# Doorways and multistep compound reactions

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The "optical background" representation of Kawai, Kerman, and McVoy for the fluctuation amplitude is written in an alternate form which has certain advantages in formulating the theory of doorway states and of multistep compound reactions. We reconsider the doorway state approach of Feshbach, Kerman, and Lemmer using the present representation of the fluctuation amplitude. Finally, we use the present representation of the fluctuation amplitude for developing a theory of multistep compound reactions following the idea of chaining due to Feshbach, Kerman, and Koonin. The advantages of the present approach are emphasized in both the cases.

[NUCLEAR REACTIONS Optical background representation of the fluctuation amplitude, doorways, preequilibrium reactions, multistep compound reactions.]

## I. INTRODUCTION

The original idea of Bohr, commonly known as the independent hypothesis,[1,2] together with the subsequent evaporation model[3] by Weisskopf is a well understood and successful formalism for explaining various aspects of compound nuclear processes. Similarly, the mechanism of direct nuclear reactions is also now well understood.[4] But the reactions which are neither purely direct nor purely compound still lack a satisfactory dynamical theory. The aim of the present paper is to improve the understanding of the mechanism of such reactions through a satisfactory dynamical theory. Such reactions include, for example, preequilibrium reactions and heavy ion deep inelastic collisions.

Feshbach, Kerman, and Koonin[5] (FKK) recently developed a dynamical theory for such processes. They generalized the doorway state approach of Feshbach, Kerman, and Lemmer[6,7] (FKL) to the case where a hierarchy of doorways are present. They proposed the chaining hypothesis where they assumed that the reaction proceeds to the most complicated compound nuclear states through different stages of increasing complexity and transition to the final state is allowed at any of the intermediate stages including the most complicated stage. FKK worked with the optical background representation of the scattering amplitude first introduced by Kawai, Kerman, and McVoy[8] (KKM) and applied the chaining hypothesis. By so doing they developed a statistical theory for multistep compound and direct reactions. They broke up both the direct and fluctuation cross sections into multistep components—the successive components involving an intermediate stage of increasing complexity. This idea of dividing the cross section into its multistep components was first used by Griffin[9] in his exciton model. Since then, before and after the work by FKK various workers[10] have divided the cross section into multistep components and experimental results have supported this idea.[11,12]

The elegant formulation of FKK defines the direct and fluctuation amplitudes using the approach of KKM. This introduces two problems into the formalism. First, the chaining condition which FKK introduces is merely a hypothesis imposed from outside, though their dynamical formulation violates it. A part of this violation is physical in origin, another part is a mathematical artifact of the special version of the optical background representation they use. FKK divide the closed (open) channel space $Q$ ($P$) into a hierarchy of orthogonal spaces $Q_n$ ($P_n$), $n=1,2,\ldots,r$, where the excitations of the $n$th stage is more complex than those of the $(n-1)$th stage. Then they assume that the residual interaction can induce transition from the $n$th stage only to the $(n\pm1)$th stages. That means the spaces $Q_n$ and $P_n$ are related by the residual interaction to $Q_{n\pm1}$ and $P_{n\pm1}$ spaces. Hence, physically, chaining hypothesis must be broken for the time evolution of the system in the $Q$ space. For example, instead of passing from the $Q_n$ space to the $Q_{n+1}$ space, the system may first pass to the $P_{n+1}$ space and then propagate to the $P_m$ space through the $P$ space and then pass to the $Q_{m\pm1}$ space. Sevgen[13] pointed out that such a violation of the chaining hypothesis must occur and found out the modifications necessary in order to incorporate such processes. We call such a violation of the chaining hypothesis physical. However, at extreme low energies where the elastic channel is the only open channel whose projection is denoted by $P_0$ in FKK one has $P=0$ and physical violations of the $Q$ space chaining through the $P$ space is not allowed. Even in this limit the chaining condition is not exact in the FKK approach. Second, the optical background representation of KKM has the nice property that the fluctuation amplitudes average to zero without requiring the property of statistical fluctuation of the form factors of certain transitions. It would be nice to maintain this property while defining multistep components of the fluctuation amplitudes. The multistep components of FKK do not satisfy this property and use the property of statistical fluctuation of certain form factors. It is the purpose of the present paper to remedy the above two limitations of the FKK approach.

We provide an alternative derivation of the KKM optical background representation and write the final result in a form which has certain advantages in defining the multistep fluctuation components. First, the unphysical violations of the chaining condition are eliminated, i.e., in the

 

extreme low energy limit ($P\to 0$) the chaining condition becomes exact. Second, the multistep fluctuation amplitudes defined from this representation average to zero without needing the property of statistical fluctuation of the form factors.

Using the present representations we derive the multistep compound cross sections in the general case where the chaining condition is not valid. Sevgen[13] considered this problem assuming that the system enters the $Q$ space always through the stage $Q_1$. We do not see any justification for imposing this constraint when the chaining condition is removed, since, physically, the system can propagate in the $P$ space until it reaches the stage $n$ and then it can pass to the $Q_{n+1}$ space. We show how this possibility can be accommodated to the considerations of Sevgen[13] using the present form of the optical background representation.

In Sec. II we derive the alternate representation for the optical background representation and show its equivalence to the KKM representation. As a warmup to the discussion of multistep compound reactions, in Sec. III we discuss the doorway state formulation of FKL using the present and the usual optical background representations and show the advantages of the present approach. In Sec. IV we define the multistep fluctuation components using the present form of the optical background representation. In Sec. V we show how the chaining condition is restored in the low energy limit where the elastic channel is the only open channel. Finally, in Sec. VI we present a brief summary.

