Two-pion-exchange three-nucleon force and the ^{3}H and ^{3}He nuclei

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We have derived a two-pion-exchange three-body force and calculated its contribution to the trinucleon ground state. The hyperspherical harmonic method has been used. Coulomb and the three-body forces have been treated nonperturbatively. Results show sensitivity to the short range part of the force. A comparative study is made with other forms of three-body forces found in the literature.

[NUCLEAR STRUCTURE Trinucleon systems, three-body force.]

I. INTRODUCTION

A better understanding of nucleon-nucleon interaction requires a deeper study of three-nucleon systems, such as ^{3}H and ^{3}He. A few realistic two-body potentials exist which can fit well two-nucleon data.[1] However, they fail to reproduce experimental data for ^{3}H and ^{3}He, although binding energies, charge form factors, rms radii, etc., reach basically a common set of values when one considers various "realistic" two-body forces (2BF) and distinct calculation techniques.[1] For instance, the experimental binding energies (BE) for ^{3}H (8.482 MeV) and ^{3}He (7.718 MeV) fail to be reproduced by about 1.5 MeV and 2.0 MeV, respectively. The charge form factors[2] $|F_{\text{ch}}(q^2)|$ show a striking disagreement with experiment: The theoretical momentum transfer at the first minimum is too high and the height of the second maximum ($F_{\max}$) too low. Those discrepancies between experimental data and theoretical calculations with realistic 2BF seem to indicate that something is still missing. One can certainly expect that meson exchange current (MEC) would play an important role in the calculation, as well as other effects (recoil effect, relativistic corrections, etc.).[3] Recent works[4–9] conjecture that the inclusion of the three-body force (3BF) should account for part of the above discrepancy in the data. The nature of this force supports this argument; namely, inclusion of the 3BF strengthens the potential minimum around the equilateral triangle configuration, and decreases the attraction in the collinear configuration.[4–9] A three-body correlation should appear in such a way that $F_{\text{ch}}(q^2)$ should be improved. But so far all the calculations are still in an initial stage, suffering from either a lack of consistency (arising from the problem of how to incorporate these effects to 2BF, choice of Feynman diagrams, approximations, etc.), or calculational limitations.[3–12]

In this work we perform an essentially exact calculation for trinucleon systems, using the hyperspherical harmonic (HH) method.[13] The Afnan-Tang $S3$ (Ref. 14) potential which is reasonably realistic, although quite simple in structure, has been chosen to represent the 2BF. The forces due to Coulomb and proper three nucleon interactions do not alter the structure of the equations in the HH method and are taken into account nonperturbatively.

In the first part of this work we derive in a pedagogical way the 3BF due to the exchange of two pions ($\pi\pi E$-3BF) using effective Lagrangians which are approximately invariant under transformations of the group $SU(2)\times SU(2)$.[15] Calculations of the $\pi\pi E$-3BF using chiral symmetry implemented by means of current algebra have already appeared in the literature.[16,17] The motivation for implementing it by means of effective Lagrangians is that we achieve a much better understanding of the dynamical origins of the various contributions, making it very easy to compare our results with other existing forms, such as the classic Fujita-Miyazawa result.[18] Another advantage of a calculation based on Feynman diagrams is that it can guide us in the evaluation of the contribution to 3BF of other diagrams, involving the exchange of heavier mesons. In this work we consider the two terms of the 3BF which are generated by the s and p waves of the virtual pions, and their relative importance.

The 3BF due to the exchange of heavier mesons or more than two pions, have shorter range and should be, to some extent, shadowed by the two-body repulsive core. This core is also required in the case when the form factor of the πNN vertex is taken to be unity because in this case, the 3BF is extremely singular at short distances and this would generate unphysical nodes in the physical radial wave function.[6] These nodes can be prevented either by means of a phenomenological cutoff parameter[6] or pion nucleon form factors,[9] and both possibilities are discussed here.

The potential corresponding to the $\pi\pi E$-3BE is derived in Sec. II, the calculation method is introduced in Sec. III, and our results and conclusions are presented in Sec. IV.

II. THREE-BODY FORCES

A. Introduction

The properties of three nucleon systems such as the ^{3}H or ^{3}He are determined by the interactions of their constituents. The forces among three nucleons are due either to

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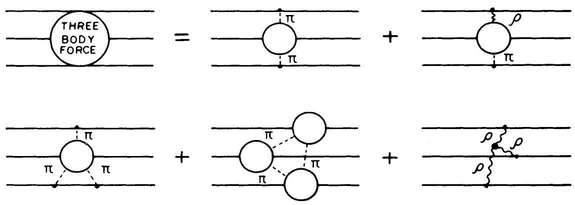

FIG. 1. Some diagrams corresponding to three-body forces.

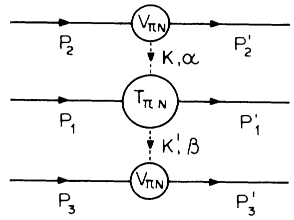

FIG. 2. Diagram corresponding to the $\pi\pi E$-3BF.

pair interactions or proper 3BF. By proper 3BF we mean processes corresponding to diagrams which cannot be separated into two pieces by cutting forward propagating nucleon lines only.

Three nucleon forces are due to exchanges of bosons and some of the corresponding processes are shown in Fig. 1. The first diagram represents the pion-pion exchange ($\pi\pi$E)-3BF, the only one to be discussed in detail in the present paper. The other diagrams correspond to forces of shorter range and represent either the exchange of heavier mesons or the exchange of more than two pions.

In order to evaluate the $\pi\pi E$-3BF, we calculate the contribution of the exchange of two pions to the elastic scattering of three unbound nucleons. This process corresponds to permutations of the diagrams of Fig. 2. In this figure the broken lines represent pions, the unbroken ones nucleons, $V_{\pi\text{N}}$ is the πNN vertex, and $T_{\pi\text{N}}$ is the amplitude for the process $\pi\text{N}\rightarrow\pi\text{N}$. The pions participating in these interactions are off shell and hence the evaluation of these amplitudes can only be performed with the help of some theory.

The most successful theory describing pion processes is based on the assumption that their interactions are approximately invariant under transformations of the group $\text{SU}(2)\times\text{SU}(2)$. The corresponding approximate symmetry, known as chiral symmetry, becomes exact when the four-momentum of the pions vanishes. There are two main approaches for applying this symmetry to processes involving pions. One of them uses the so-called current algebra whereas the other is based upon effective Lagrangians. They are physically equivalent, but correspond to rather different calculational techniques. The former approach has, as pointed out by Weinberg,[19] the disadvantages of requiring much algebraic effort when the number of pions is not small, and of hiding the dynamical implications of the soft pion limit.

These problems are not present in the alternative approach which is based on effective or phenomenological Lagrangians which are built in such a way as to reproduce the results of current algebra when used in lowest order in perturbation theory.[19] It is important to stress that these Lagrangians, which constitute the basis of the so-called chiral dynamics, are different in spirit, for instance, from those appearing in quantum electrodynamics. Rather than being fundamental objects, they are quick and efficient tools for implementing chiral symmetry. The use of effective Lagrangians should not, therefore, be seen as an attempt to apply ordinary perturbation theory in calculation of strong processes.

There are at least two possible ways of constructing effective Lagrangians which are approximately chiral invariant. The first one corresponds to the linear σ model, containing four spinless boson fields, three of which are pseudoscalar and associated to the pions, the fourth being scalar and called σ.[20] This model, although adequate for implementing chiral symmetry, has problems in providing a reasonable description of nature, since no serious candidate for the σ seems to exist among the known particles. Moreoever, this model predicts that the nucleon mass should vanish in the limit of exact symmetry. Hence, drastic changes in the nucleon mass would be due to a small breaking term in the Lagrangian. It is important to notice that the linear σ model prescribes the pseudoscalar coupling between pions and nucleons.

Chiral symmetry can also be implemented by means of nonlinear effective Lagrangians containing only physical fields. In the present work a nonlinear Lagrangian is used in the evaluation of $V_{\pi\text{N}}$ and $T_{\pi\text{N}}$ which contribute to $\pi\pi E$-3BF. The πN system has been extensively studied by means of chiral symmetry and agreement with experiment is good both below threshold and for pion energies up to 350 MeV.[22] In the nonlinear Lagrangian approach the πN coupling is of the pseudovector type and the amplitude for the process $\pi\text{N}\rightarrow\pi\text{N}$ is assumed to be given by the diagram of Fig. 3.

The vertices in these diagrams are extracted from the following terms of the effective nonlinear Lagrangian:

$$L_{\pi\text{NN}}=\frac{g}{2m}\{\bar{N}\gamma_\mu\gamma_5\vec{\tau}\text{N}\}\cdot\partial^\mu\vec{\phi}\ , \tag{1}$$

$$L_{\pi\text{N}\Delta}=g_\Delta\{\bar{\Delta}^\mu[g_{\mu\nu}-(Z+\tfrac{1}{2})\gamma_\mu\gamma_\nu]\vec{\text{M}}\text{N}\}\cdot\partial^\nu\vec{\phi}+\text{H.c.}\ , \tag{2}$$

$$L_{\rho\text{NN}}=\frac{\gamma_0}{2}(\bar{\text{N}}\gamma_\mu\vec{\tau}\text{N})\vec{\rho}^{\,\mu}+\frac{\gamma_0}{2}\left[\bar{\text{N}}\left[\frac{\mu_\text{p}-\mu_\text{n}}{4m}\right]i\sigma_{\mu\nu}\vec{\tau}N\right]\cdot(\partial^\mu\vec{\rho}^{\,\nu}-\partial^\nu\vec{\rho}^{\,\mu})\ , \tag{3}$$

$$L_{\rho\pi\pi}=\gamma_0\vec{\rho}_\mu\cdot(\vec{\phi}\times\partial^\mu\vec{\phi})+\frac{\gamma_0}{4m_\rho^2}(\delta-1)(\partial_\mu\vec{\rho}_\nu-\partial_\nu\vec{\rho}_\mu)\cdot(\partial^\mu\vec{\phi}\times\partial^\nu\vec{\phi})\ . \tag{4}$$

