Low and intermediate energy nucleus-nucleus elastic scattering and the optical limit of Glauber theory

J. Chauvin, D. Lebrun,* A. Lounis, and M. Buenerd
Institut des Sciences Nucléaires, 38026 Grenoble Cedex, France
(Received 3 June 1983)

It is shown that a simplified Glauber approach using an experimental nucleon-nucleon forward amplitude and a Gaussian nucleus density reproduces fairly well both nucleus-nucleus reaction cross sections and elastic scattering differential cross sections over a broad energy range.

[NUCLEAR REACTIONS $^{12}C+^{12}C$ elastic scattering, $E/A=25$, 30, and 85 MeV, optical limit of Glauber theory, reaction cross section.]

I. INTRODUCTION

Over the last few years the dominant role played by the nucleon-nucleon cross section on the projectile-nucleus reaction cross section (σ_R) has been brought to light for light hadrons and a ^{12}C projectile.[1] In particular, it has been shown that Glauber-type approaches to describe the dependence of σ_R upon the incident energy are quite successful.[2,3] Even the simplest approach, in which Coulomb effects, Fermi motion, and Pauli blocking are ignored, leads to a good description of the experimental results,[2] although this could be due to some cancellation effects.[4]

Some time ago, an analytical formulation of the problem was proposed by Karol[2] for high-energy nucleus-nucleus reaction cross sections. The only inputs of the calculation are the rms radii of the two nuclei and the experimental nucleon-nucleon total cross section. Applying this formulation to the available data on σ_R for ^{12}C projectiles at $E/A=30$ MeV (Ref. 5) and 85 MeV (Ref. 6) leads to a fairly good agreement. In order to investigate how far such an approach can describe the heavy ion scattering data, we have extended the method to the description of the elastic scattering. The final formulation is equivalent to the optical limit of the Glauber approximation and it provides a surprisingly good description of the data for $^{12}C+^{12}C$ at $E_{\rm lab}=300$, 360, and 1010 MeV. This suggests that this simple model could provide good predictions for nucleus-nucleus scattering over a large energy range, since only the experimental nucleon-nucleon cross section and the rms radii of the nuclei are required.

II. FORMALISM

Karol used the semiclassical approach of Fernbach, Serber, and Taylor.[7] The reaction cross section could be deduced from a transparency function $T(b)$, or from the probability that the projectile undergoes no interaction at impact parameter b, through the integral

$$\sigma_R=2\pi\int_0^{\infty} b\,db[1-T(b)]\ . \qquad (1)$$

Assuming that the flux attenuation in the elastic channel occurs by means of nucleon-nucleon collisions along a straight line trajectory (Fig. 1), the transparency function reads as

$$T(b)=\exp\left[-\bar{\sigma}_{\rm NN}\int_{-\infty}^{+\infty}\rho_T(b,z)\rho_P(b,z)dt\right]\ , \qquad (2)$$

where the integral represents the overlap of the projectile (subscript P) and target (subscript T) densities.

$\bar{\sigma}_{\rm NN}$ is an average nucleon-nucleon total cross section obtained from the experimental proton-proton and neutron-proton total cross section through

$$A_TA_P\bar{\sigma}_{\rm NN}=(Z_TZ_P+N_TN_P)\sigma_{\rm pp}+(Z_TN_P+Z_PN_T)\sigma_{\rm np}\ . \qquad (3)$$

The experimental values of $\sigma_{\rm nn}$ and $\sigma_{\rm pp}$ are taken for $E_{\rm N}=E_P/A_P$.[8] Then, assuming a Gaussian shape for the nuclei densities, the following analytical form for the transparency function is obtained[2]:

$$\begin{aligned}\ln T(b)&=-\bar{\sigma}_{\rm NN}\pi^2\rho_P(0)\rho_T(0)\frac{a_T^3a_P^3}{(a_P^2+a_T^2)}\\&\quad\times\exp[-b^2/(a_P^2+a_T^2)]\\&=-\bar{\sigma}_{\rm NN}0_v(b)\end{aligned} \qquad (4)$$

with $\bar{\sigma}_{\rm NN}$ in fm^2 and

$$\begin{aligned}&\rho(r)=\rho(0)\exp(-r^2/a^2)\ ,\\&a={\rm rms}(1.5)^{-1/2}\ ,\\&\rho(0)=A/(a\sqrt{\pi})^3\ .\end{aligned} \qquad (5)$$

This leads straightforwardly to the value of σ_R using relation (1).