## II. THE OPTICAL BACKGROUND REPRESENTATION

For the sake of simplicity we consider the problem of elastic scattering of two nuclei. Following Feshbach[14] we separate the transition amplitude into its slowly and rapidly varying components as follows:

$$T=\langle\phi\,|\,(V+VG_{PP}V)\,|\,\phi\rangle + \langle\phi\,|\,(1+VG_{PP})H\mathscr{G}_{QQ}H(1+G_{PP}V)\,|\,\phi\rangle\ , \tag{2.1}$$

where

$$G_{PP}=P(E-H_{PP}+i\epsilon)^{-1}P\ , \tag{2.2}$$

$$\mathscr{G}_{QQ}=Q(E-H_{QQ}-H_{QP}G_{PP}H_{PQ}+i\epsilon)^{-1}Q\ , \tag{2.3}$$

and where $V$ is the channel interaction, $\phi$ is the incident channel state, and $H$ is the Hamiltonian of the system. In Eqs. (2.1)–(2.3) we use the notation $H_{PP}=PHP$, $H_{PQ}=PHQ$, etc., where $P$ and $Q$ refer, as usual, to open and closed channel spaces. The first term on the right-hand side of (2.1) varies slowly with energy while the last term varies rapidly with energy. However, the average of the last term in (2.1) is not zero. This problem can be easily avoided by defining

$$T=T_{\rm op}+T_{\rm fl}\ , \tag{2.4}$$

where

$$T_{\rm op}=\langle\phi\,|\,(V+VG_{PP}V)\,|\,\phi\rangle + \langle\phi\,|\,(1+VG_{PP})H\langle\mathscr{G}_{QQ}\rangle_I H(1+G_{PP}V)\,|\,\phi\rangle \tag{2.5}$$

and

$$T_{\rm fl}=\langle\phi\,|\,(1+VG_{PP})H(\mathscr{G}_{QQ}-\langle\mathscr{G}_{QQ}\rangle_I)H(1+G_{PP}V)\,|\,\phi\rangle\ , \tag{2.6}$$

where, as in *KKM*, $\langle\ \rangle_I$ denotes averaging over an energy interval $I$. As in KKM we use the Lorentzian averaging function which yields

$$\langle\mathscr{G}_{QQ}\rangle_I=Q[E-H_{QQ}-H_{QP}G_{PP}H_{PQ}+i(I/2)]^{-1}Q\ . \tag{2.7}$$

Insofar as $\langle\mathscr{G}_{QQ}\rangle_I$ is unchanged by reaveraging the energy average of fluctuation amplitude $T_{\rm fl}$ is zero, $\langle T_{\rm fl}\rangle_I\cong 0$. Hence, with the separation given by (2.4) the compound-direct interference is eliminated in the average cross section. Using the following iterative series for $\langle\mathscr{G}_{QQ}\rangle_I$,

$$\langle\mathscr{G}_{QQ}\rangle_I=\langle G_{QQ}\rangle_I+\langle G_{QQ}\rangle_I HG_{PP}H\langle G_{QQ}\rangle_I+\cdots\ , \tag{2.8}$$

where

$$\langle G_{QQ}\rangle_I=Q[E-H_{QQ}+i(I/2)]^{-1}Q\ , \tag{2.9}$$

in Eq. (2.5) and rearranging terms it is not difficult to see that $T_{\rm op}$ can be written in the form of the first term on the right-hand side of Eq. (2.1):

$$T_{\rm op}=\langle\phi\,|\,\mathscr{U}+\mathscr{U}\mathscr{G}_{\rm op}\mathscr{U})\,|\,\phi\rangle\ , \tag{2.10}$$

where

$$\mathscr{G}_{\rm op}=P(E-H_{PP}-H_{PQ}\langle G_{QQ}\rangle_I H_{QP})^{-1}P \tag{2.11}$$

and

$$\mathscr{U}=V+H\langle G_{QQ}\rangle_I H\ . \tag{2.12}$$

The resolvent operator $\mathscr{G}_{\rm op}$ was introduced by KKM in their formulation of the optical background representation. It is not difficult to realize that $T_{\rm op}$ given by Eq. (2.10) is the optical transition amplitude implicit in the work of KKM. Using Eq. (2.7), Eq. (2.6) can be written as

$$T_{\rm fl}=\langle\phi\,|\,(1+VG_{PP})H\langle\mathscr{G}_{QQ}\rangle_I i(I/2)\mathscr{G}_{QQ}H(1+G_{PP}V)\,|\,\phi\rangle\ . \tag{2.13}$$

Equations (2.4), (2.10), and (2.13) constitute the present form of the optical background representation. In the representation of KKM Eq. (2.13) for the fluctuation amplitude is replaced by

$$T_{\rm fl}=\langle\phi\,|\,(1+\mathscr{U}\mathscr{G}_{\rm op})V_{PQ}\mathscr{G}_{QQ}^{\rm KKM}V_{QP}(1+\mathscr{G}_{\rm op}\mathscr{U})\,|\,\phi\rangle\ , \tag{2.14}$$

where

$$\mathscr{G}_{QQ}^{\rm KKM}=Q(E-H_{QQ}-V_{QP}\mathscr{G}_{\rm op}V_{PQ})^{-1}Q\ , \tag{2.15}$$

$$V_{PQ}=H_{PQ}[i(I/2)\langle G_{QQ}\rangle_I]^{1/2}\ , \tag{2.16}$$

and

$$V_{QP}=[i(I/2)\langle G_{QQ}\rangle_I]^{1/2}H_{QP}\ . \tag{2.17}$$

Since the optical amplitudes of KKM and the present work are identical it may not be surprising to predict that Eqs. (2.13) and (2.14) yield equivalent results, since in both the cases the optical and fluctuation amplitudes must sum up to the full amplitude given by Eq. (2.1). Indeed the results given by Eqs. (2.13) and (2.14) are equivalent. The equivalence can be proved by using the following easily verified identites:

$$B\equiv V_{PQ}\mathcal{G}^{\rm KKM}_{QQ}V_{QP}$$
$$=PH[i(I/2)\langle G_{QQ}\rangle_I]^{1/2}(G_{QQ}+\mathcal{G}^{\rm KKM}_{QQ}V_{QP}\mathcal{G}_{\rm op}V_{PQ}G_{QQ})[i(I/2)\langle G_{QQ}\rangle_I]^{1/2}HP \tag{2.18}$$
$$=H_{PQ}i(I/2)\langle G_{QQ}\rangle_I(G_{QQ}+G_{QQ}H_{QP}[\mathcal{G}^{-1}_{\rm op}-H_{PQ}\langle G_{QQ}\rangle_I i(I/2)G_{QQ}H_{QP}]^{-1}H_{PQ}i(I/2)\langle G_{QQ}\rangle_I G_{QQ})H_{QP} \tag{2.19}$$
$$=H_{PQ}i(I/2)\langle G_{QQ}\rangle_I[G_{QQ}+G_{QQ}H_{QP}\mathcal{G}_{PP}H_{PQ}(G_{QQ}-\langle G_{QQ}\rangle_I)]H_{QP} \tag{2.20}$$
$$=H_{PQ}i(I/2)\langle G_{QQ}\rangle_I\mathcal{G}_{QQ}(H-HG_{PP}H\langle G_{QQ}\rangle_I H)P\ , \tag{2.21}$$