The symbols $\vec{\phi}$, N, Δ, and $\vec{\rho}$ denote, respectively, the pion, nucleon, delta, and rho fields, whose masses are μ, m, M_Δ, and m_ρ. The matrices $\vec{\tau}$ and $\vec{M}$ combine two nucleons, and a nucleon and a delta into isospin 1 states. The parameters μ_p and μ_n represent the proton and neutron anomalous magnetic moments whereas δ can be measured in the decay $\rho\rightarrow\pi\pi$. The parameter Z appearing in Eq. (2) represents the possibility of spin $\frac{1}{2}$ components into the off-pole delta wave function.[24] This form of the $\pi N\Delta$ coupling corresponds to the delta propagator given by[25]

$$iG_{\mu\nu}(p)=-i\frac{(\not p+M_\Delta)}{p^2-M_\Delta^2}\left[g_{\mu\nu}-\tfrac{1}{3}\gamma_\mu\gamma_\nu-\frac{\gamma_\mu p_\nu}{3M_\Delta}+\frac{p_\mu\gamma_\nu}{3M_\Delta}-\frac{2p_\mu p_\nu}{3M_\Delta^2}\right]. \tag{5}$$

The interaction Lagrangians allow us to evaluate the amplitude $T_{\pi N}$, describing the process $\pi N\rightarrow\pi N$. When the nucleons are on shell, this amplitude can be parametrized as

$$T_{\pi N}^{ab}=\bar{u}(\vec{p}\,'_1)\left[\left[A^++\frac{\not k'+\not k}{2}B^+\right]\delta_{ab}+\left[A^-+\frac{\not k'+\not k}{2}B^-\right]i\epsilon_{bac}\tau^c\right]u(\vec{p}_1). \tag{6}$$

The first two diagrams of Fig. 3 correspond to nucleon poles in the s and u channels. Their contribution to $T_{\pi N}$ is given by

$$A_N^+=\frac{g^2}{m}, \tag{7}$$

$$B_N^+=\frac{g^2}{m}\frac{\nu}{(k\cdot k'/2m)^2-\nu^2}, \tag{8}$$

$$A_N^-=0, \tag{9}$$

$$B_N^-=\frac{-g^2}{2m^2}-\frac{g^2}{m}\frac{(k\cdot k'/2m)}{(k\cdot k'/2m)^2-\nu^2}, \tag{10}$$

where ν is defined as

$$\nu\equiv(p_1+p'_1)(k+k')/4m. \tag{11}$$

The diagram corresponding to rho exchange yields the following values for $A^\pm$ and $B^\pm$:

$$A_\rho^+=B_\rho^+=0, \tag{12}$$

$$A_\rho^-=-\frac{\gamma_0^2}{m_\rho^2}(\mu_p-\mu_n)\nu\frac{1+(\delta-1)t/4m_\rho^2}{1-t/m_\rho^2}, \tag{13}$$

$$B_\rho^-=\frac{\gamma_0^2}{m_\rho^2}(1+\mu_p-\mu_n)\frac{1+(\delta-1)t/4m_\rho^2}{1-t/m_\rho^2}. \tag{14}$$

The contribution of the delta pole to the amplitude $T_{\pi N}$ is given by

$$A_\Delta^+=\frac{2g_\Delta^2}{9m}\left\{\frac{\nu_\Delta}{\nu_\Delta^2-\nu^2}\hat{A}-m\frac{(m+M_\Delta)}{M_\Delta^2}(2M_\Delta^2+mM_\Delta-m^2+k^2+k'^2)+4m\frac{k\cdot k'}{M_\Delta^2}[(M_\Delta+m)Z+(2M_\Delta+m)Z^2]\right\}, \tag{15}$$

$$B_\Delta^+=\frac{2g_\Delta^2}{9m}\left[\frac{\nu}{\nu_\Delta^2-\nu^2}\hat{B}-\frac{8m^2\nu}{M_\Delta^2}Z^2\right], \tag{16}$$

$$A_\Delta^-=-\frac{g_\Delta^2}{9m}\left\{\frac{\nu}{\nu_\Delta^2-\nu^2}\hat{A}+\frac{8m^2\nu}{M_\Delta^2}[(m+M_\Delta)Z+(2M_\Delta+m)Z^2]\right\}, \tag{17}$$

$$B_\Delta^-=-\frac{g_\Delta^2}{9m}\left[\frac{\nu_\Delta}{\nu_\Delta^2-\nu^2}\hat{B}-m\frac{(m+M_\Delta)^2}{M_\Delta^2}-\frac{4m}{M_\Delta^2}(2m^2+2mM_\Delta-k^2-k'^2)Z-\frac{4m}{M_\Delta^2}(4mM_\Delta+2m^2+k\cdot k')Z^2\right], \tag{18}$$

where

$$\nu_\Delta=\left[\frac{M_\Delta^2-m^2-k\cdot k'}{2m}\right], \tag{19}$$

$$\hat{A}=\left[\frac{(m+M_\Delta)^2}{2M_\Delta^2}(2M_\Delta-m)(M_\Delta^2-m^2)-3(m+M_\Delta)k\cdot k'+\frac{k^2+k'^2}{2M_\Delta^2}(m+M_\Delta)(M_\Delta^2-m^2)+\frac{k^2k'^2}{2M_\Delta^2}(2M_\Delta+m)\right], \tag{20}$$

$$\hat{B}=\left[\frac{(m+M_\Delta)^2}{2M_\Delta^2}(M_\Delta^2+m^2-4mM_\Delta)-3k\cdot k'+\frac{k^2+k'^2}{2M_\Delta^2}(2M_\Delta^2-m^2+mM_\Delta)+\frac{k^2k'^2}{2M_\Delta^2}\right] . \tag{21}$$

The last contribution to $T_{\pi N}$ comes from the σ term. This term is produced in the current algebra approach by the commutator between the axial charge and the pion field. In the present calculation we consider it as a correction to the amplitude and parametrize it as follows[21]:

$$A_\sigma^+=\alpha_\sigma+\beta_\sigma k\cdot k' \tag{22}$$

and

$$B_\sigma^+=A_\sigma^-=B_\sigma^-=0 , \tag{23}$$

where α_σ and β_σ are constants that can be extracted from experiment. They are related to the so called nucleon σ term by[22]

$$\sigma_{\rm NN}=f_\pi^2\alpha_\sigma , \tag{24}$$

where f_π is the pion decay constant.

The preceding results allow us to evaluate the amplitude T_{3N}, describing the contribution of the $\pi\pi$E to the three-nucleon interaction,

$$T_{3N}=[\bar{u}(\vec{p}\,'_2)\not{k}\gamma_5\tau^a u(\vec{p}_2)]\frac{g/2m}{k^2-\mu^2}\left\{\bar{u}(\vec{p}\,'_1)\left[\left[A^++\frac{\not{k}'+\not{k}}{2}B^+\right]\delta_{ab}+\left[A^-+\frac{\not{k}'+\not{k}}{2}B^-\right]i\epsilon_{bac}\tau^c\right]u(\vec{p}_1)\right\}$$

$$\times\frac{g/2m}{k'^2-\mu^2}[\bar{u}(\vec{p}\,'_3)\not{k}'\gamma_5\tau^b u(\vec{p}_3)] . \tag{25}$$

In the above expression the amplitudes $A^\pm$ and $B^\pm$ receive contributions from the nucleon and delta poles and the ρ and σ exchanges. In order to write down the expression for the potential in momentum space one has to subtract the part of the amplitude representing an iteration of the two nucleon potential and to perform a nonrelativistic reduction of Eq. (25).

B. The potential in momentum space

The potential in momentum space is expressed in terms of nonrelativistic nucleons and therefore we neglect terms of order $(\vec{p}\,^2/m^2)$ in Eq. (25), where $\vec{p}$ is a typical nucleon momentum which we assume to be of order of μ, the pion mass. The approximate expressions for the dynamical variables appearing in the elastic three nucleon amplitude are given below. The momentum p of a nucleon is

$$p=(E,\vec{p})\cong\left[m+\frac{\vec{p}\,^2}{2m},\vec{p}\right] . \tag{26}$$

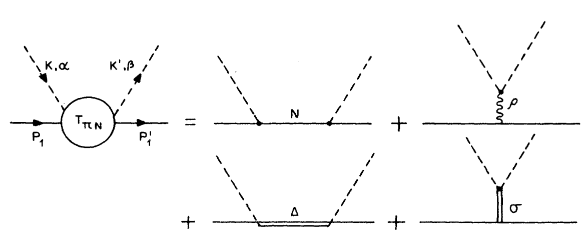

FIG. 3. Diagrams contributing to πN scattering.

The momenta of the π are

$$k=(\omega,\vec{k})\cong\left[\frac{\vec{p}\,_2^2-\vec{p}\,_2'^2}{2m},\vec{p}_2-\vec{p}\,'_2\right] , \tag{27}$$

$$k'=(\omega',\vec{k}\,')\cong\left[\frac{\vec{p}\,_3'^2-\vec{p}\,_3^2}{2m},\vec{p}\,'_3-\vec{p}_3\right] . \tag{28}$$

The covariant variables ν and s are approximated by

$$\nu\cong\frac{1}{4m}[2m(\omega+\omega')-(\vec{p}_1+\vec{p}\,'_1)\cdot(\vec{k}+\vec{k}\,')] , \tag{29}$$

$$s\cong m^2+2m\nu-\vec{k}\cdot\vec{k}\,' . \tag{30}$$

We use the following form for the Dirac spinors:

$$u(\vec{p})=\frac{1}{\sqrt{E+m}}\begin{bmatrix}E+m\\ \vec{\sigma}\cdot\vec{p}\end{bmatrix}\chi , \tag{31}$$

where $\vec{\sigma}$ and χ are Pauli matrices and spinors, respectively. The γ matrices are such that

$$\not{a}=\begin{bmatrix}a_0 & -\vec{\sigma}\cdot\vec{a}\\ \vec{\sigma}\cdot\vec{a} & -a_0\end{bmatrix} . \tag{32}$$

When the above approximations are used in Eq. (25) and only leading terms are kept, we obtain the nonrelativistic reduction of the three nucleon amplitude,

$$t_{3N}=\frac{g}{\vec{k}^2+\mu^2}\frac{g}{\vec{k}'^2+\mu^2}(\vec{\sigma}^{(2)}\cdot\vec{k})(\vec{\sigma}^{(3)}\cdot\vec{k}')\tau_a^{(2)}\tau_b^{(3)}$$
$$\times\{[2mf^+ + i\vec{\sigma}\cdot(\vec{k}'\times\vec{k})b^+]\delta_{ab}+[2mf^- + i\vec{\sigma}\cdot(\vec{k}'\times\vec{k})b^-]i\epsilon_{bac}\tau^c\}\ , \tag{33}$$

where $\vec{\sigma}^{(i)}$ and $\vec{\tau}^{(i)}$ indicate expectation values,

$$f^{\pm}=a^{\pm}+b^{\pm}\nu\ , \tag{34}$$

and $a^{\pm}$ and $b^{\pm}$ are the nonrelativistic reduction of the amplitudes $A^{\pm}$ and $B^{\pm}$.

