\pi=0.084$.) Note they are virtually indistinguishable up to 20 deg.

Neglecting Coulomb distortion for the moment, one has the *on-shell* substitution rule for π^+d elastic scattering,

$$e^{i\delta_{33}}\sin\delta_{33} \rightarrow \tfrac{1}{4}e^{i\delta_{33}}\sin\delta_{33} + \left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3}{m_\pi^2}\frac{[(M_{\Delta^{++}}-M_p)+(M_n-M_p)]}{(\omega^*+M_n-M_p)\left[(M_{\Delta^{++}}-M_p)-\omega^*-i\dfrac{\Gamma^{++}}{2}\right]}\,, \tag{32}$$

where

$$\Gamma^{++} = \frac{8}{3}\left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3}{m_\pi^2}\frac{[(M_{\Delta^{++}}-M_p)+(M_n-M_p)]}{(\omega^*+M_n-M_p)}\,, \tag{33}$$

together with the analogous rule for π^-d elastic scattering,

$$e^{i\delta_{33}}\sin\delta_{33} \rightarrow \tfrac{1}{4}e^{i\delta_{33}}\sin\delta_{33} + \left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3}{m_\pi^2}\frac{[(M_{\Delta^{-}}-M_n)+(M_p-M_n)]}{(\omega^*+M_p-M_n)\left[(M_{\Delta^{-}}-M_n)-\omega^*-i\dfrac{\Gamma^{-}}{2}\right]}\,, \tag{34}$$

where

$$\Gamma^{-} = \frac{8}{3}\left[\frac{f^2}{4\pi}\right]\frac{\kappa_0^3}{m_\pi^2}\frac{[(M_{\Delta^{-}}-M_n)+(M_p-M_n)]}{(\omega^*+M_p-M_n)}\,. \tag{35}$$

In Fig. 8, we show a plot of $A(\theta_L)$ (dot-dashed curve) which includes both the effects of Coulomb distortion and of baryon mass differences. Note that the predicted $A(\theta_L)$ shows considerable sensitivity to the latter of these effects for $\theta_L \gtrsim 80°$. In this angular domain, theory predicts a smooth variation with θ_L which is consistent only in an average sense with the structure exhibited by the data. On the other hand, we observe that even this qualitative agreement disappears when one includes the effects of Coulomb distortion in the ($\pi^{\pm}$n) channels according to Ref. 1. [See Fig. 9 (solid curve).] (This may indicate that *a larger effective neutron charge radius would be more realistic here*.)

We conclude our discussion of perturbations on $A(\theta_L)$ with an elementary treatment of the internal Coulomb corrections in terms of the IA formalism. We confine ourselves to those arising from intermediate states in the

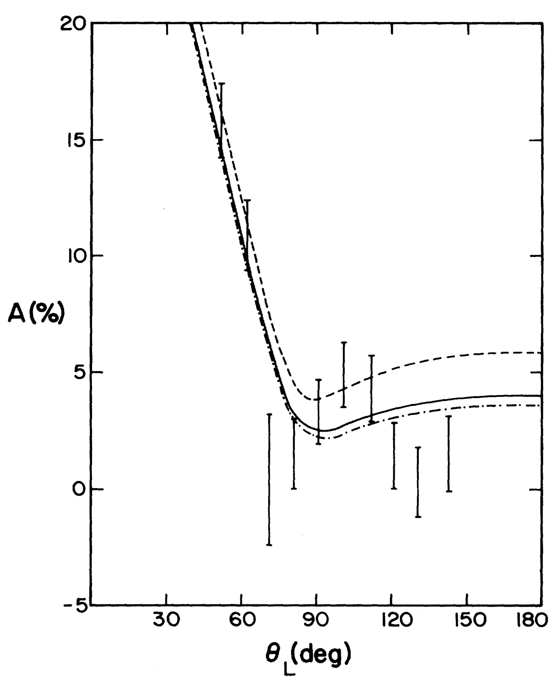

FIG. 8. Comparison of theoretical predictions of $A(\theta_L)$ at 65 MeV with Saclay data (Ref. 3): (i) Coulomb distortion effects in the two-body channels ($\pi^{\pm}$p) only according to Ref. 1 (dashed curve); (ii) Coulomb distortion effects as in (i) together with Δ-mass perturbation ($M_{\Delta^-}-M_{\Delta^{++}}=5.9$ MeV) and spectator mass-difference perturbation (dot-dashed curve); (iii) Coulomb distortion effects and mass perturbation [as in (ii)] *and* internal Coulomb corrections from a simplified cutoff model (see text) in the case of π^-d scattering (solid curve).