where

$$\mathcal{G}_{PP}=P(E-H_{PP}-H_{PQ}G_{QQ}H_{QP})^{-1}P\ . \tag{2.22}$$

Equation (2.19) follows from (2.18) by considering the Neumann series of the resolvent operators of both sides. Equation (2.20) follows by using the explicit form (2.9). Finally, Eq. (2.21) follows by using Eq. (2.3). Using Eq. (2.11) we can rewrite Eq. (2.21) as

$$B=\mathcal{G}^{-1}_{\rm op}\mathcal{G}_{\rm op}H\langle G_{QQ}\rangle_I i(I/2)\mathcal{G}_{QQ}HG_{PP}\mathcal{G}^{-1}_{\rm op}\ , \tag{2.23}$$
$$=\mathcal{G}^{-1}_{\rm op}G_{PP}H\langle \mathcal{G}_{QQ}\rangle_I i(I/2)\mathcal{G}_{QQ}HG_{PP}\mathcal{G}^{-1}_{\rm op}\ , \tag{2.24}$$

where Eq. (2.24) follows by considering Neumann series solutions of the relevant resolvent operators in (2.23) and (2.24). Substituting Eq. (2.24) in Eq. (2.14) one arrives at Eq. (2.13) with the use of the identities

$$(1+\mathcal{U}\mathcal{G}_{\rm op})\mathcal{G}^{-1}_{\rm op}G_{PP}=(1+VG_{PP})\ ,$$
$$G_{PP}\mathcal{G}^{-1}_{\rm op}(1+\mathcal{G}_{\rm op}\mathcal{U})=(1+G_{PP}V)\ . \tag{2.25}$$

Hence, the equivalence of Eqs. (2.13) and (2.14) is established.

Although Eqs. (2.13) and (2.14) are equivalent we believe that Eq. (2.13) has certain advantages, especially in the formulation of doorway states and multistep compound reactions. In order to understand these advantages let us consider the effective Hamiltonians of $\mathcal{G}_{QQ}$ and $\mathcal{G}^{\rm KKM}_{QQ}$ given by

$$\mathcal{H}_{QQ}=H_{QQ}+H_{QP}G_{PP}H_{PQ} \tag{2.26}$$

and

$$\mathcal{H}^{\rm KKM}_{QQ}=H_{QQ}+V_{QP}\mathcal{G}_{\rm op}V_{PQ}\ , \tag{2.27}$$

respectively. It is easy to see that both of these effective interactions violate the chaining condition in general even when the Hamiltonian $H$ obeys the chaining condition. But in the extreme low energy limit when the elastic channel is the only open channel the chaining condition can be restored via Eqs. (2.13) and (2.26), whereas it will be always violated, as in the approaches of FKK and Sevgen, while using Eqs. (2.14) and (2.27). It is this violation of the chaining condition in the approaches of FKK and Sevgen through the $Q$ space which we call unphysical in the Introduction. There is another problem with Eq. (2.14) if one uses it to formulate the theories of doorway states or multistep compound reactions. The essence of Eq. (2.14) is that the $T_{\rm fl}$ defined by it averages to zero without using the property of statistical fluctuation of certain form factors. This property cannot be maintained in the multistep components defined via (2.14), whereas it can be maintained in the multistep components defined via (2.13). Although Eqs. (2.13) and (2.14) are equivalent, the multistep components defined from these equations cease to be equivalent. We discuss these aspects first in the doorway state model of FKL in the next section before discussing multistep compound reactions in Sec. IV.

## III. DOORWAY STATES

We introduce the projection operator corresponding to the doorway state by $d$ such that[6]

$$Q=d+q\ , \tag{3.1}$$
$$PQ=Pd=Pq=dq=0\ . \tag{3.2}$$

In addition, following FKL (Ref. 6) one assumes, as usual,

$$H_{Pd}\neq 0,\quad H_{dq}\neq 0\ , \tag{3.3}$$

but

$$H_{Pq}=0\ , \tag{3.4}$$

so that

$$H_{PQ}=H_{Pd},\quad H_{QP}=H_{dP}\ . \tag{3.5}$$

With these definitions $T_{\rm fl}$ defined by Eq. (2.13) naturally breaks up into two parts:

$$T_{\rm fl}=T_d+T_q\ , \tag{3.6}$$

where

$$T_d=\langle\phi\,|\,(1+VG_{PP})H\langle\mathcal{G}_{dd}\rangle_I i(I/2)\mathcal{G}_{dd}H(1+G_{PP}V)\,|\,\phi\rangle \tag{3.7}$$

and

$$T_q=\langle\phi\,|\,(1+VG_{PP})H\langle\mathcal{G}_{dq}\rangle_I i(I/2)\mathcal{G}_{qd}H(1+G_{PP}V)\,|\,\phi\rangle \tag{3.8}$$

with

$$\mathcal{G}_{dd} \equiv d\frac{1}{E-H_{QQ}-W_{dd}}d$$
$$=d\frac{1}{E-H_{dd}-W_{dd}-H_{dq}e_q^{-1}H_{qd}}d \tag{3.9}$$

and

$$\mathcal{G}_{qd}=qe_q^{-1}H_{qd}\mathcal{G}_{dd}\ , \tag{3.10}$$

where

$$e_q=(E-H_{qq}+i\epsilon) \tag{3.11}$$

and

$$W=HG_{PP}H\ . \tag{3.12}$$

In Eq. (3.6) $T_d$ is the doorway state part and is expected to yield the doorway space resonances once the $q$ space is averaged. This can be done with respect to a Lorentzian averaging function of width $I'$ large compared to the width and spacing of $q$ space states. In Eq. (3.6) $I$ is supposed to be large compared to the width and spacing of doorway states. It is not difficult to see that $T_d$ averages to zero when averaged with respect to an averaging function of width $I$ large compared to width and spacing of doorway states. This is because averaging Eq. (3.7) one has