The final form of the nonrelativistic three nucleon amplitude is determined by the contributions of the various diagrams of Fig. 3 to $f^{\pm}$ and $b^{\pm}$. These contributions are displayed below.

Nucleon. The iteration of the one-pion exchange two-nucleon potential corresponds to intermediate nucleons propagating forward in time. In order to subtract this contribution from the amplitude $T_{\pi N}$ it is convenient to decompose the numerator of the nucleon propagators as follows:

$$(\not p+m)=\tfrac{1}{2}\left[\left[1+\frac{p_0}{E}\right]u(\vec{p})\bar{u}(\vec{p})\right.$$
$$\left.-\left[1-\frac{p_0}{E}\right]v(-\vec{p})\bar{v}(-\vec{p})\right]\ , \tag{35}$$

where p_0 is the energy component of p, the four-momentum of the propagating nucleon, and

$$E=(m^2+\vec{p}^{\,2})^{1/2}\ . \tag{36}$$

In this expression forward propagation in time is represented by the term proportional to $u\bar{u}$, which can be written as

$$(\not p+m)_F\equiv\tfrac{1}{2}\left[1+\frac{p_0}{E}\right]\bar{u}(\vec{p})\bar{u}(\vec{p})$$
$$=\tfrac{1}{2}\left[1+\frac{p_0}{E}\right](\hat{\not p}+m)\ , \tag{37}$$

where the four vector $\hat{p}$ is given by

$$\hat{p}=(E,\vec{p})\ . \tag{38}$$

Thus backward propagation in time is represented by

$$(\not p+m)_B\equiv(\not p+m)-(\not p+m)_F$$
$$=\tfrac{1}{2}\left[1+\frac{p_0}{E}\right](\not p-\hat{\not p})+\tfrac{1}{2}\left[1-\frac{p_0}{E}\right](\not p+m)\ . \tag{39}$$

The contribution of a backward propagating nucleon to the direct πN amplitude is given by the nucleon pole diagrams with the numerator of the propagator replaced by Eq. (39). When the terms proportional to $\not p$ are eliminated by means of the Dirac equation, we obtain (where DB stands for direct and backward)

$$T_{DB}^{ab}=\frac{g^2}{4m^2}(\delta_{ab}+i\epsilon_{bac}\tau^c)\frac{1}{2E(p_0+E)}$$
$$\times\bar{u}(\vec{p}\,'_1)\left[-2m(s-m^2)+(s+3m^2)\frac{\not k+\not k'}{2}-(E+p_0)\not k'\gamma_0\not k\right]u(\vec{p}_1)\ . \tag{40}$$

The nonrelativistic limit of this equation is

$$t_{DB}^{ab}=0\ . \tag{41}$$

Rho. The contribution of the ρ exchange is given by Eqs. (12)—(14). When writing their nonrelativistic limits, we use the equality[26]

$$\gamma_0^2/m_\rho^2=\tfrac{1}{2}f_\pi^2\ . \tag{42}$$

Thus we obtain

$$f_\rho^+=b_\rho^+=0\ , \tag{43}$$
$$f_\rho^-=\frac{1}{2f_\pi^2}\ , \tag{44}$$
$$b_\rho^-=\frac{1}{2f_\pi^2}(1+\mu_p-\mu_n)\ . \tag{45}$$

Delta. The nonrelativistic reduction of Eqs. (15)—(18) produces

$$f_\Delta^+=\frac{2g_\Delta^2}{9M_\Delta^2}\frac{\vec{k}\cdot\vec{k}'}{M_\Delta-m}[4M_\Delta^2-mM_\Delta+m^2-4(M_\Delta^2-m^2)Z-4(2M_\Delta^2-mM_\Delta-m^2)Z^2]\ , \tag{46}$$

$$b_\Delta^+=f_\Delta^-=0\ , \tag{47}$$

$$b_\Delta^- = \frac{g_\Delta^2}{9M_\Delta^2}\frac{2m}{M_\Delta - m}[(2M_\Delta^2 + mM_\Delta - m^2) + 4(M_\Delta^2 - m^2)Z + 4(2M_\Delta^2 - mM_\Delta - m^2)Z^2] \ . \tag{48}$$

The value of the parameter Z can be extracted from the subthreshold πN amplitude. The analysis of Olsson and Osypowski[22] has shown that the experimental results are compatible with the value $Z = -\frac{1}{2}$, which will be adopted in the present work. Thus Eqs. (46)—(48) become

$$f_\Delta^+ = \frac{8g_\Delta^2}{9(M_\Delta - m)}\vec{k}\cdot\vec{k}' \ , \tag{49}$$

$$b_\Delta^- = \frac{4g_\Delta^2 m}{g(M_\Delta - m)} \ . \tag{50}$$

Sigma. Finally, the nonrelativistic contribution of the σ term is

$$f_\sigma^+ = \alpha_\sigma - \beta_\sigma \vec{k}\cdot\vec{k}' \ , \tag{51}$$

$$b_\sigma^+ = f_\sigma^- = b_\sigma^- = 0 \ . \tag{52}$$

Using the above results in Eq. (30) we obtain the final form for the nonrelativistic three-nucleon scattering amplitude in momentum space,

$$\begin{aligned} t_{3\mathrm{N}} = 2m & \left[\frac{g}{\vec{k}^2+\mu^2}\right]\left[\frac{g}{\vec{k}'^2+\mu^2}\right](\vec{\sigma}^{(2)}\cdot\vec{k})(\vec{\sigma}^{(3)}\cdot\vec{k}') \\ & \times \left\{ \vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}\left[\alpha_\sigma + \left[\frac{8g_\Delta^2}{9(M_\Delta - m)} - \beta_0\right](\vec{k}\cdot\vec{k}')\right] - i\vec{\tau}^{(1)}\cdot(\vec{\tau}^{(2)}\times\vec{\tau}^{(3)})\frac{1}{2f_\pi^2}v \right. \\ & \left. - \vec{\tau}^{(1)}\cdot(\vec{\tau}^{(2)}\times\vec{\tau}^{(3)})\left[\frac{1}{2f_\pi^2}\frac{(1+\mu_\mathrm{p}-\mu_\mathrm{n})}{2m} + \frac{2g_\Delta^2}{9(M_\Delta - m)}\right]\vec{\sigma}^{(1)}\cdot(\vec{k}\times\vec{k}')\right\} \ . \end{aligned} \tag{53}$$

The part of the amplitude proportional to v is velocity dependent and corresponds to nonlocal terms in the potential. Besides, as will be seen later, its coefficient is smaller than those of the other two terms. Hence, we will not consider this term in the present work.

At this point, it is convenient to discuss the introduction of form factors, which are not predicted by chiral symmetry used in tree approximations. Rather, they can be understood as phenomenological corrections to the vertices and must be considered in realistic calculations. Their formal inclusion in Eq. (53) is very simple, since it would correspond to allowing the coupling constants to become dependent on the four-momentum of the pion. In the present work the πNN form factor for a pion of four momentum k is included in the following form:

$$g \to g(\vec{k}^2) = g \times \left[\frac{\Lambda^2 - \mu^2}{\Lambda^2 - \vec{k}^2}\right]^n \ . \tag{54}$$

This parametrization is such that the form factor becomes equal to the coupling constant when the pion is on-shell. The value of the parameter Λ can be determined if we assume that $g(k^2)$ satisfies the Goldberger-Treiman[27] relation when $k^2=0$. This would mean that $g(\mu^2) \cong (1+0.06/n)g(0)$ (Ref. 28) and therefore $\Lambda \sim 4\sqrt{n}\,\mu$. This value of Λ shows that relativistic corrections are of the same order of magnitude as the ones arising from form factors. For instance, in the assessment of the delta contribution to Eq. (53) we have neglected terms of order $\mu^2/(M_\Delta^2 - m^2)$, which are comparable to μ^2/Λ^2. Thus it would seem to be an inconsistent procedure to keep the form factors while neglecting relativistic corrections. Nevertheless in this work we do keep the corrections to the πNN coupling constant. This is done in order to ensure the convergence of the Fourier transforms of the amplitudes for any values of the relative distance between two nucleons, as we will discuss in the next subsection.

Before proceeding to the transition to coordinate space, however, it is convenient to establish the relationship between the amplitude $t_{3\mathrm{N}}$ and the three-body potential.

The three nucleon potential in momentum space is defined as[11]

$$\langle \vec{p}\,'_1\vec{p}\,'_2\vec{p}\,'_3 | W^{123} | \vec{p}_1\vec{p}_2\vec{p}_3\rangle = -(2\pi)^3\delta^3(\vec{p}_f - \vec{p}_i)\frac{1}{8m^3}t_{3\mathrm{N}} \ . \tag{55}$$

The factor $1/8m^3$ has been introduced because the momentum space states are normalized as

$$\langle \vec{p}\,' | \vec{p}\rangle = (2\pi)^3\delta^3(\vec{p}\,' - \vec{p}) \ . \tag{56}$$

C. The potential in coordinate space

The potential in coordinate space is obtained by Fourier tranforming the momentum space one. Thus

$$\langle \vec{r}\,'_1,\vec{r}\,'_2,\vec{r}\,'_3 | W^{123} | \vec{r}_1,\vec{r}_2,\vec{r}_3\rangle = -\frac{(2\pi)^3}{(2m)^3}\int \frac{d\vec{p}\,'_1}{(2\pi)^3}\cdots\frac{d\vec{p}_3}{(2\pi)^3}\delta^3(\vec{p}\,'_1+\vec{p}\,'_2+\vec{p}\,'_3-\vec{p}_1-\vec{p}_2-\vec{p}_3)e^{-i\vec{p}\,'_1\cdot\vec{r}\,'_1}\cdots e^{i\vec{p}_3\cdot\vec{r}_3}t_{3\mathrm{N}} \ . \tag{57}$$

Using the expression for t_{3N} given by Eq. (53) we have

$$\langle \vec{r}\,'_1,\vec{r}\,'_2,\vec{r}\,'_3 \mid W^{123} \mid \vec{r}_1,\vec{r}_2,\vec{r}_3 \rangle \equiv \delta^3(\vec{r}_1-\vec{r}\,'_1)\delta^3(\vec{r}_2-\vec{r}\,'_2)\delta^3(\vec{r}_3-\vec{r}\,'_3)W(1)\,, \tag{58}$$

where

$$\begin{aligned} W(1)= \int \frac{d\vec{k}\,'}{(2\pi)^3}\frac{d\vec{k}}{(2\pi)^3} e^{i\vec{k}\,'\cdot\vec{r}_{31}} e^{i\vec{k}\cdot\vec{r}_{12}} \left[\frac{g(\vec{k}^{\,2})/2m}{\vec{k}^{\,2}+\mu^2}\right]\left[\frac{g(\vec{k}\,'^2)/2m}{\vec{k}\,'^2+\mu^2}\right](\vec{\sigma}^{(2)}\cdot\vec{k})(\sigma^{(3)}\cdot\vec{k}\,') \\ \times\left\{-\vec{\tau}^{(2)}\cdot\vec{\tau}^{(3)}\left[\alpha_\sigma+\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right](\vec{k}\cdot\vec{k}\,')\right]\right. \\ \left. +\,\vec{\tau}^{(1)}\cdot(\vec{\tau}^{(2)}\times\vec{\tau}^{(3)})\vec{\sigma}^{(1)}\cdot(\vec{k}\times\vec{k}\,')\left[\frac{1}{2f_\pi^2}\cdot\frac{(1+\mu_{\rm p}-\mu_{\rm n})}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]\right\}. \end{aligned} \tag{59}$$

The variables $\vec{r}_{ij}$ are defined as

$$\vec{r}_{ij}=\vec{r}_i-\vec{r}_j\,, \tag{60}$$

and i, j, and k correspond to cyclic permutations of the integers 1, 2, and 3.