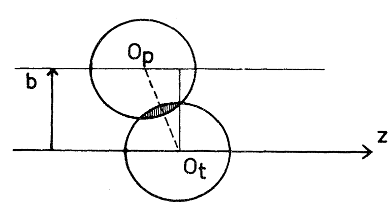

FIG. 1. The projectile 0_p is assumed to have straight line trajectories located at distance b from the z axis passing through the center of mass of the target 0_t.

To extend the model to describe the differential cross section, one uses the correspondence between quantal and semiclassical expressions. Relation (1) could be deduced from the usual quantal relation

$$\sigma_R = \pi\lambda^2 \sum_l (2l+1)\{1 - |S_l|^2\} \tag{6}$$

with $b = \lambda(l+1/2)$,

$$T(b) = |S_l|^2 , \quad \int dl = \sum_l . \tag{7}$$

If one introduces the complex phase shift Δ_l related to the scattering matrix S_l through

$$S_l = e^{2i\Delta_l} , \tag{8}$$

from the correspondence $T(b) = |S_l|^2$, one gets

$$\ln T(b) = -4\,\mathrm{Im}\Delta_l$$

and, using relation (4),

$$\mathrm{Im}\Delta_l = \tfrac{1}{4}\bar{\sigma}_{\mathrm{NN}} 0_v(b) .$$

With the help of the optical theorem for nucleon-nucleon scattering, one replaces $\bar{\sigma}_{\mathrm{NN}}$ by

$$\bar{\sigma}_{\mathrm{NN}} = 4\pi\lambda_{\mathrm{NN}} \mathrm{Im} f_{\mathrm{NN}}(0) \tag{9}$$

and then

$$\mathrm{Im}\Delta_l = \pi\lambda_{\mathrm{NN}} 0_v(b) \mathrm{Im} f_{\mathrm{NN}}(0) . \tag{10}$$

Postulating the same relationship as (10) for the real parts, the nucleus-nucleus scattering matrix S_l is expressed in terms of the nucleon-nucleon forward amplitude and of the overlap density function as

$$S_l = \exp[2\pi i \lambda_{\mathrm{NN}} f_{\mathrm{NN}}(0) 0_v(b)] . \tag{11}$$

This result is the optical limit of Glauber theory,[9] which is generally obtained in a more rigorous procedure from the basic assumptions of the Glauber theory, $E \gg V$ and $\lambda_p \ll a$, in which a is a distance on which the potential V exhibits significant variation (the diffuseness of the optical potential, for example). Two further approximations must be made to derive the optical limit: that the nucleons have independent motion in the nucleus and that the size of the nucleus be large compared to the range of the nucleon-nucleon force. The nucleus-nucleus differential cross section $d\sigma/d\omega$ is deduced in the standard way from the scattering matrix S_l or complex phase shift Δ_l:

$$\frac{d\sigma}{d\omega} = |f(\Theta)|^2 \tag{12a}$$

for different spinless nuclei or

$$\frac{d\sigma}{d\omega} = |f(\Theta) + f(\pi - \Theta)|^2 \tag{12b}$$

for identical spinless nuclei, with the usual expression for the scattering amplitude $f(\Theta)$ for charged particles

$$f(\Theta) = f_{\mathrm{Coul}}(\Theta) + \lambda \sum_l (2l+1) P_l(\cos\Theta) e^{2i\sigma_l}(e^{2i\Delta_l} - 1)/2i . \tag{13}$$