$$\langle T_d\rangle_I \sim \sum_{d'}\frac{f_{d'}f_{d'}i(I/2)}{[E-\epsilon_{d'}-\langle d'\,|\,(W_{dd}+H_{dq}\langle e_q^{-1}\rangle_I H_{qd})\,|\,d'\rangle+i(I/2)]^2}\ , \tag{3.13}$$

with

$$f_{d'}=\langle\phi\,|\,(1+VG_{PP})H\,|\,d'\rangle\ , \tag{3.14}$$

where we have employed an eigenfunction expansion in terms of eigenfunctions of $H_{dd}$ with eigenvalue $\epsilon_d$. As $I$ is large compared to the spacing and width of doorway states the matrix element of the denominator can be considered to be real. In a large nuclear system the sum in (3.13) will extend over many states, and because of the squared term in the denominator, as argued by KKM, $T_d$ averages to zero without needing the (questionable) property of statistical fluctuation of the form factors $f_d$.

In Eq. (3.6) $T_q$ is the rapidly varying part corresponding to $q$ space fluctuations. Again using the forms given by Eq. (3.10) for the resolvent operator $\mathcal{G}_{qd}$ it is easy to see that $\langle T_q\rangle_I\cong 0$. In fact, $\langle T_q\rangle_{I'}\cong 0$, where $I'$ is just sufficient to wash away the $q$ space fluctuations while retaining the doorway space fluctuations ($I>I'$). In other words $I'$ is large compared to the width and spacing of states in the $q$ space but can be comparable to the width and spacing of states in the doorway space. Using Eq. (3.10) it is easy to see that

$$\langle T_q\rangle_{I'}\sim\sum_q\frac{f_q'\langle f_q'\rangle_I i(I/2)}{[E-\epsilon_q+i(I/2)][E-\epsilon_q+i(I'/2)]}\ , \tag{3.15}$$

where

$$f_q'=\langle\phi\,|\,(1+VG_{PP})H\mathcal{G}_{dd}H\,|\,q\rangle\ . \tag{3.16}$$

Here we again have employed an eigenfunction expansion in terms of eigenfunctions $q$ of $H_{qq}$ with eigenvalue $\epsilon_q$. Again it can be verified that as $\epsilon_q$ moves along the real axis the product

$$[E-\epsilon_q+i(I/2)]^{-1}[E-\epsilon_q+i(I'/2)]^{-1} \tag{3.17}$$

moves around a heart shaped curve centered at the origin. Hence, as pointed out by KKM, even without the assistance of random phases of $f_q$'s the factor (3.17) assures that the phases of the terms in the sum will distribute themselves between 0 and $2\pi$, in such a way that the sum in (3.15) vanishes.

Although $T_d$ and $T_q$ average to zero it does not mean that the corresponding cross sections are zero. In fact they will contribute to $d$ and $q$ space cross sections. But since both $T_d$ and $T_q$ average to zero the energy averaged cross section will have no interference terms between $d$ and $q$ spaces:

$$\langle\sigma_{\rm fl}\rangle_I=\langle\sigma_d\rangle_I+\langle\sigma_q\rangle_I\ . \tag{3.18}$$

In Eq. (3.18) $\langle\sigma_d\rangle$ is the $d$ space contribution to the cross section, and $\langle\sigma_q\rangle$ is the $q$ space contribution. From Eqs. (3.7) and (3.8) it is easy to see that both the terms in (3.18) vary on the same scale as the doorway space fluctuations when $I$ is big enough to wash away just the $q$ space fluctuations.

We could have tried to carry on the same analysis starting with $T_{\rm fl}$ defined by Eq. (2.14). With conditions (3.1)—(3.5) $T_{\rm fl}$ of (2.14) naturally breaks up into four pieces:

$$T_{\rm fl}=T_{dd}+T_{dq}+T_{qd}+T_{qq}\ , \tag{3.19}$$

where $T_{ij}$ satisfies (2.14) with $\mathcal{G}_{QQ}^{\rm KKM}$ replaced by $i\mathcal{G}_{QQ}^{\rm KKM}j$, $i,j=d,q$. The main difficulty with the separation (3.19) is that although $\langle T_{\rm fl}\rangle_I=0$, $\langle T_{ij}\rangle_I\neq 0$, because in $T_{ij}$ one does not have a product of two resolvent operators of the form $(\langle G\rangle_I G)$, as in Eqs. (3.7) and (3.8), which will guarantee the vanishing of the average. It is also not possible to extract the doorway space resonances in a simple way from the energy averaged cross section using (3.19), because of the appearance of interference terms between various pieces. This problem will be more serious if one uses Eq. (2.14) to define the multistep components of the fluctuation cross section as in Refs. 5 and 13.

## IV. MULTISTEP COMPOUND REACTIONS

In this section we use representation (2.13) for the fluctuation amplitude in order to define its multistep components. As in FKK we divide the $Q$ space into a set of orthogonal subspaces $Q_n$ such that

$$Q=\sum_{n=1}^{r}Q_n,\quad Q_nQ_m=\delta_{nm}\ , \tag{4.1}$$

$$Q_nP=PQ_n=0\ , \tag{4.2}$$

and

$$H_{n,m}\equiv Q_nHQ_m=0,\quad |n-m|>1\ . \tag{4.3}$$

The spaces $Q_n$, $n=1,2,\ldots,r$, define a hierarchy of doorway states such that for $m>n$, $Q_m$ is more complex than $Q_n$. The spaces $Q_n$ should be compared with $d$ and $q$ of Sec. III. Now $T_{\rm fl}$ of Eq. (2.13) can be naturally broken up into the following components:

$$T_{\rm fl}=\sum_{n=1}^{r}\sum_{m=1}^{r}T_{n,m}\ , \tag{4.4}$$

where $T_{n,m}$ satisfies Eq. (2.13) with $\mathscr{G}_{QQ}$ replaced by $\Delta_{n,m}$ defined by

$$\Delta_{n,m}=Q_n\mathscr{G}_{QQ}Q_m\ . \tag{4.5}$$

In their treatment of multistep fluctuation components FKK and Sevgen took $m=1$, which is a good approximation in the absence of many open channels. But if we have several open channels the system can propagate through the $P$ space to a complicated configuration and then enter the $Q$ space at a stage $m\neq 1$. So, physically, in the presence of other open channels one will in general have contributions to $T_{n,m}$ for $m\neq 1$. We shall see below that

$$\langle T_{n,m}\rangle_I=0\ , \tag{4.6}$$

for each $m$ and $n$. We shall see that $T_{n,m}$ contributes to fluctuations of space $Q_m$ when $m\geq n$, and to fluctuations of $Q_n$ when $n>m$. As each component of Eq. (4.4) averages to zero, crossed terms will be absent in the cross section.