Equation (59) may be rewritten as

$$\begin{aligned} W(k)= \left[\frac{C_s}{\mu^2}\right](\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})U(r_{ki})U(r_{jk}) \\ +\left[\frac{C_p}{\mu^4}\right](\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})(\vec{\nabla}_{ki}\cdot\vec{\nabla}_{jk})U(r_{ki})U(r_{jk}) \\ -\left[\frac{C_p'}{\mu^4}\right](\vec{\tau}^{(i)}\times\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})(\vec{\sigma}^{(k)}\cdot\vec{\nabla}_{ki}\times\vec{\nabla}_{jk})U(r_{ki})U(r_{jk})\,. \end{aligned} \tag{61}$$

In this work we adopt the above form for the potential. However, for the sake of completeness, it is worth noting that the usual properties of Pauli matrices allow us to cast this expression in the form used by Fujita and Miyazawa[18]:

$$\begin{aligned} W(k)= \left\{\frac{C_s}{\mu^2}(\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})\right. \\ +\frac{1}{4\mu^4}[(C_p+C_p')(\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})(\vec{\tau}^{(k)}\cdot\vec{\tau}^{(i)})+(C_p-C_p')(\vec{\tau}^{(k)}\cdot\vec{\tau}^{(i)})(\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})] \\ \times(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})(\vec{\sigma}^{(k)}\cdot\vec{\nabla}_{jk})(\vec{\sigma}^{(k)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki}) \\ +\frac{1}{4\mu^4}[(C_p-C_p')(\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})(\vec{\tau}^{(k)}\cdot\vec{\tau}^{(i)})+(C_p+C_p')(\vec{\tau}^{(k)}\cdot\vec{\tau}^{(i)})(\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})] \\ \left.\times(\vec{\sigma}^{(i)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(k)}\cdot\vec{\nabla}_{ki})(\vec{\sigma}^{(k)}\cdot\vec{\nabla}_{jk})(\vec{\sigma}^{(j)}\cdot\vec{\nabla}_{jk})\right\}U(r_{ki})U(r_{jk})\,. \end{aligned} \tag{62}$$

The strength parameters in the potential are given by

$$\begin{aligned} C_s&=+\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^2\alpha_\sigma\,, \\ C_p&=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^4\left[\frac{8g_\Delta^2}{9(M_\Delta-m)}-\beta_\sigma\right], \\ C_p'&=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^4\left[\frac{1}{2f_\pi^2}\frac{1+\mu_{\rm p}-\mu_{\rm n}}{2m}+\frac{2g_\Delta^2}{9(M_\Delta-m)}\right]. \end{aligned} \tag{63}$$

The function $U(x)$ is given by

$$\begin{aligned} U(x)&=\frac{4\pi}{\mu}\int\frac{d\vec{k}}{(2\pi)^3}\frac{e^{-i\vec{k}\cdot\vec{x}}}{\vec{k}^{\,2}+\mu^2}\left[\frac{g(k^2)}{g}\right] \\ &=\frac{4\pi}{\mu}\int\frac{d\vec{k}}{(2\pi)^3}\frac{e^{-i\vec{k}\cdot\vec{x}}}{\vec{k}^{\,2}+\mu^2}\left[\frac{\Lambda^2-\mu^2}{\Lambda^2+\vec{k}^{\,2}}\right]^n. \end{aligned} \tag{64}$$

The integrals on the variables $\vec{k}$ and $\vec{k}\,'$ are the Fourier transforms of the pion propagator multiplied by the πN form factor. As we have mentioned before, these form factors have been kept in order to ensure the convergence of the integrals for high momenta when $\vec{x}$ is close to zero. Therefore, the form factor is effectively used as a cutoff in the momenta. By performing a power counting, one notes that the integrals are convergent for values of n greater than 1. In this work we adopt the value $n=2$, and the function $U(x)$ becomes

$$U(x)=\frac{e^{-\mu x}}{\mu x}-\frac{\Lambda}{\mu}\frac{e^{-\Lambda x}}{\Lambda x}-\frac{1}{2}\frac{\mu}{\Lambda}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x} . \tag{65}$$

In the literature[6,7] other procedures for dealing with the mathematical short distance problem can be found. For instance, the short distance repulsion between nucleons can be simulated by the introduction of a cutoff parameter, x_0 (in coordinate space), which can be defined as follows:

$$\bar{U}(x)=\begin{cases}U(x_0), & x<x_0,\\ U(x), & x\geq x_0,\end{cases} \tag{66}$$

with $\Lambda=\infty$. It is worth pointing out that form factors [Eq. (54)] and the short distance repulsion are due to distinct physical mechanisms and hence are not mutually exclusive.

The form of the function $U(x)$ given by Eq. (65) allows us to rewrite the potential as follows:

$$\begin{aligned}W(k)={}&C_s(\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})U_1(r_{ki})U_1(r_{jk})+\frac{C_p}{9}(\vec{\tau}^{(i)}\cdot\vec{\tau}^{(j)})\\
&\times\{\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}U_0(r_{ki})U_0(r_{jk})+[3(\vec{\sigma}^{(i)}\cdot\hat{r}_{jk})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}]U_0(r_{ki})U_2(r_{jk})\\
&+[3(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{ki})-\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}]U_2(r_{ki})U_0(r_{jk})\\
&+[9\cos\theta_k(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-3(\vec{\sigma}^{(i)}\cdot\hat{r}_{jk})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-3(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{ki})+\vec{\sigma}^{(i)}\cdot\vec{\sigma}^{(j)}]U_2(r_{ki})U_2(r_{jk})\}\\
&+\frac{C_p'}{9}(\vec{\tau}^{(i)}\times\vec{\tau}^{(j)}\cdot\vec{\tau}^{(k)})\{\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\cdot\vec{\sigma}^{(k)}U_0(r_{ki})U_0(r_{jk})\\
&\qquad+[3(\vec{\sigma}^{(k)}\times\vec{\sigma}^{(i)}\cdot\hat{r}_{jk})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\cdot\vec{\sigma}^{(k)}]U_0(r_{ki})U_2(r_{jk})\\
&\qquad+[3(\vec{\sigma}^{(j)}\times\vec{\sigma}^{(k)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})-\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\vec{\sigma}^{(k)}]U_2(r_{ki})U_0(r_{jk})\\
&\qquad+[9\vec{\sigma}^{(k)}\cdot(\hat{r}_{ki}\times\hat{r}_{jk})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})-3(\vec{\sigma}^{(k)}\times\vec{\sigma}^{(i)}\cdot\hat{r}_{jk})(\vec{\sigma}^{(j)}\cdot\hat{r}_{jk})\\
&\qquad-3(\vec{\sigma}^{(j)}\times\sigma^{(k)}\cdot\hat{r}_{ki})(\vec{\sigma}^{(i)}\cdot\hat{r}_{ki})+\vec{\sigma}^{(i)}\times\vec{\sigma}^{(j)}\cdot\vec{\sigma}^{(k)}]U_2(r_{ki})U_2(r_{jk})\} ,\end{aligned} \tag{67}$$

where

$$\cos\theta_k=\frac{\vec{\mathbf{r}}_{ki}\cdot\vec{\mathbf{r}}_{jk}}{r_{ki}r_{jk}} , \tag{68}$$

$$U_0(x)=\frac{e^{-\mu x}}{\mu x}-\frac{\Lambda}{\mu}\frac{e^{-\Lambda x}}{\Lambda x}-\frac{1}{2}\frac{\Lambda}{\mu}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x} , \tag{69}$$

$$U_1(x)=-\frac{e^{-\mu x}}{\mu x}\left[1+\frac{1}{\mu x}\right]+\frac{\Lambda^2}{\mu^2}\left[1+\frac{1}{\Lambda x}\right]\frac{e^{-\Lambda x}}{\Lambda x}+\frac{1}{2}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x} , \tag{70}$$

and

$$U_2(x)=\frac{e^{-\mu x}}{\mu x}\left[1+\frac{3}{\mu x}+\frac{3}{\mu^2x^2}\right]-\frac{\Lambda^2}{\mu^2}\frac{e^{-\Lambda x}}{\mu x}\left[1+\frac{3}{\Lambda x}+\frac{3}{\Lambda^2x^2}\right]-\frac{1}{2}\frac{\Lambda}{\mu}\left[\frac{\Lambda^2}{\mu^2}-1\right]e^{-\Lambda x}\left[1+\frac{1}{\Lambda x}\right] . \tag{71}$$

In the remainder of this work we assess the importance of this three-body force to the properties of ^{3}H and ^{3}He. In order to do this we consider only the leading contribution to the trinucleon wave function, which is given by the totally symmetric S wave.[6,9] This choice makes our results not fully realistic, since D waves can produce significant effects. This fact does not prevent, however, the usefulness of a detailed study of the S wave, because these results can provide a reliable point of departure for the inclusion of other effects. With this purpose in mind, we evaluate the expectation value of Eq. (67) between totally antisymmetric spin and isospin states[9,13] which are given by

$$|A\rangle=\frac{1}{\sqrt{2}}[\,|(t=0,\tfrac{1}{2})T=\tfrac{1}{2}\rangle\,|(s=1,\tfrac{1}{2})S=\tfrac{1}{2}\rangle-|(t=1,\tfrac{1}{2})T=\tfrac{1}{2}\rangle\,|(s=0,\tfrac{1}{2})S=\tfrac{1}{2}\rangle] , \tag{72}$$

where T and S are the isospin and spin of the system, while t and s are the corresponding quantities for the two-nucleon pair. This expectation value is the following:

$$W=\sum_{k=1}^{3} W(k)=C_0U_0(r_{ki})U_0(r_{jk})+C_1\cos\theta_k U_1(r_{ki})U_1(r_{jk})$$
$$+C_2(3\cos^2\theta_k-1)U_2(r_{ki})U_2(r_{jk})+(\text{cyclic permutations})\,. \tag{73}$$