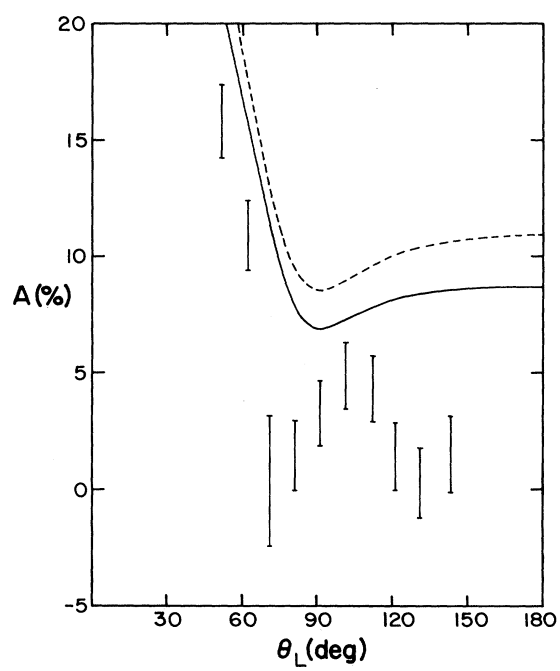

FIG. 9. Comparison of theoretical predictions of $A(\theta_L)$ at 65 MeV with Saclay data (Ref. 3): (i) Coulomb distortion effects in *all* two-body channels (both $\pi^{\pm}$p and $\pi^{\pm}$n) according to Ref. 1 (dashed curve); (ii) Coulomb distortion effects as in (i) together with Δ-mass perturbation and spectator mass-difference perturbation (solid curve).

(π^-n) channel. [Observe that the spectator baryon is *neutral* in the case of the (π^+p) channel.] These corrections comprise the attractive (Δ^-p) interaction and the repulsive (pp) interaction (from the crossed nucleon pole). Since the resonance and crossed nucleon pole contributions to δ_{33} are nearly equal in size at threshold, *we expect the internal Coulomb corrections to nearly cancel in the low-energy region.* Quantitatively, the internal Coulomb correction (for *point charges*) is characterized by

$$I=\int_0^{q_{\max}}\frac{d\vec{q}}{(2\pi)^3}\frac{e^2}{\vec{q}^{\,2}}=\frac{2\alpha}{\pi}q_{\max}\simeq 1\ \text{MeV}\ ,$$

for a cutoff of 1 fm^{-1}. After making the appropriate mass shifts in our model expressions (32) and (33) we see that the perturbation translates into the small effect ($\lesssim 15\%$) exhibited in Fig. 8 (solid curve). In short, the apparent discrepancy between theory and experiment noted earlier persists. It is hoped that with the further acquisition of elastic $\pi^{\pm}$d scattering data at energies below $T_\pi = 100$ MeV, this puzzle will be resolved.

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[11]In this case, $\vec{k}_{av}$ is the momentum which makes the *S*-state average of the second term in the Taylor expansion about $\vec{k}_{av}$ of the integrals of the non-spin-flip amplitudes vanish. It is also the case that the resulting amplitudes are on shell when the deuteron is taken to be a nonrelativistically recoiling particle of mass $2M$, where M is the nucleon mass.

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$$A=\left[j_2(kr_{\rm N})+\frac{1}{kr_{\rm N}}j_1(kr_{\rm N})\right]\cos\delta_{33} - \left[n_2(kr_{\rm N})+\frac{1}{kr_{\rm N}}n_1(kr_{\rm N})\right]\sin\delta_{33}\ ,$$

$$B=\frac{1}{kr_{\rm N}}[n_2(kr_{\rm N})\sin\delta_{33}-j_2(kr_{\rm N})\cos\delta_{33}]\ .$$

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