Following Sevgen we define $W(n)$ through the following nonlinear recursion relations

$$W(n)=W(n+1)+W(n+1)G_{n+1}W(n+1)\ , \tag{4.7}$$

with

$$W(r)=\mathscr{H} \tag{4.8}$$

and

$$W(r-1)=\mathscr{H}+\mathscr{H}G_r\mathscr{H}\ , \tag{4.9}$$

etc., where

$$G_n=Q_n[E-Q_nW(n)Q_n]^{-1}Q_n\ , \tag{4.10}$$

with

$$G_1=\Delta_{1,1}\ . \tag{4.11}$$

With these definitions one can evaluate the various $\Delta_{n,j}$ through

$$\Delta_{n,j}=G_nW(n)\sum_{m<n}\Delta_{m,j},\quad n>j \tag{4.12}$$

with

$$\Delta_{j,n}=\Delta_{n,j}^{T}\ , \tag{4.13}$$

where $T$ represents transposed. Any of the $\Delta_{n,m}$'s can be calculated using Eqs. (4.12) and (4.13). Explicitly, one has

$$\Delta_{n,j}=G_nW(n)\Bigg[1+\sum_{n>m_1>1}G_{m_1}W(m_1)+\sum_{n>m_2>m_1>1}G_{m_2}W(m_2)G_{m_1}W(m_1)+\cdots +G_{n-1}W(n-1)G_{n-2}W(n-2)\cdots G_3W(3)G_2W(2)\Bigg]\Delta_{1,j},\quad n>j\ . \tag{4.14}$$

Using (4.11), (4.13), and (4.14) all the $\Delta_{n,j}$'s can be found out. For example,

$$\Delta_{4,3}=G_4W(4)[1+G_3W(3)+G_2W(2)+G_3W(3)G_2W(2)]G_1[W(2)G_2+1]W(3)G_3\ . \tag{4.15}$$

It is interesting to realize that $\Delta_{n,m}$ starts with the propagator $G_n$ and ends on the propagator $G_m$. From Eqs. (2.13), (4.4), and (4.5) it is easy to see that $T_{n,m}$ has the form

$$T_{n,m}=\sum_{j=1}^{r}T_{n,m}^{(j)}\equiv\sum_{j=1}^{r}\langle\psi^{(-)}|H\langle\Delta_{j,n}\rangle_I i(I/2)\Delta_{n,m}H|\psi^{(+)}\rangle\ , \tag{4.16}$$

where

$$|\psi^{(+)}\rangle=(1+G_{PP}V)|\phi\rangle\ ,$$

and $\langle\psi^{(-)}|$ is defined analogously. Let us now study (4.16) and prove (4.6). We note using Eqs. (4.11), (4.13), and (4.14) that $\Delta_{j,n}$ and $\Delta_{n,m}$ have the form

$$\Delta_{j,n}=A_{j,n}G_n \tag{4.17a}$$

and

$$\Delta_{n,m}=G_nA_{m,n}^{T}\ , \tag{4.17b}$$

where an explicit expression for $A$ can be easily written down. [For example, one can easily read off the expression for $A_{4,3}$ using Eq. (4.15)]. Now substituting (4.17) into Eq. (4.16) one has

$$T_{n,m}=\sum_{j=1}^{r}\langle\psi^{(-)}|H\langle A_{j,n}\rangle_I\langle G_n\rangle_I i(I/2)G_nA_{m,n}^{T}H|\psi^{(+)}\rangle\ . \tag{4.18}$$

In Eq. (4.18) we have the product of two resolvent operators of the form $(\langle G_n\rangle_I G_n)$, and, if one can employ the usual spectral representation of $G_n$,

$$G_n=\sum_{n}\frac{|\phi_n\rangle\langle\bar{\phi}_n|}{E-\epsilon_n}\ , \tag{4.19}$$

where $\phi_n$ and $\bar{\phi}_n$ are biorthogonal states of the operator $Q_nW(n)Q_n$ with (energy-independent) eigenvalue $\epsilon_n$, the usual arguments of KKM can be used to verify (4.6). The expression (4.19) is, however, not in general valid because

$Q_n W(n) Q_n$ of Eq. (4.10) is strongly energy dependent in general.

In the following we find the conditions under which Eq. (4.19) can be justified so that Eq. (4.6) is valid. Using Eqs. (4.7)—(4.9) it is easy to see that $Q_n W(n) Q_n$ can be written as

$$Q_n W(n) Q_n = Q_n \mathscr{H} Q_n + Q_n U(n) Q_n \ , \tag{4.20}$$

where the first term on the right-hand side alone will contribute to poles appearing in $G_n$ of (4.10) at a rate given by $D_n$—the spacing of states in the $n$th space. However, from the definition of $Q_n W(n) Q_n$ it can be easily realized that the last term of (4.20) contains the sum of terms each of which contains at least one of the resolvent operators $G_i$, $i=(n+1), (n+2), \ldots, r$, and hence appears to contribute to poles appearing at a rate given by $D_i$. Hence, the last term of (4.20) cannot be taken as slowly varying. Now extending the arguments of FKK and making eigenfunction expansions of the resolvent operators $G_i$'s, successively, in the last term of (4.20) it is easy to realize that an approximate energy independent diagonalization of (4.20), needed for justifying (4.19), is possible if the average width $\Gamma_i$ of the levels in the space $i$ is large compared to the average spacing of poles $D_n$ of the $n$th space[5,13]:

$$\Gamma_i \gg D_n, \quad \text{all } i > n \ . \tag{4.21}$$

We also assume that $\Gamma_n \gg \Gamma_{n+1}$. If (4.21) is satisfied the energy independent diagonalization (4.19) can be justified and $G_n$ will contain poles at a rate given by $D_n$. Equation (4.18) has exactly the form of Eq. (2.13) and hence using (4.19) in (4.18) it is easy to see that the product of the energy denominators of $\langle T_{n,m} \rangle_I$, given by

$$\langle T_{n,m} \rangle_I = \sum_{j=1}^{r} \sum_{n'} \frac{\langle \psi^{(-)} | H \langle A_{j,n} \rangle_I | \phi_{n'} \rangle i(I/2) \langle \bar{\phi}_{n'} | A^T_{m,n} \rangle_I H | \psi^{(+)} \rangle}{(E - \epsilon_{n'} + iI/2)^2} \ , \tag{4.22}$$

will guarantee (4.6). The present averaging technique is very different from the arguments of self-averaging used by FKK.