The coefficients of the potential are given by

$$C_0=-(C_p-4C_p')/3=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^4\left[\frac{2}{3f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{\beta_\sigma}{3}\right], \tag{74}$$

$$C_1=-C_s=-\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^2\alpha_\sigma\,, \tag{75}$$

$$C_2=-(C_p+2C_p')/3=\left[\frac{1}{4\pi}\right]^2\left[\frac{g\mu}{2m}\right]^2\mu^4\left[\frac{1}{3f_\pi^2}\frac{(1+\mu_p-\mu_n)}{2m}+\frac{4g_\Delta^2}{9(M_\Delta-m)}-\frac{\beta_\sigma}{3}\right]. \tag{76}$$

In order to complete the evaluation of the 3BF we use the following values for the parameters appearing in the above equations: $g=13.39$,[29] $m=938.28$ MeV,[30] $\mu=139.57$ MeV,[30] $f_\pi=93$ MeV,[30] $\alpha_\sigma=1.05\mu^{-1}$,[30] $\beta_\sigma=-0.80\mu^{-3}$,[22] $\mu_p-\mu_n=3.706$, $g_\Delta=1.84\mu^{-1}$,[22] $M_\Delta=1220$ MeV.[22] Thus we obtain

$$C_0=-(0.46-0.23)\text{ MeV}=-0.23\text{ MeV}\,,$$
$$C_1=-0.92\text{ MeV}\,,$$
$$C_2=(0.23+0.65+0.23)\text{ MeV}=1.11\text{ MeV}\,.$$

The figures, within parentheses, correspond to partial contributions in Eqs. (74)—(76), and are displayed in order to indicate the relative importance of the σ, ρ, and Δ contributions to the intermediate πN amplitude in Fig. 2.

D. Comparison with other results

In this subsection we compare our results with the historical Fujita-Miyazawa[18] potential as well as with some more recent ones. There are two main reasons for the differences found among the various works. The first one regards the values of experimental quantities such as masses, coupling constants, and scattering lengths that have been used as input in the calculations. Differences in the 3BF due to these values reflect the experimental situation at a given time and will not be considered here.

The second source of divergence among the various works is a much more serious one, since it is due to the theoretical treatment of the intermediate πN scattering amplitude. In any realistic calculation we must require this amplitude to reproduce on-shell πN data as well as to be suitable for off-shell extrapolation. It is the failure of meeting one of these requirements that makes some of the potentials discussed below different from the one derived in this work.

In the work of Fujita and Miyazawa,[18] for instance, the πN amplitude was assumed to be dominated by the excitation of the Δ resonance. Besides, they also assumed that the background amplitude could be properly represented by the isospin symmetric scattering length. In hindsight we realize that both assumptions are not free of problems. First, we note that the Δ dominance of the πN amplitude occurs only when the total center of mass energy is close to the mass of the resonance, which is clearly not the case in the present problem. Our calculation, for instance, shows that the contributions due to σ and ρ exchanges are as important as the one coming from the Δ pole.

The use of scattering lengths poses problems of a different nature. Nowadays it is well known that πN scattering lengths and volumes receive contributions from the nucleon and delta poles besides the σ and ρ exchanges.[21] Therefore, when scattering lengths are used we must subtract the contribution of the nucleon pole and be careful not to double count the delta. A further problem associated with the use of scattering lengths and volumes comes from the fact that they are on-shell amplitudes evaluated at threshold whereas 3BF's require off-shell amplitudes below threshold. This region can be reached either by means of dispersion relations[31] or chiral symmetry[21] and both methods show that these amplitudes can differ appreciably. We can see, thus, that the smallness of the isospin symmetric scattering length does not mean that the corresponding s-wave amplitude for off-shell pions is small. In order to illustrate this point, we consider the case of the isospin even scattering length, which is given by

$$a_0^+=\frac{m}{4\pi(m+\mu)}C^+(\mu,0)\,, \tag{77}$$

where the function $C^+(\nu,t)$ is related to the πN amplitudes A^+ and B^+ by[21]

$$C^+(\nu,t)=A^+(\nu,t)+\frac{\nu}{(1-t/4m^2)}B^+(\nu,t)\,. \tag{78}$$

We now define a new function $\bar{C}^+(\nu,t)$ by subtracting the nucleon contribution from $C^+(\nu,t)$. This new function can be expanded in a power series of ν and t,[31]

$$\bar{C}^+(\nu,t)=(c_1^+ +c_2^+t)+(c_3^+ +c_4^+t)\nu^2+(c_5^+ +c_6^+t)\nu^4\,. \tag{79}$$

In Ref. 30 we find the following values for the on-shell coefficients:

$$c_1^+=-1.50\mu^{-1},\quad c_2^+=1.14\mu^{-3}\,,$$
$$c_3^+=1.12\mu^{-3},\quad c_4^+=0.15\mu^{-5}\,,$$
$$c_5^+=0.20\mu^{-5},\quad c_6^+=0.03\mu^{-7}\,.$$

The value of this amplitude at threshold is

$$\bar{C}^{+}(\mu,0)\cong c_1^{+}+c_3^{+}\mu^2+c_5^{+}\mu^4=0.18\mu^{-1} . \quad (80)$$

The 3BF, on the other hand, requires the use of this amplitude in a different kinematic region, namely, one in which $\nu \sim \mu^2/4m$. In this region we have

$$\bar{C}^{+}(\nu\equiv\mu^2/4m,t)\equiv(c_1^{+})^{\text{off}}+(c_2^{+}t)^{\text{off}} , \quad (81)$$

where the superscript "off" stands for off-shell pions. Thus $\bar{C}^{+}(\nu\equiv\mu/4m,t)$ is quite different from $\bar{C}^{+}(\mu,0)$ and hence the use of this amplitude in the appropriate kinematical region is very important. Thus, the potentials derived by the Tucson group[16] and by ourselves show that the contributions of both s and p waves to the intermediate πN amplitude, represented by the coefficients C_s, C_p, and C_p', should be considered.

The above comments show that the πN amplitude which serves as the basis for the Fujita-Miyazawa (FM) potential[18] does not reproduce on-shell data and is not suitable for off-shell extrapolation. These limitations cause this potential to be only partially realistic. It is instructive to note that the FM potential formally follows from Eq. (73) when we consider only the Δ contribution in Eqs. (74)—(76). For comparative purposes we also include in this work an evaluation of the contribution of the FM force to the properties of trinucleons.

Attempts to increase the dynamical content of the intermediate πN amplitude can be found in the works of Loiseau and Nogami,[32] where the σ-exchange contribution has been considered explicitly, and of Loiseau, Nogami, and Ross,[33] who treated the ϵ exchange. These authors did so, however, assuming that the cancellations that make the isospin symmetric scattering length small do persist for off-shell pions. Besides, they assume that σ or ϵ exchanges contribute only to s waves, whereas in the present work we show that the σ contribution to p waves, represented by β_σ, has an important role in the final result. Also, they do not consider the ρ exchange, which contributes as much as the Δ to the isospin odd amplitude. Thus, the refinements introduced by these authors were not enough to make the 3BF fully realistic.

The intermediate πN amplitude used in the work of Yang[11] is based on chiral symmetry and is therefore appropriate for off-shell extrapolation. It has the problem, however, of not reproducing well the on-shell πN scattering data, since he treated the ρ exchange as a contact interaction, which is equivalent to disregarding the nucleon magnetic momenta. He also did not consider the σ term and hence his results can be formally obtained from ours by making $\alpha_\sigma=\beta_\sigma=\mu_{\text{p}}=\mu_{\text{n}}=0$ into Eqs. (74)—(76).

The potential derived by the Tucson group[16] and by Coon and Glöckle[17] is based on an off-shell πN amplitude which was constructed by means of chiral symmetry and reproduces well on-shell data; this is the reason why our expressions are essentially the same as theirs. So, the main difference between both works is not in the expressions, but rather, in the method employed to obtain them. The Tucson group has used a πN amplitude derived by means of current algebra whereas we have used effective Lagrangians. As we pointed out before, both methods are equally appropriate for implementing chiral symmetry, the advantage of the latter being that it is much simpler and makes explicit the dynamical implications of the model.

It is worth pointing out that the term of the potential which appears in Refs. 9 and 17 proportional to C_s has one more term than ours, since their authors used a different parametrization for the σ term. This term arises from the πN form factor, and it is associated with the relativistic inconsistencies mentioned above. Our numerical results are slightly different from those of Refs. 9, 16, and 17 due to the "experimental" input. The accuracy of this input is about 10% if one assumes it to be of the same order of magnitude as that of the σ term determined in Ref. 30.

Knowledge of dynamics is crucial in the few-body problem. For instance, exchange currents, which play important roles in electromagnetic form factors, and scattering amplitudes are due to processes in which the external probe is able to "see" the vertices and propagators of the interactions corresponding to the potentials. Therefore, the description of virtual processes by means of effective Lagrangians is a very convenient one.

For the purpose of this work we consider the 3BF given by Eq. (73) and compare our result with previous works.