Here, the l (or b) variation of the phase shift is given by the Gaussian overlap function

$$\Delta_l = \pi\lambda_{\mathrm{NN}} f_{\mathrm{NN}}(0°) 0_v(b) . \tag{14}$$

Next, one needs both the imaginary part (i.e., σ_{NN}) and the real part of the nucleon-nucleon scattering amplitude. However, to our knowledge, no procedure to extract this average nucleon-nucleon scattering amplitude from proton-proton and neutron-proton data is discussed in the literature. At very high energies, the real part vanishes, as does the problem[10]: We obtain the usual black sphere diffraction pattern (as in the Blair model) which, here, is not compatible with experimental data (see Fig. 3). We propose a simple procedure based on the fact that a low energy nucleon-nucleon collision is determined only by S waves. Following Glauber, we ignore the spin-isospin structure of the nucleon-nucleon interaction and suppose that there exists some effective average interaction $V_{\mathrm{NN}}(r)$ acting only in the S state at low energies. Then the nucleon-nucleon scattering amplitude reduces to

$$f_{\mathrm{NN}}(0°) = \tfrac{1}{2}\lambda_{\mathrm{NN}}[\sin 2\bar{\delta}_0 + i(1 - \cos 2\bar{\delta}_0)] , \tag{15}$$

which is fully determined by the average phase shift $\bar{\delta}_0$. This phase shift is constrained by the optical theorem to give the total average cross section

$$\bar{\sigma}_{\mathrm{NN}} = 2\pi\lambda_{\mathrm{NN}}(1 - \cos 2\bar{\delta}_0) \tag{16}$$

and, then the knowledge of σ_{NN} gives both the real and the imaginary parts of $f_{\mathrm{NN}}(0°)$. The S wave approximation could be tested on the proton-proton system by taking standard values[11] for the $^1S_0$ phase shift. At $E_p = 30$ MeV, the calculated σ_{pp} value is 104 mb compared to the 100 mb experimental value. At $E/A = 85$ MeV, the S-wave approximation is not very justified. As is usually done, the complex scattering amplitude $f_{\mathrm{NN}}(0°)$ is given in Table I by $\bar{\sigma}_{\mathrm{NN}}$ (imaginary part) and the ratio of its real to imaginary part by $\bar{\alpha}_{\mathrm{NN}}$. Here, in the optical limit, this ratio is also the ratio of the real to the imaginary part for the nucleus-nucleus complex phase shift Δ_l and the same for the optical potential.[9] It expresses the relative importance of refraction compared to absorption in the nucleus-nucleus collision.

A calculation of the same type was reported earlier, using the optical limit of Glauber and Gaussian densities.[12] However, due to the fact that no experimental data were available, only nucleus-nucleus elastic scattering below 10 MeV per nucleon was investigated. Starting from experimental values, the authors treated as adjustable parameters the nucleon-nucleon forward scattering amplitude and the nuclear densities distributions and obtained good fits to the data for different parameter families. Unfortunately, they ignored the nucleus-nucleus reaction cross section, which represents a strong constraint on $\bar{\sigma}_{\mathrm{NN}}$. Surprisingly, the values given for the experimental nucleon-nucleon data in this reference are far from ours. For $E_{\mathrm{N}} = 34$ MeV, the authors took $\bar{\sigma}_{\mathrm{NN}} = \frac{1}{2}(\sigma_{pp} + \sigma_{np}) = 300$ mb and $\bar{\alpha}_{\mathrm{NN}} = 0.3$; for this energy we find $\bar{\sigma}_{\mathrm{NN}} = 170$ mb and $\bar{\alpha}_{\mathrm{NN}} = 0.9$. We must note that these very different values for $\bar{\sigma}_{\mathrm{NN}}$ are extracted from the same Hess compilation.[8] It is important to clarify this point since the good agree-

TABLE I. Nucleon-nucleon data and ^{12}C-^{12}C reaction cross sections. The experimental nucleus-nucleus σ_R were extracted from optical model analysis (Refs. 5, 6, and 13) (in parenthesis) and from direct measurements (Ref. 15).