Expanding the $\Delta_{n,m}$ of Eq. (4.16) [as in Eq. (4.15)] it is easy to see that the expression for $T_{n,m}$ given by (4.16) will contain all $G_i$'s from $i=1$ to $k$ where $k$ is the larger of $n$ and $m$, assuming that the averaging interval $I$ is large enough to wash away rapid variations in $\langle \Delta_{j,n} \rangle_I$. As (4.21) is assumed to be satisfied, $G_i$ contains poles appearing at a rate given by $D_i$, and $G_k$ (where $k$ is the larger of $n$ and $m$) will determine the most rapidly varying fluctuations of $T_{n,m}$. Hence $T_{n,m}$ will contribute to $n$ space fluctuations if $n \geq m$ and to $m$ space fluctuations otherwise.

So, in brief, one of the major achievements of the present work is Eq. (4.6), showing that doorway classes can be defined consistently with (4.21), whose interference terms automatically average to zero, so that the multistep compound cross section is a simple sum of contributions, one from each class. The present result is significantly different from that of Friedman *et al.*[10] who achieved the same goal imposing the condition $\Gamma_{n+1} \ll \Gamma_n$ for the width of the successive classes. From Eqs. (4.4) and (4.16) it is easy to see that the compound nuclear cross section relating channels $c$ and $c'$ will have the form

$$\sigma_{\mathrm{fl},cc'} = \sum_{n=1}^{r} \sum_{m=1}^{r} \tau_{n,c} \Pi_{n,m} \tau_{m,c'} \ , \tag{4.23}$$

which is the form obtained by Weidenmüller *et al.*[10] Equation (4.23) has the same physical content as Eq. (4.4). The factor $\tau_{n,c}$ in each element of $\sigma$ gives the probability of reaching the $n$th space from channel $c$, $\Pi_{n,m}$ denotes the propagation from stage $n$ to $m$, and finally $\tau_{m,c'}$ denotes the escape to channel $c'$ from the $m$th space. Finally, one has to sum over all possible $m$ and $n$ from 1 to $r$.

It is, however, also possible to write the final result (4.23) in the form given by FKK and Sevgen. From an expansion of $\Delta$ [of the form given by Eqs. (4.14) and (4.15)] it is easy to realize that the probability flow factors of FKK and Sevgen becomes very complicated and can be read off from Eq. (4.14). Instead of giving a detailed discussion of the calculation of cross sections in the most general case we shall limit ourselves to the extreme low energy limit in Sec. V (assuming no exoergic reaction channels) in order to see the advantages of the present approach.

## V. EXTREME LOW ENERGY LIMIT

Since the calculation of the cross section is in general complicated, we consider the extreme low energy limit which will clearly show the advantages of the present approach over that of FKK (Ref. 5) and Sevgen.[13] In the case of the neutron-nucleus scattering the incident neutron energy is assumed to be so low that the only open channel is the elastic channel (assuming no exoergic channels). As the neutron gets trapped it passes through a hierarchy of doorway hallway states all of which belong to the $Q$ space. Hence, the violation of the chaining condition through the $P$ space is not allowed. The whole $P$ space now corresponds to the entrance channel space $P_0$ which is connected by the residual interaction to the chain

$$Q_1 \rightarrow Q_2 \rightarrow Q_3 \rightarrow \cdots \rightarrow Q_{r-1} \rightarrow Q_r \ ,$$

where the chain is terminated at the $r$th stage which has a very complicated mode of excitation. The depletion factor, as we shall show below, exhibits such a chaining for the multistep components of the fluctuation cross section. The work of FKK and Segven, however, fails to reproduce such a chaining because of unphysical violation of the chaining through the $Q$ space. The effective Hamiltonian $\mathscr{H}_{QQ}$ of (2.26) obeys the chaining condition (4.3) as

$$Q_n H_{QP} P = P H_{PQ} Q_n = 0, \quad n \neq 1 \ . \tag{5.1}$$

But the effective Hamiltonian $\mathscr{H}^{\mathrm{KKM}}_{QQ}$ of (2.27) violates the chaining condition (4.3) because of the presence of the square root factors in the $V$'s. It is this violation of the chaining condition through the $Q$ space which we call unphysical since in the extreme low energy limit the physical system obeys the chaining condition.

In the present case $T_{fl}$ of Eq. (2.13) can be naturally broken up into the following multistep components:

$$T_{fl}=\sum_{n=1}^{r} T_{n,1} , \tag{5.2}$$

where

$$T_{n,1}=\langle \psi^{(-)} | H\langle \Delta_{1,n}\rangle_I i(I/2)\Delta_{n,1}H | \psi^{(+)}\rangle , \tag{5.3}$$

and where the $\Delta$'s are again defined by Eq. (4.5). Now the effective Hamiltonian $\mathscr{H}$ of $\mathscr{G}_{QQ}$ defined by Eq. (2.26) obeys the chaining condition, and hence one has

$$\Delta_{n,1}=G_nHG_{n-1}H\cdots HG_3HG_2HG_1 \tag{5.4}$$

and

$$\Delta_{1,n}=G_1HG_2HG_3\cdots HG_{n-1}HG_n . \tag{5.5}$$

Now Eqs. (4.7)—(4.11) are valid with $\mathscr{H}$ replaced by $H$ as

$$\mathscr{H}_{n,m}=H_{n,m}=0, \quad |n-m|>1 . \tag{5.6}$$

Also, one has

$$\Delta_{n,1}=G_nH\Delta_{(n-1),1} \tag{5.7}$$

and

$$\Delta_{1,n}=\Delta_{1,(n-1)}HG_n . \tag{5.8}$$

Substituting Eqs. (5.7) and (5.8) in (5.3) and following the arguments given after Eq. (4.16) it is easy to see that