III. THE HYPERSPHERICAL HARMONIC APPROACH

In this work we use an HH expansion of the wave function for solving the nonrelativistic Schrödinger equation for three nucleons of mass m,

$$\left[-\frac{\hbar^2}{m}(\nabla^2_{\vec{x}_i}+\nabla^2_{\vec{y}_i})+V_{123}(\vec{x}_i,\vec{y}_i)\right]\Psi(\vec{x}_i,\vec{y}_i) = E\Psi(\vec{x}_i,\vec{y}_i) , \quad (82)$$

written in terms of the Jacobi coordinates (not unique)

$$\vec{x}_i=\vec{r}_j-\vec{r}_k ,\qquad \vec{y}_i=\frac{2}{\sqrt{3}}\left[\vec{r}_i-\frac{\vec{r}_j+\vec{r}_k}{2}\right] , \quad (83)$$

where $\vec{r}_i$ are the particle coordinates while V_{123} is the interaction between the three nucleons. This interaction consists of a sum of a two-body force (2BF) V (which is the sum of three pairwise interactions), and a three-body force (3BF) W (constituted by three terms). Equation (83) defines three equivalent sets of coordinates ($i=1,2,3$) for the description of the three-body problem [cyclic permutations in the indices $(i,j,k)=(1,2,3)$]. Equation (83) can be solved using, for example, the hyperspherical harmonic approach[13] in which the wave function Ψ is expanded in a complete orthonormal set of hyperspherical functions in the following way:

$$\Psi(\vec{x}_i,\vec{y}_i)=\sum_{K\alpha_i} r^{-5/2}\Phi_{K\alpha_i}(r)Y_{2K,\alpha_i}(\hat{x}_i,\hat{y}_i,\phi_i) , \quad (84)$$

where

$$Y_{2K,\alpha_i}(\Omega_i)\equiv Y^{LM}_{2K,l_{x_i}l_{y_i}}(\Omega_i) ,$$

$$[\alpha_i \rightarrow l_{x_i},l_{y_i},L,M] ,$$

and

$$r^2=x_1^2+y_1^2=x_2^2+y_2^2=x_3^2+y_3^2 \ ,$$

$$x_i=r\cos\phi_i \ , \ y_i=r\sin\phi_i \quad \left[0\le\phi_i\le\frac{\pi}{2}\right] \ .$$

The notation $\hat{x}_i$ and $\hat{y}_i$ means

$$\hat{x}_i\equiv(\theta_{x_i},\phi_{x_i}),\ \hat{y}_i\equiv(\theta_{y_i},\phi_{y_i}) \ ,$$

respectively. The complete orthonormal set

$$\{Y^{LM}_{2K,l_{x_i}l_{y_i}}(\Omega_i)\}$$

is the angular part of homogeneous harmonic polynomials of degree $2K$ ($K=0,1,2,\ldots,\infty$) in the six-dimensional space. Substitution of Eq. (84) into Eq. (83) leads to a system of coupled differential equations,[13]

$$\left[-\frac{d^2}{dr^2}+\frac{\mathscr{L}_K(\mathscr{L}_K+1)}{r^2}+\kappa^2\right]\Phi_{K\alpha_i}(r)+\sum_{K'\alpha_i'}\langle K\alpha_i\,|\,v_{123}\,|\,K'\alpha_i'\rangle\Phi_{K'\alpha_i'}(r)=0 \ , \tag{85}$$

where $\mathscr{L}_K=K+\frac{3}{2}$, $\kappa^2=-m/\hbar^2E$ ($E<0$ for bound states), $v_{123}=m/\hbar^2V_{123}$, and $\langle v_{123}\rangle$ is integrated over the five angles, resulting in a function of r. The elements of the angular basis with total angular momentum (L,M), related to the angular coordinates $(\hat{x}_i,\hat{y}_i,\phi_i)$, are

$$Y^{LM}_{2K,l_{x_i}l_{y_i}}(\Omega_i)=\sum_{m_{x_i}m_{y_i}}\langle l_{x_i}l_{y_i}m_{x_i}m_{y_i}\,|\,LM\rangle Y_{[2K]}(\Omega_i) \ , \tag{86}$$

where

$$Y_{(2K)}(\Omega_i)=Y_{l_{x_i}m_{x_i}}(\hat{x}_i)Y_{l_{y_i}m_{y_i}}(\hat{y}_i)\,{}^{(2)}P_{2K}^{l_{y_i}l_{x_i}}(\phi_i) \tag{87}$$

and

$${}^{(2)}P_{2K}^{l_{y_i}l_{x_i}}(\phi_i)=\left[\frac{4(K+1)n!(n+l_{x_i}+l_{y_i}+1)!}{\Gamma(n+l_{x_i}+\frac{3}{2})\Gamma(n+l_{y_i}+\frac{3}{2})}\right]^{1/2}(\cos\phi_i)^{l_{y_i}}(\sin\phi_i)^{l_{x_i}}P_n^{(l_{x_i}+1/2,\,l_{y_i}+1/2)}(\cos2\phi_i) \ , \tag{88}$$

where n is the integer

$$\tfrac{1}{2}(2K-l_{x_i}-l_{y_i})$$

and $P_n^{(a,b)}(x)$ is the usual Jacobi polynomial. Since the hyperspherical basis $Y^{LM}_{2K,l_{x_i}l_{y_i}}$ forms a complete orthonormal set for any i, we can choose this index arbitrarily. For the sake of simplicity, we will drop from now on the subscripts in the quantum numbers and variables. When v_{123} contains spin, isospin, etc., operators, the associated quantum numbers are included into the α label and in the sum in the matrix elements of Eq. (85).

A. Matrix elements of the interactions

In this work we restrict ourselves to the space symmetrical S state (for which

$$L=M=0 \Longrightarrow l_x=l_y=l;\ \{\alpha\}\rightarrow\{0,0,l,l\}) \ .$$

Let $|A(S,T)\rangle$ be the fully antisymmetric spin-isospin state for total spin (isospin) $S=\frac{1}{2}$ ($T=\frac{1}{2}$). The wave function of the completely space symmetry S is given by

$$\Psi(\vec{x},\vec{y})=|A(S,T)\rangle\sum_{K=0}^{\infty}[r^{-5/2}\Phi_K(r)]P^{(S)}_{2K}(\Omega) \ , \tag{89}$$

where the normalized symmetric HH's are

$$\mathscr{P}^{(S)}_{2K}(\Omega)=\sum_{\substack{l=0\\ \text{even}}}a^0_{2K,l}Y^{00}_{2K,l,l}(\Omega) \ , \tag{90}$$

where

$$a^0_{2K,l}=C_{2K}\sqrt{2l+1}S_0({}^{(2)}P^{ll}_{2K}(\phi)) \ ,$$

the operator S_0 defined by

$$S_0(f(\phi))\equiv\tfrac{1}{3}[f(0)+f(2\pi/3)+f(-2\pi/3)] \ , \tag{91}$$

and the normalization coefficient

$$(C_{2K})^{-2}=\sum_{\substack{l=0\\ \text{even}}}^{K}(2l+1)[S_0({}^{(2)}P^{ll}_{2K}(\phi))]^2 \ . \tag{92}$$

To reduce the number of coupled differential equations to a smaller number of significant coupled equations we restrict the angular basis to the "optimal subset."[13] The optimal subset is defined as the subset of angular basis that is directly connected through the potential to $\mathscr{P}^{(S)}_0(\Omega)$ which has the predominant contribution to the ground state, i.e.,

$$\langle\mathscr{P}^{(S)}_{2K}\,|\,v^{123}\,|\,\mathscr{P}^{(S)}_0\rangle\neq0 \ . \tag{93}$$

In practice we are obliged to consider two separate expansions, one for the 2BF and another for the 3BF. The expansions should be in terms of the same basis. They are given by[13]

$$\langle A \mid \begin{Bmatrix} \sum_{i<j} v_{ij}(r_{ij}) \\ \sum_{k=1}^{3} w(k) \end{Bmatrix} \mid A\rangle = 3\pi^{3/2} \sum_{K=0}^{\infty} (-)^K \langle 0 \mid K \mid K\rangle \mathscr{P}_{2K}^{(S)}(\Omega) \begin{Bmatrix} v_{2K}(r) \\ w_{2K}(r) \end{Bmatrix} . \tag{94}$$

The matrix element of 2BF between two HH's is given by

$$\langle K\alpha \mid v \mid K'\alpha'\rangle = \left\langle K\alpha \left| \sum_{i<j} v_{ij}(r_{ij}) \right| K'\alpha' \right\rangle = 3\langle K\alpha \mid v(x_1) \mid K'\alpha'\rangle \delta_{\alpha'\alpha} = 3 \sum_{K''=0}^{\infty} (-)^{K''} \langle K \mid K'' \mid K'\rangle v_{2K''}(r) , \tag{95}$$

where

$$V_{2K''}^{(\lambda)}(r) = \frac{2K!\Gamma\left[3+\frac{\lambda}{2}\right]}{\Gamma\left[\frac{\lambda+3}{2}\right]\Gamma(K+\frac{3}{2})} \int_0^1 V(r'r) P_K^{\lambda+(1/2),(1/2)}(1-2r'^2)(1-r'^2)^{1/2} r'^{\lambda+2} dr' \tag{96}$$

are the so-called potential multipoles.[13] Here λ refers to the rank of the tensor character of the force (for central forces, $\lambda=0$) and

$$v_{2K''}^{(\lambda)}(r) = \left[\frac{m}{\hbar^2}\right] V_{2K''}^{(\lambda)}(r) .$$

The factors $\langle K \mid K'' \mid K'\rangle$ are geometrical coefficients independent of the shape of the interaction[13] and are given by

$$\langle K \mid K'' \mid K'\rangle = \frac{\pi}{16} C_{2K} C_{2K'} {}^{(2)}P_{2K''}^{00}(0) \sum_{l=0}^{\min(K,K')} (2l+1) S_0({}^{(2)}P_{2K}^{ll}(\phi)) S_0({}^{(2)}P_{2K'}^{ll}(\phi)) \langle {}^{(2)}P_{2K}^{ll} \mid {}^{(2)}P_{2K''}^{00} \mid {}^{(2)}P_{2K'}^{ll}\rangle . \tag{97}$$

The last $3P$ matrix element in Eq. (97) is defined as[13]

$$\langle {}^{(2)}P_{2K}^{ll} \mid {}^{(2)}P_{2K''}^{l_2''l_1''} \mid P_{2K'}^{l_2'l_1'}\rangle = \int_0^{\pi/2} d\phi \sin^2\phi \cos^2\phi \, {}^{(2)}P_{2K}^{ll}(\phi) {}^{(2)}P_{2K''}^{l_2''l_1''}(\phi) {}^{(2)}P_{2K'}^{l_2'l_1'}(\phi) . \tag{98}$$

The effective potential $v(x_1)$ is half the sum of the singlet and triplet even central potentials, for the space symmetric S state.