E/A (MeV)	λ_{NN} (fm)	σ_{NN} (mb)	α_{NN}	E (MeV)	λ (fm)	Experimental σ_R (mb)	Calculated σ_R (mb)	$b_{1/2}$ (fm)	$\pi b_{1/2}^2$ (mb)
25.	1.82	241.	0.85	300.	0.152	(1300)	1311	6.4	1287
30.	1.66	196.	0.87	360	0.14	1315±40 (1260)	1262	6.2	1208
85.	1.	61.	1.	1020	0.082	960±25 (1000)	988.	5.5	950

ment presented here (see below) would be destroyed if the nucleon-nucleon input were so drastically changed.

III. RESULTS

First, we apply the preceding relations to the symmetric system $^{12}C+^{12}C$. A large variety of experimental data is available from CERN at $E/A=85$ MeV (Ref. 6), from Grenoble SARA facilities at $E/A=30$ MeV (Ref. 5), and from Hahn-Meitner-Institut (HMI) at $E/A=25$ MeV.[13] It is well known that nucleus-nucleus elastic scattering is essentially determined by a small range of impact parameters, corresponding to surface collisions. The lower partial waves are totally absorbed as illustrated by the transparency function $T(b)$ (see Fig. 2). In the surface region, the nuclear potential and also the Coulomb potential (about 8 MeV) are small compared to the kinetic energy. Therefore, the straight line trajectory assumption is certainly a reasonable approximation.

Taking the rms ^{12}C radius as 2.37 fm from electron scattering data[14] the ^{12}C density reads as

$$\rho(r)=0.2974\exp-(x/1.935)^2\ \mathrm{fm}^{-3}\ . \tag{17}$$

The nucleon-nucleon data are listed in Table I. With these inputs, the $^{12}C+^{12}C$ reaction cross section is well reproduced by the model (Table I). It must be noted that the

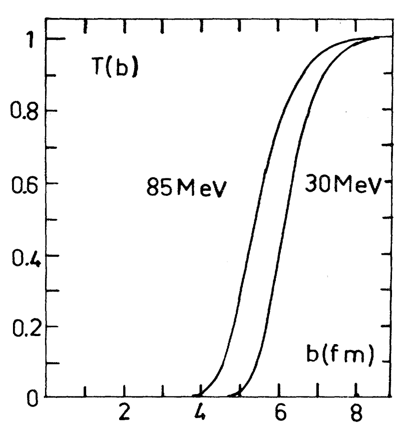

FIG. 2. $^{12}C+^{12}C$ transparency function $T(b)$ as deduced from the analytical formulation of Ref. 2. Optical model analysis of $d\sigma/d\omega$ leads to identical curves, and then to identical reaction cross sections.

same agreement could be obtained for a great variety of ^{12}C induced reactions in this energy range.[15]

The strong absorption impact parameter $b_{1/2}$, for which $T(b)=\frac{1}{2}$, provides a good estimate of the reaction cross section $\pi b_{1/2}^2$. This effective surface becomes smaller as the energy increases, or in semiclassical terms, the nuclear surface becomes more and more transparent.[1]

The most surprising agreement is obtained for the elastic differential cross section (Fig. 3) with no need for renormalization. Although the agreement is not perfect, the cross section at forward angles is nicely reproduced for the three energies.

At large angles, the model produces too much diffractive structure and overestimates the cross section at $E/A=25$ and 85 MeV. The poor agreement at large angles can be understood qualitatively. Let us consider the deflection function as defined from the Wentzel-Kramers-Brillouin (WKB) approximation by

$$\Theta(l)=2\frac{d}{dl}(\mathrm{Re}\Delta_l+\sigma_l)\ . \tag{18}$$

Note that this relationship holds only for not too strong absorption.[16] Rainbows occur at the extrema of this deflection function; the Coulomb rainbow is usually observed for the systems dominated by Coulomb interaction as in Fig. 5, whereas the nuclear rainbow located at smaller impact parameters is much more elusive and generally obscured by the absorption.[6] If one neglects in relation (18) the Coulomb part (much smaller than the nuclear part in the region of the nuclear rainbow), the latter is obtained for an impact parameter