$$\langle T_{n,1}\rangle_I=0 , \tag{5.9}$$

i.e., each of the multistep components of the fluctuation amplitude averages to zero. Now one can calculate the multistep components of the fluctuation cross section[15]:

$$\sigma_{fl}=\frac{4\pi^3}{k^2}\sum_{n=1}^{r}\langle |T_{n,1}|^2\rangle , \tag{5.10}$$

where

$$\langle |T_{n,1}|^2\rangle\cong(I/2)^2\left\langle\sum_n \frac{|\langle\psi^{(-)}|H\langle\Delta_{1,(n-1)}\rangle_I H|n\alpha\rangle|^2|\langle n\tilde{\alpha}|H\Delta_{(n-1),1}H|\psi^{(+)}\rangle|^2}{|E-\epsilon_n|^2|E-\epsilon_n+i(I/2)|^2}\right\rangle_I , \tag{5.11}$$

where we have used the following eigenfunction expansion of $G_n$ (Refs. 5 and 13):

$$G_n=\sum_n \frac{|n\alpha\rangle\langle n\tilde{\alpha}|}{E-\epsilon_n} . \tag{5.12}$$

Now one can simplify (5.11) as follows[15]:

$$\langle |T_{n,1}|^2\rangle\cong\langle |\langle\psi^{(-)}|H\Delta_{1,(n-1)}H|n\alpha\rangle|^2\rangle_\alpha\frac{2\pi}{\Gamma_nD_n}\langle |\langle n\tilde{\alpha}|H\Delta_{(n-1),1}H|\psi^{(+)}\rangle|^2\rangle_\alpha \tag{5.13}$$

$$\cong\frac{2\pi\langle |\langle\psi^{(-)}|H\Delta_{1,(n-1)}H|n\alpha\rangle|^2\rangle_\alpha}{D_n}\left[\frac{D_n}{2\pi\Gamma_n}\right]\frac{2\pi\langle |\langle n\tilde{\alpha}|H\Delta_{(n-1),1}H|\psi^{(+)}\rangle|^2\rangle_\alpha}{D_n} , \tag{5.14}$$

where $\Gamma_n$ and $D_n$ are the average width and spacing of state of space $Q_n$. Now one can go on using the chaining identity (5.4) successively in

$$X\equiv 2\pi D_n^{-1}\langle |\langle\psi^{(-)}|H\Delta_{1,(n-1)}H|n\alpha\rangle|^2\rangle_\alpha \tag{5.15}$$

as in FKK, and arrive at

$$X=2\pi D_{n-1}^{-1}\langle |\langle\psi^{(-)}|H\Delta_{1,n-2}H|(n-1)\beta\rangle|^2\rangle_\beta\Gamma_{n-1}^{-1}2\pi D_n^{-1}\langle |\langle(n\tilde{-}1)\beta|H|n\alpha\rangle|^2\rangle_{\alpha,\beta} . \tag{5.16}$$

Defining

$$\Gamma_{n-1}^{\downarrow}=2\pi D_n^{-1}\langle |\langle(n\tilde{-}1)\beta|H|n\alpha\rangle|^2\rangle_{\alpha,\beta} \tag{5.17}$$

and iterating this process we get

$$X=(\Gamma_1D_1^{-1})\left[\prod_{k=1}^{n-1}\Gamma_k^{\downarrow}\Gamma_k^{-1}\right] , \tag{5.18}$$

where

$$\Gamma_1=2\pi\langle |\langle\psi^{(-)}|H|1\gamma\rangle|^2\rangle_\gamma . \tag{5.19}$$

From Eqs. (5.12), (5.14), and (5.18) one has

$$\langle |T_{n,1}|^2\rangle=\frac{\Gamma_1}{D_1}\left[\prod_{k=1}^{n-1}\frac{\Gamma_k^{\downarrow}}{\Gamma_k}\right]\frac{D_n}{2\pi\Gamma_n}\left[\prod_{k=1}^{n-1}\frac{\Gamma_k^{\downarrow}}{\Gamma_k}\right]\frac{\Gamma_1}{D_1} , \tag{5.20}$$

and from Eqs. (5.10) and (5.20) one has

$$\sigma_{fl}=\frac{\pi}{k^2}\sum_{n=1}^{r}T_n\frac{1}{2\pi D_n^{-1}\Gamma_n}T_n , \tag{5.21}$$

where

$$T_n=2\pi D_1^{-1}\Gamma_1\left[\prod_{k=1}^{n-1}\Gamma_k^{\downarrow}\Gamma_k^{-1}\right] . \tag{5.22}$$

Equation (5.21) gives the desired result for the fluctuation cross section in the present case. Each partial cross section first contains the usual factor $2\pi D_1^{-1}\Gamma_1$ representing the probability for formation of the first stage—the doorway space $Q_1$. Then a product of depletion factors $\Gamma_k^{\downarrow}\Gamma_k^{-1}$ appears expressing, as in FKK, propagation to some final space $Q_n$. The factor $D_n(2\pi\Gamma_n)^{-1}$ represents propagation in $Q_n$ space, and then the product of depletion factors $\Gamma_k^{\downarrow}\Gamma_k^{-1}$ represents propagation to the doorway space $Q_1$ again. Finally, the factor $2\pi D_1^{-1}\Gamma_1$

represents the escape to the (elastic) exit channel.

So in the extreme low energy limit we have symmetry between the exit and the entrance channels. The explicit symmetry between the entrance and exit channels in Eq. (5.21) is, of course, physical in origin since the chaining condition is exact in this limit. Because of the unphysical violation of the chaining condition through the $Q$ space, as is obvious from Eq. (2.27), the FKK result fails to possess this symmetry. Here we have used an apparently asymmetric fluctuation amplitude $T_{fi}$ given by Eq. (2.13), which yields a symmetric multistep cross section, whereas FKK use the manifestly symmetric optical background representation of KKM given by Eq. (2.14) and obtain an asymmetric multistep cross section. As pointed out by FKK and Sevgen the various terms in the summation in Eq. (5.21) contribute to precompound emission from stage $n$, and the contribution from the $r$th space has a symmetric Hauser-Feshbach form[16] as has been pointed out by Satchler[17] in the limit of only one open channel.