A similar procedure is used to obtain the matrix elements of 3BF. It is given by

$$\langle K\alpha \mid w \mid K'\alpha'\rangle = 3 \sum_{K''=0}^{\infty} (-)^{K''} \langle K \mid K'' \mid K'\rangle w_{2K''}(r) , \tag{99}$$

where the potential multipoles are calculated for each term of w. In order to do this we should notice that in Eq. (73) the tensor characters of the first, second, and third terms correspond to $\lambda=0$, 1, and 2, respectively. The corresponding multipoles are

$$\lambda=0 \rightarrow W_{2K''}^{0}(r) = C_0 \frac{\pi^{1/2}}{256} \frac{1}{K''+1} \sum_{\substack{l=0 \\ \text{even}}}^{K''} (2l+1) A_{2K''} \sum_{K_1K_2} (-)^{K_1+K_2} F_{K_1K_2}^{K''}(l,l) U_{2K_1}^{(0)}(r) U_{2K_2}^{(0)}(r) , \tag{100}$$

where $U_{2K}^{(0)}(r)$ is given by Eq. (96) with $V(r'r) \rightarrow U_0(r'r)$;

$$\lambda=1 \rightarrow W_{2K''}^{1}(r) = C_1(\tfrac{4}{15})^2 \pi^{-1/2} \frac{1}{K''+1} \sum_{\substack{l=0 \\ \text{even}}}^{K''} (2l+1) A_{2K''} \sum_{K_1K_2} (-)^{K_1+K_2} U_{2K_1}^{(1)}(r) U_{2K_2}^{(1)}(r) \times \sum_{l'} (2l'+1)(-)^{l'+1} \begin{bmatrix} 1 & l' & l \\ 0 & 0 & 0 \end{bmatrix}^2 F_{K_1+1,K_2+2}^{K''}(l',l) , \tag{101}$$

where $U_{2K}^{(1)}(r)$ is given by Eq. (96) with $V(r'r) \rightarrow U_1(r'r)$;

$$\lambda=2 \rightarrow W_{2K''}^{2}(r) = C_2 \frac{\pi^{3/2}}{128} \frac{1}{K''+1} \sum_{\substack{l=0 \\ \text{even}}}^{K''} \sqrt{2l+1} A_{2K''} \times \sum_{K_1K_2} (-)^{K_1+K_2} U_{2K_1}^{(2)}(r) U_{2K_2}^{(2)}(r) \sum_{l'} (2l'+1) \begin{bmatrix} 2 & l & l' \\ 0 & 0 & 0 \end{bmatrix}^2 F_{K_1+2,K_2+2}^{K''}(l',l) \tag{102}$$

where $U_{2K}^{(2)}(r)$ is given by Eq. (96) with $V(r'r) \to U_2(r'r)$. The expressions for $A_{2K''}$ and F are given by

$$A_{2K''} = S_0({}^{(2)}P_{2K''}^{ll}(\phi))/S_0({}^{(2)}P_{2K''}^{00}(\phi)) \ , \tag{103}$$

and

$$F_{K_1+\lambda,K_2+\lambda}^{K''}(l',l) = \langle {}^{(2)}P_{2K_1+\lambda}^{\lambda 0} \mid {}^{(2)}P_{2K''}^{ll} \mid {}^{(2)}P_{2K_2+\lambda}^{l'l} \rangle {}^{(2)}P_{2K_1+\lambda}^{\lambda 0}(0) \, {}^{(2)}P_{2K_2+\lambda}^{l'l}(-2\pi/3) \ . \tag{104}$$

In an appendix we indicate how to obtain the above 3BF multipoles.

From Eq. (85) we can write the system of coupled differential equations where 2BF and 3BF are taken into consideration:

$$\left[-\frac{d^2}{dr^2} + \frac{\mathscr{L}_K(\mathscr{L}_K+1)}{r^2} + \kappa^2 \right] \Phi_K(r) + 3 \sum_{K',K''=0}^{\infty} (-)^{K''} \langle K \mid K'' \mid K' \rangle [v_{2K''}(r) + w_{2K''}(r)] \Phi_{K'}(r) = 0 \ , \tag{105}$$

where

$$w_{2K''}(r) = [W_{2K''}^0(r) + W_{2K''}^1(r) + W_{2K''}^2(r)] \frac{m}{\hbar^2} \ .$$

For practical purpose, the set of equations (105) is truncated to a finite number of partial waves (the upper values of the K quantum numbers are estimated after convergence of the solution is reached.[13]) This set can be solved exactly numerically. However, in order to reduce time and memory requirements we then solved it using the uncoupled adiabatic approximation[34] (UAA).

IV. RESULTS AND CONCLUSIONS

The Afnan-Tang $S3$ potential[14] has been chosen to represent the 2BF. The $\pi\pi E$-3BF is given by Eq. (73), whose coefficients have been obtained by means of effective Lagrangians that are approximately invariant under chiral transformations. We have argued that this approach to the problem is a very convenient one, since it allows a clear understanding of the dynamical origin of the various contributions to the strength parameters C_0, C_1, and C_2, defined by Eqs. (74)–(76).

Those equations show that these parameters receive contributions from the Δ pole and ρ and σ exchanges, there being no reason for the isolated consideration of some of them. Other forms of 3BF, such as those derived by the Tucson (T) group,[16,17] Yang[11] (Y), Fujita and Miyazawa[18] (FM), and Loiseau, Nogami, and Ross[33] (LNR) can be formally obtained from Eq. (73), by choosing convenient values for the dynamical parameters. [Notice that Eq. (73) is an S-wave reduction of Eq. 61]. In the calculations we have taken 12 multipoles of 2BF and 5 multipoles of 3BF, which give reasonably convergent results.[6] In Tables I and II we present the main results of the calculation. In Table I we illustrate the role of the cutoff parameter x_0 defined by Eq. (66), in the case of the FM force. Results of this calculation[6] show that both BE and $P_{\mathrm{ch}}(q^2)$ depend strongly on this cutoff parameter. As shown in Ref. 6, discontinuities in BE and in the first maximum of $|F_{\mathrm{ch}}(q^2)|$ (called $F_{\max}$) for ^{3}He appear at small values of x_0. Those discontinuities are caused by nodes near the origin in the radial wave function.[6] Since those nodes are not expected to appear in the ground state wave function, this somehow sets a criterion for the value of x_0, which should be around the hard core radius of the Reid hard core potential. Thus we cannot take x_0 as a free phenomenological parameter to fit experimental data.[6,31,32] A similar statement can be made for the LNR potential.[33] For all the above 3BF's, results are quite sensitive to the variation of the parameters.[6] Since the accuracy of the "experimental" input is of the order of 10%, Table II shows results of various combinations of the strength parameters. This table also shows the relative contribution of the different terms in Eq. (73). We omit the calculated rms radius since it is well within the experimental error bar. In Fig. 4 we plot $|P_{\mathrm{ch}}(q^2)|$ for ^{3}He, using different forms of 3BF.

Some important features should be mentioned about the calculations. The forces derived by the Tucson group and by ourselves are more realistic than the FM force. Nevertheless, the latter seems to be more "effective" in filling the gap between the experimental values of the binding energy and the 2BF contribution. A possible

TABLE I. FM force ($C_0 = C_1 = 0$) results of the calculation for the bound states of the trinucleon system. ($\mu = 0.7$ fm^{-1} and no nodes near origin.) In this table we have used the regularization scheme at short distances defined by Eq. (66).

Calculation	C_2 (MeV)	x_0 (fm)	BE (MeV)	Value at $q^2=1$ fm^{-2}	$\lvert F_{\mathrm{ch}}(q^2)\rvert$ Position of first minimum (fm^{-2})	$F_{\max} \times 10^{+3}$	rms charge radius (fm)
^{3}H (2BF)			6.489	0.590	15.98	1.50	1.817
^{3}H (2BF + 3BF)	0.9	0.340	7.658	0.617	16.47	1.94	1.737
^{3}He (2BF)			5.789	0.565	15.91	1.06	1.889
^{3}He (2BF + 3BF)	0.9	0.340	6.922	0.592	16.39	1.39	1.810
	0.46	0.277	6.485	0.581	15.54	1.58	1.841
^{3}H (expt)			8.482	0.622			1.70±0.05
^{3}He (expt)			7.718	0.576	11.8	~5	1.84±0.03

TABLE II. Increases of binding energy [ΔBE=BE(2BF + 3BF)−BE(2BF)], position of first minimum (q_0^2), and F_{max} generated by the introduction of 3BF, for the parameters indicated in the first columns. (See note in Ref. 43.) In this table we have used the regularization scheme at short distances defined by Eq. (54).

Nucleus	Description	No. of order	C_0 (MeV)	C_1 (MeV)	C_2 (MeV)	Λ^2 (fm^{-2})	ΔBE (MeV)	q_0^2 (fm^{-2})	$F_{max}\times 10^3$
^{3}H	2BF + 3BF	1	−0.23	−0.92	1.11	25	0.010	15.80	1.59
		2		−0.92		25	−0.203	15.87	1.47
		3		−0.92	1.11	25	0.064	15.83	1.61
		4	−0.23	−0.92	1.11	17	−0.087	15.73	1.55
		5		−0.92	1.11	17	−0.068	15.71	1.57
		6	−0.12	−0.76	1.20	25	0.067	15.80	1.61
^{3}He	2BF + 3BF	7	−0.23	−0.92	1.11	25	0.010	15.70	1.55
		8		−0.92		25	−0.197	15.80	1.42
		9	−0.23	−0.92	1.11	17	−0.084	15.69	1.51
		10	−0.12	−0.76	1.20	17	−0.049	15.71	1.52

reason for such a behavior is that different regularization schemes have been used in both cases. This result shows that the short distance behavior of the 3BF is important and that it should be treated very carefully in realistic calculations. Our results allow us to conclude that cutoffs in the 3BF, in both configuration and momentum spaces, eliminate the nodes in the radial wave function. In the latter case this happens because the πNN form factor is taken not to be one, as in the case of the FM force. We

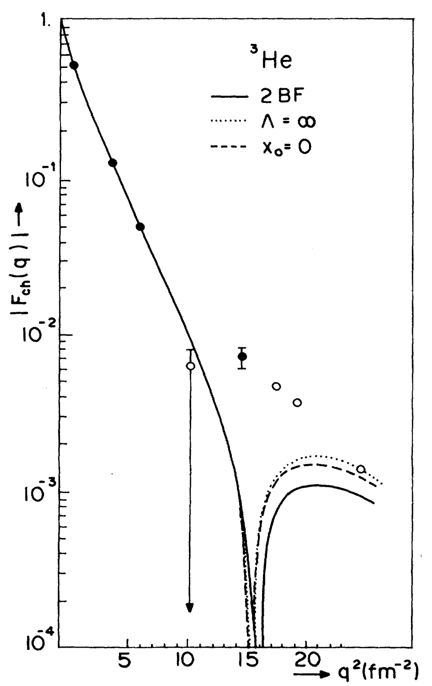

FIG. 4. Calculated charge form factor for ^{3}He using Eq. (66), T, and FM forces. For FM force C_2=0.9 MeV, x_0=0.277 fm ($\Lambda=\infty$). For our work and the T (x_0=0) force the curves are quite close and for Λ^2=17. and 25. fm^{-2} they are also too close to be drawn separately. Only the contribution of the p-wave part of those forces were considered in the plot. For all forces μ=0.7 fm^{-1}. The experimental data were taken from Refs. 41 and 37 (closed circles) and from Ref. 38 (open circles).

use values of Λ^2 ranging from 17. to 25., as suggested in Ref. 9. The overall effect of reducing Λ^2 from 25. to 17. is to reduce the effects of the 3BF. The Λ dependence decreases rapidly as the interparticle distances increase. On the other hand, F_{max} seems to be of the same order of magnitude for all 3BF's.