$$b_R=a_{^{12}C}=1.935\ \mathrm{fm}\ .$$

This value is determined by the ^{12}C rms radius as given by relation (5). No rainbow scattering can be predicted by our model at such a small impact parameter where the absorption is total. The comparison with the results of an optical model analysis shows that the phase shifts obtained in the two types of approach have the same l dependence only in the asymptotic zone ($b\gtrsim 5$ fm). This is due to the optical limit assumption used here, i.e., the same l dependence and the same value ($\bar{\alpha}_{NN}\sim 1$) of the real and imaginary parts of the phase shifts. For $E/A=25$ and 30 MeV, optical model analysis shows that the absorption is so strong below $b=5$ fm that partial waves in the region of the nuclear rainbow can hardly

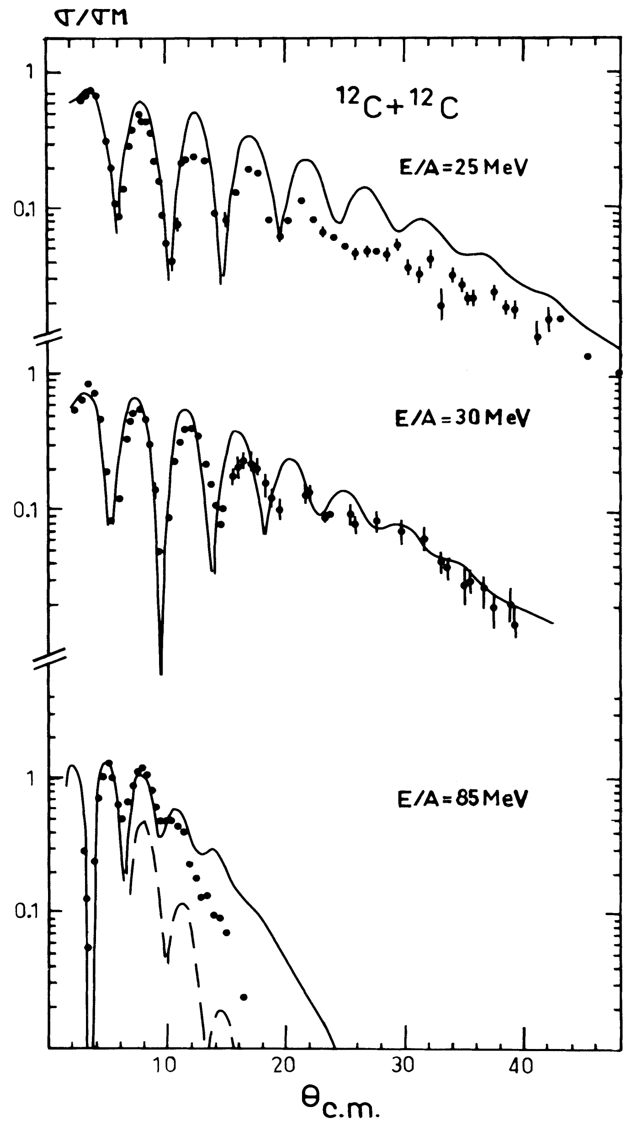

FIG. 3. Elastic scattering differential cross sections for the $^{12}C+^{12}C$ system at $E/A=25$, 30, and 85 MeV compared to the calculations. The dashed line for $E/A=85$ MeV illustrates the black-sphere diffractive pattern obtained when refraction is neglected ($\alpha_{NN}=0$).

contribute to the cross section.[5,13] The situation is rather different at $E/A=85$ MeV where total absorption takes place at smaller impact parameters[6] and unmasks the region where our approximation becomes bad. This probably explains the larger disagreement between our calculation and the data at this latter incident energy (Fig. 3).