In this section we have calculated the multistep fluctuation cross section in the extreme low energy limit. A similar calculation can be carried out starting from Eq. (4.16) in the general case. One should use expansions of the type given by Eq. (4.14) for both $\langle \Delta_{j,n} \rangle_I$ and $\Delta_{n,m}$ in Eq. (4.16) and proceed as in the present discussion which is valid for the extreme low energy limit. Then the product of depletion factors may produce a complicated route for probability flow. There are two essential differences between probability flow in the present case and that obtained by FKK and Sevgen. First, because of the physical violation of the chaining condition through the open channel $P$ space the system can bypass any number of doorway hallway states in the $Q$ space and enter the $Q$ space at stage $m$. How large $m$ is will depend on how large the $P$ space is. At low energy we have seen that $m=1$. At higher energies more and more complicated states may belong to the $P$ space. Hence the system may propagate to a complicated configuration through the $P$ space and enter the $Q$ space at stage $m,m>1$. This possibility was not considered by FKK and Segven and is allowed in the present approach. Second, in the approach of FKK and Segven in the $n$th multistep fluctuation component the system always moves directly from the $n$th stage to the exist channel space. In the present approach, apart from having a transition from the $n$th stage to the exit channel space, the system may take various other routes to the exit channel space, as can be easily understood from Eq. (4.16). An explicit calculation of the various multistep components of the fluctuation amplitude is complicated and not very interesting physically and hence is not performed in the present work.

## VI. SUMMARY

We provide an alternative derivation of the KKM optical background representation and write the final result in a form which has certain advantages in defining multistep fluctuation components of the cross section. First, the unphysical violations of chaining in multistep fluctuation components through the $Q$ space is eliminated; i.e., at low energies, as expected, the present theory obeys chaining assuming no exoergic reaction channels. Second, the multistep fluctuation amplitudes of the present approach average to zero without needing the property of statistical fluctuations of the form factors. Using the present approach we derive expressions for the multistep fluctuation amplitudes in the general case. We calculate the multistep fluctuation cross sections in the extreme low energy limit and find that the chaining condition is restored, though in the most general case the chaining condition is expected to be violated. The final result in this special case is symmetric with respect to the incident and the exit channels in agreement with physical expectations. The result of FKK and Segven in this special case is asymmetric and violates the chaining condition in contradiction with physical expectations.

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[1]See, for example, J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1952).

[2]N. Bohr, Nature 137, 344 (1936).

[3]V. F. Weisskopf, Phys. Rev. 52, 295 (1937).

[4]See, for example, W. Tobocman, *Theory of Direct Nuclear Reactions* (Oxford University Press, London, 1961); N. Austern, *Direct Nuclear Reaction Theory* (Wiley, New York, 1970).

[5]H. Feshbach, A. K. Kerman, and S. Koonin, Ann. Phys. (N.Y.) 125, 429 (1980), referred to as FKK in the text.

[6]H. Feshbach, A. K. Kerman, and R. H. Lemmer, Ann. Phys. (N.Y.) 41, 230 (1967), referred to as FKL in the text.

[7]See, however, also, C. Mahaux and H. A. Weidenmüller, *Shell-Model Approach to Nuclear Reactions* (North-Holland, Amsterdam, 1969); Nucl. Phys. A91, 241 (1967); B. Block and H. Feshbach, Ann. Phys. (N.Y.) 23, 47 (1963); A. K. Kerman, L. Rodberg, and J. E. Young, Phys. Rev. Lett. 11, 422 (1963); R. A. Ferrell and W. M. MacDonald, *ibid.* 16, 187 (1965); W. M. MacDonald and A. Mekjian, Phys. Rev. 160, 730 (1967); A. F. R. de Toledo Piza and A. K. Kerman, Ann. Phys. (N.Y.) 43, 363 (1967); 48, 173 (1968).

[8]M. Kawai, A. K. Kerman, and K. W. McVoy, Ann. Phys. (N.Y.) 75, 156 (1973), referred to as KKM in the text.

[9]J. J. Griffin, Phys. Rev. Lett. 17, 478 (1966); Phys. Lett. 24B, 5 (1967); M. Blann, Annu. Rev. Nucl. Sci. 25, 123 (1975).

[10]W. A. Friedman, M. S. Hussein, K. W. McVoy, and P. A. Mello, Phys. Rep. 77, 47 (1981); D. Agassi, H. A. Weidenmüller, and G. Mantzouranis, *ibid.* 22C, 145 (1975); M. S. Hussein and K. W. McVoy, Phys. Rev. Lett. 43, 1645 (1979); W. A. Friedman, Phys. Rev. C 22, 1 (1980).

[11]Evidence for multistep direct processes are given in R. J. Ascuitto and N. K. Glendenning, Phys. Rev. C 2, 415 (1970); 2, 1260 (1970); T. Tamura and T. Udagawa, *ibid.* 5, 1127 (1972);

Phys. Lett. 71B, 273 (1977); 78B, 189 (1978); T. Tamura, T. Udagawa, and H. Lenske, Phys. Rev. C 26, 379 (1982); R. Bonetti, M. Camnasio, L. Colli Millazo, and P. E. Hodgson, *ibid.* 24, 71 (1981).

[12]Evidence for multistep compound processes are given in C. Holbrow and H. Barschall, Nucl. Phys. 42, 264 (1963); R. Wood, R. Borchers, and H. Barschall, *ibid.* 71, 529 (1965); R. Bonetti *et al.*, Phys. Rev. C 25, 717 (1982); R. Bonetti, L. Colli Millazo, M. Melanotte, and M. S. Hussein, *ibid.*, 25, 1406 (1982).

[13]A. Sevgen, Phys. Lett. 102B, 102 (1981).

[14]H. Feshbach, Ann. Phys. (N.Y.) 5, 357 (1958); 19, 287 (1962).

[15]H. A. Weidenmüller, Phys. Lett. 42B, 304 (1972).

[16]W. Hauser and H. Feshbach, Phys. Rev. 87, 366 (1952); L. Wolfenstein, *ibid.* 82, 690 (1951).

[17]G. R. Satchler, Phys. Lett. 7, 55 (1963).