In this work we have derived a $\pi\pi E$-3BF. We have also calculated and compared the contributions of different forms of 3BF to the trinucleon ground state. However, the very short range behavior of 3BF's is still uncertain due possibly to other neglected diagrams. Only the totally symmetric S part of the ^{3}H wave function (which is responsible for almost 90% of the trinucleon ground state) is kept in the solution of the Schrödinger equation using the hyperspherical harmonic method. We are aware[4] that the realistic 2BF plus 3BF strongly couple the S and D parts of the trinucleon wave function. It seems that the neglect of this coupling affects directly the calculation of the binding energy contribution. Computer limitations led us to consider only S waves in this work. However, we believe that our results constitute a suitable departure point for the inclusion of contributions of other effects. Enhancements of binding energy (BE) and $F_{ch}(q^2)$ are in the right direction compared with experimental data, although that of $F_{ch}(q^2)$ is small. This observation agrees basically with previous approximate or model calculations[3–9,36] However, calculations in this direction are still in an initial stage, and it is too early to draw definite conclusions.[39–42] In order to improve our results one should consider realistic 2BF's—as well as including the contributions of the D component to the total wave function—other dynamical effects such as π-ρ and ρ-ρ exchanges,[44] relativistic corrections, and MEC's to $F_{ch}(q^2)$. Calculations along these lines are at present being considered by us.

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APPENDIX A: MATRIX ELEMENTS FOR THE THREE-BODY FORCE

To calculate the matrix element

$$\langle \mathscr{P}^{(S)}_{2K}(\Omega) \mid w \mid \mathscr{P}^{(S)}_{2K'}(\Omega)\rangle$$

of the 3BF between two fully space symmetric HH's, we must expand the 3BF in terms of the same basis. We notice that there is a single HH in this symmetry for each of the lowest values $2K$ for $K=0,2,3,4,5$ (for $K=1$, there are only mixed symmetry states).[13] Therefore, the HH decomposition is unique for $K<6$. In agreement with Eq. (89), the HH expansion of a 2BF is given by[13]

$$\langle A \mid v_{ij}(r_{ij}) \mid A\rangle = \pi^{3/2}\sum_{\lambda\mu}\sum_{K=0}^{\infty}(-)^{K}\left\{A_{\lambda\mu}(i,j)\frac{\Gamma[(\lambda+3)/2]}{\Gamma[(\lambda/2)+3]}\sum_{[2K+\lambda]}Y^{\lambda\mu}_{[2K+\lambda]}(\phi)Y_{[2K+\lambda]}(\Omega)\right\}v^{(\lambda)}_{2K}(r)\ , \tag{A1}$$

where the multipoles $v^{(\lambda)}_{2K}(r)$ are given by Eq. (96), $[2K+\lambda]\equiv 2K+\lambda, l_1, m_1, l_2, m_2$ [see Eq. (87)], and $A_{\lambda\mu}(i,j)$ is a coefficient which may include spin, isospin, or other nonspatial degrees of freedom. For the $\lambda=0$, 1, and 2 terms of the 3BF, given by Eq. (73), we may expand them in HH by using Eq. (A1).

For instance, for the $\lambda=0$ terms, we write

$$U_0(x_i) = \pi^{3/2}\frac{\Gamma(3/2)}{\Gamma(3)}\sum_{K=0}^{\infty}(-)^{K}\sum_{[2K]}Y^{00}_{[2K]}(\phi_{jk})Y_{[2k]}(\Omega)U^{(0)}_{2K}(r)\ , \tag{A2}$$

where

$$Y^{00}_{[2K]}(\phi) = {}^{(2)}P^{l_2 l_1}_{2K}(\phi_{ij})\int d\hat{q}\, Y_{00}(\hat{q})Y^{*}_{l_1 m_1}(\hat{q})Y^{*}_{l_2 m_2}(\hat{q})\ . \tag{A3}$$

A similar expression holds for $U(x_j)$. After taking the product of both of them and some algebraic manipulation, the result projected on the basis $Y^{00}_{2K'',ll}(\Omega)$ and summed over a cyclic permutation on the indices (i,j,k), we obtain

$$\begin{aligned}\sum_{\text{cyclic}}\langle Y^{00}_{2K'',ll} \mid U_0(x_i)U_0(x_j)\rangle &= \frac{\pi^2}{256}\sqrt{2l+1}\sum_{K_1K_2}(-)^{K_1+K_2}U^{(0)}_{2K_1}(r)U^{(0)}_{2K_2}(r)\\ &\times\sum_{l_1l_2l_1'l_2'}(2l_1+1)(2l_2+1)\\ &\times(2l_1'+1)(2l_2'+1)\begin{bmatrix} l_1 & l_2 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} l_1' & l_2' & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} l_1 & l_1' & l\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix} l_2 & l_2' & l\\ 0 & 0 & 0\end{bmatrix}\\ &\times\begin{Bmatrix} l_2 & l_1 & 0\\ l_1' & l_2' & l\end{Bmatrix}\langle {}^{(2)}P^{l_2l_1}_{2K_1} \mid {}^{(2)}P^{ll}_{2K''} \mid {}^{(2)}P^{l_2'l_1'}_{2K_2}\rangle\sum_{\text{cyclic}}{}^{(2)}P^{l_2l_1}_{2K_1}(\phi_{ik}){}^{(2)}P^{l_2'l_1'}_{2K_2}(\phi_{kj})\ ,\end{aligned} \tag{A4}$$

where $\phi_{ik}=0$, $-2\pi/3$, $2\pi/3$ correspond to the interparticle distances r_{12}, r_{23}, r_{31}, respectively, and $U^{(0)}_{2K}(r)$ are the $\lambda=0$ tensor multipoles in the six dimensional space, given by Eq. (96). In choosing $i=1$, $j=3$, and $k=2$, which correspond to $\phi_{ik}=0$ and $\phi_{kj}=-2\pi/3$, the above equation is simplified because $l_1=0$ (due to the nature of ${}^{(2)}P^{l_2l_1}_{2K_1}$); we then obtain

$$\begin{aligned}C_0\sum_{\text{cyclic}}\langle AY^{00}_{2K'',l,l} \mid U_0(x_1)U_0(x_j)A\rangle &= \frac{3\pi^2}{256}C_0\sqrt{2l+1}\sum_{K_1K_2}(-)^{K_1+K_2}U^{(0)}_{2K_1}(r)U^{(0)}_{2K_2}(r)\\ &\times\langle {}^{(2)}P^{00}_{2K_1} \mid {}^{(2)}P^{ll}_{2K''} \mid {}^{(2)}P^{ll}_{2K_2}\rangle {}^{(2)}P^{00}_{2K_1}(0){}^{(2)}P_{2K_2}(-2\pi/3)\ .\end{aligned} \tag{A5}$$

For the $\lambda=1$ term, we notice that

$$\cos\theta_k = \frac{4\pi}{3}\sum_{\mu=-1}^{1}(-)^{\mu}Y_{1\mu}(\hat{r}_{ik})Y_{1,-\mu}(\hat{r}_{kj})\ ,$$

and from Eq. (73) we get

$$W^1(k) = C_1\frac{4\pi}{3}\sum_{\mu}(-)^{\mu}[Y_{1\mu}(\hat{r}_{ik})U_1(x_i)][Y_{1,-\mu}(\hat{r}_{kj})U_1(x_j)]\ . \tag{A6}$$

By expanding the terms between square brackets in HH, with the use of Eq. (A1), and following the same procedure as for the $\lambda=0$ case, we obtain

$$\sum_{\text{cyclic}} \langle AY^{00}_{2K'',l,l} \mid C_1 \cos\theta_k U_1(x_i)U(x_j)A\rangle = 3(\tfrac{4}{15})^2\pi C_1\sqrt{2l+1}(-)^{l+1}\sum_{K_1K_2}(-)^{K_1+K_2}U^{(1)}_{2K_1}(r)U^{(1)}_{2K_2}(r) \times \sum_{l'}(2l'+1)(-)^{l'}\begin{bmatrix}1 & l' & l\\ 0 & 0 & 0\end{bmatrix}F^{K''}_{K_1+1,K_2+1}(l',l) \quad . \tag{A7}$$

For the $\lambda=2$ term, we notice that

$$3\cos^2\theta_k - 1 = \frac{8\pi}{5}\sum_{\mu=-2}^{2}(-)^{\mu}Y_{2\mu}(\hat{r}_{ik})Y_{2,-\mu}(\hat{r}_{kj}) \quad ,$$

and from Eq. (73) we get

$$W^2(k) = C_2\frac{8\pi}{5}\sum_{\mu}(-)^{\mu}[Y_{2\mu}(\hat{r}_{ik})U_2(x_i)][Y_{2,-\mu}(\hat{r}_{kj})U_2(x_j)] \quad . \tag{A8}$$

By expanding the terms between square brackets in HH, with the use of Eq. (A1), we finally obtain

$$\sum_{\text{cyclic}} \langle A^{00}_{2K'',l,l} \mid C_2(3\cos^2\theta_k - 1)U_2(x_i)U_2(x_j)A\rangle = 3.2\pi\left[\frac{\pi}{16}\right]^2 C_2\sum_{K_1K_2}(-)^{K_1+K_2}U^{(2)}_{2K_1}(r)U^{(2)}_{2K_2}(r)\sum_{l'=l-2}^{l+2}(2l'+1)\begin{bmatrix}2 & l & l'\\ 0 & 0 & 0\end{bmatrix}^2 F^{K''}_{K_1+2,K_2+2}(l',l) \quad . \tag{A9}$$

The overall factor 3 appearing in Eqs. (A5), (A7), and (A9) has been factorized out in Eq. (99). In order to obtain the potential multipoles corresponding to Eqs. (A5), (A7), and (A9), we use the following expression[13]:

$$W_{2K}(r) = \pi^{-3/2}\frac{1}{K+1}\sum_{\substack{l=0\\ \text{even}}}^{K}\sqrt{2l+1}A_{2K}\langle Y^{00}_{2K,ll}(\Omega) \mid \sum_{\text{cyclic}}\langle A \mid W \mid A\rangle\rangle \quad , \tag{A10}$$

where $W = W^0 + W^1 + W^2$.

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