Adjusting α_{NN} cannot solve this problem, since $\mathrm{Re}\Delta_l$ and $\mathrm{Im}\Delta_l$ obtained from optical model analysis[5,6] exhibit different b dependence below 5 fm. In other words, at $E/A=85$ MeV the nuclei densities have more overlap (Fig. 4) and then, higher order effects, such as nucleon correlation, probably become important.[9] This means that for the energy range $E/A=100$–200 MeV where the transparency of the nuclei is supposed to increase,[1–4] the

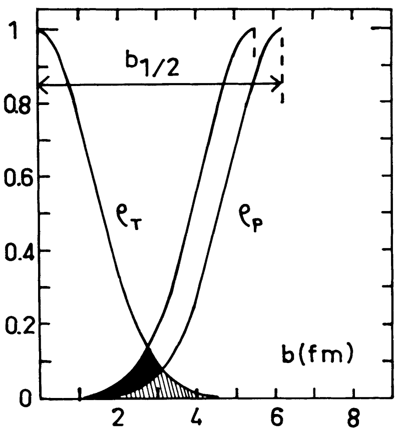

FIG. 4. Overlap of the projectile and target densities—here $^{12}C+^{12}C$—when the distance between the two nuclei is set equal to the strong absorption distance $b_{1/2}$. This overlap is smaller at $E/A=30$ MeV (dashed area) than at $E/A=85$ MeV (dashed plus black area).

present simple optical limit formulation cannot account for the large angle data.

Calculations for $^{12}C+^{208}Pb$ at $E/A=30$ MeV are shown in Fig. 5. The ^{208}Pb Gaussian density has been adjusted by Karol to reproduce the surface tail of the density. The differential cross section is compared with experimental $^{13}C+^{208}Pb$ data at the same E/A. (As in the previous calculations, partial waves up to 500 have been taken into account.) The fit is good but the reaction cross section is about 20% larger than the value deduced from a

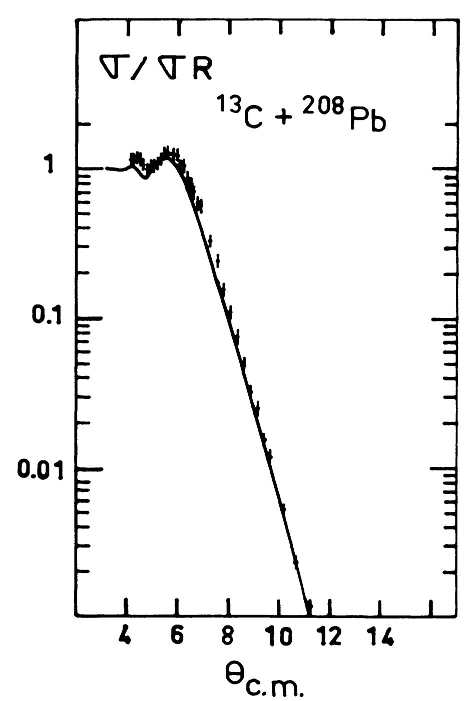

FIG. 5. Elastic scattering differential cross sections for the $^{13}C+^{208}Pb$ at $E/A=30$ MeV. The calculation uses ^{12}C density [relation (17)].

preliminary optical model analysis.[17] It must be noted that, here, Coulomb effects are not negligible since the Coulomb barrier is about 60 MeV and produces appreciable Coulomb deflection of the trajectories, which is not taken into account in our model. The straight line trajectory assumption made here is probably responsible for the disagreement on σ_R.

IV. SUMMARY

The simple approach of Karol has been extended to the description of elastic scattering at low and intermediate energies. The rather good quality of the predictions suggests that this simple model could be used to understand the main features of nucleus-nucleus elastic scattering over a broad range of incident energies.

Comparison with the results of optical model analysis could be enriching. However, more data are necessary with different projectiles and at higher energies to test to what extent the free nucleon-nucleon scattering determines nucleus-nucleus reaction and elastic scattering cross sections.

*Present address: Los Alamos Scientific Laboratory, Los Alamos, NM 87545.

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