Need of repulsion in form factors for separable two-nucleon forces in conjunction with the two-pion-exchange three-nucleon force

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The bound state Faddeev equations in the well-known simplified form for separable two-nucleon forces are extended to include a three-nucleon force. A numerical study for the two-pion-exchange three-nucleon force in conjunction with commonly used spin dependent s-wave NN forces is presented. The violent short-range behavior of the two-pion-exchange three-nucleon force requires the choice of form factors for the two-nucleon forces which have a repulsion built in.

[NUCLEAR STRUCTURE Triton calculation, interplay of two- and three-nucleon force.]

The solution of the three-body Faddeev equations is greatly simplified by using separable potentials. Also, useful physical insight has been gained from studies based on that type of interaction. Therefore it may also be useful to study possible effects of the two-pion-exchange three-nucleon force (TPTNF) within that framework. This paper describes the rather straightforward extension of the Faddeev equations incorporating a three-nucleon force and presents as a first example for this type of study triton binding energy calculations based on commonly used s-wave NN interactions. Some time ago Phillips[1] undertook a related study, restricted, however, to a separable three-body force, which was chosen in an *ad hoc* manner.

Let us consider the triton with a three-nucleon force included. The Faddeev decomposition of the bound state

$$\Psi=(1+P)\psi+\psi_4 , \tag{1}$$

where P denotes the two cyclical permutations

$$P=P_{12}P_{23}+P_{13}P_{23} , \tag{2}$$

leads to the Faddeev equations

$$\begin{aligned}\psi&=G_0t_1P\psi+G_0t_1P\psi_4 ,\\ \psi_4&=G_0t_4(1+P)\psi .\end{aligned} \tag{3}$$

Eliminating the fourth component ψ_4 one gets

$$\psi=G_0t_1P\psi+G_0t_1G_0t_4(1+P)\psi . \tag{4}$$

Here t_1 and t_4 are the two- and three-body t matrices linked to the two- and three-body forces $V_1\equiv V_{23}$ and V_4, respectively. We restrict this study to pure s-wave forces V_1 and rank 1 separable t_1 matrices. The standard reduction turns (4) into a coupled set of one-dimensional integral equations

$$F_\alpha(q)=\sum_{\alpha'}\int_0^\infty dq'q'^2(\langle g_\alpha q\alpha\,|\,PG_0\,|\,g_{\alpha'}q'\alpha'\rangle+\langle g_\alpha q\alpha\,|\,G_0t_4(1+P)G_0\,|\,g_{\alpha'}q'\alpha'\rangle)\tau_{\alpha'}(E-\tfrac{3}{4}q'^2)F_{\alpha'}(q') . \tag{5}$$

Here q is the Jacobi momentum of the "spectator" particle, $g_\alpha(p)$ the form factor defined by

$$\langle p\,|\,t_{1,\alpha}(z)\,|\,p'\rangle=g_\alpha(p)\tau_\alpha(z)g_\alpha(p') , \tag{6}$$

and α stands for the discrete quantum numbers of LS coupling:

$$|\,pq\alpha\rangle\equiv|\,pq(l\lambda)L(s\tfrac{1}{2})S(LS)\mathscr{J}(t\tfrac{1}{2})T\rangle . \tag{7}$$

Because of our restricted choice of V_1 there are only two channels α. The orbital angular momentum l in the 2-3 subsystem is zero, therefore the angular momentum λ linked to q is equal to the total orbital angular momentum L. For the 2-3 subsystem spin $s=0$ the total spin S is $S=\frac{1}{2}$, which requires $L=0$ to match with $\mathcal{J}=\frac{1}{2}$. This is the first channel. The second channel arises choosing $s=1$, $S=\frac{1}{2}$, and $L=0$. Clearly these two channels belong to $t=1$ and $t=0$, respectively, and of course to total isospin $T=\frac{1}{2}$. Finally, there is a third possibility $L=\lambda=2$, $s=1$, and $S=\frac{3}{2}$, which, however, is not connected to the first two channels in our case, as will be shown below.

The standard transition potential in (5) is given as

$$\langle g_\alpha q\alpha \,|\, PG_0 \,|\, g_{\alpha'}q'\alpha'\rangle = \mathcal{T}^{(1)}_{\alpha\alpha'} \int_{-1}^{1} dx \frac{g_\alpha(|\frac{1}{2}\vec{q}+\vec{q}\,'|)g_{\alpha'}(|\frac{1}{2}\vec{q}\,'+\vec{q}|)}{E-q^2-q'^2-qq'x} \tag{8}$$

with

$$\mathcal{T}^{(1)}_{\alpha\alpha'} = \begin{bmatrix} \frac{1}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{1}{4} \end{bmatrix} . \tag{9}$$

That potential is diagonal in L and therefore does not couple $L=0$ to $L=2$. Thus without t_4 one ends up with the familiar two coupled equations.

The two-pion-exchange three-nucleon force is believed to be on a relatively firm theoretical basis.[2] From the experience gained up to now with that force,[3] it is to be expected that it can be treated in perturbation theory. Therefore we restrict ourselves in this qualitative study to

$$\begin{aligned} t_4 \approx V_4 = v_4 + P_{12}P_{23}v_4P_{13}P_{23} \\ + P_{13}P_{23}v_4P_{12}P_{23} \; . \end{aligned} \tag{10}$$

The second equality describes the decomposition of V_4 into three cyclical terms as shown in Fig. 1. Therefore one can write

$$V_4(1+P) = (1+P)v_4(1+P) \; , \tag{11}$$

and the new term in (5) will be

$$\langle g_\alpha q\alpha \,|\, G_0(1+P)v_4(1+P)G_0 \,|\, g_{\alpha'}q'\alpha'\rangle \; . \tag{12}$$

In the spirit of pure s-wave interactions (no tensor force), and because of simplifications, we replace V_4 by the following force

$$\bar{V}_4 \equiv |\xi_a\rangle\langle\xi_a|V_4|\xi_a\rangle\langle\xi_a| \; , \tag{13}$$

where $|\xi_a\rangle$ is the totally antisymmetric state in spin-isospin space (to $S=T=\frac{1}{2}$). Thus $\bar{V}_4$ acts only in that part of the wave function which is totally symmetric in the space part. For the triton this is known to be about 90% of the wave function. That spin-isospin averaged three nucleon force was studied recently.[4] It is easy to see that choosing $\bar{V}_4$ [Eq. (11)] simplifies to

$$\bar{V}_4(1+P) = |\xi_a\rangle(1+P_{\rm sp})v_4^{\rm sp}(1+P_{\rm sp})\langle\xi_a| \; , \tag{14}$$

where the operators with the index sp act only in normal space and

$$v_4^{\rm sp} \equiv \langle\xi_a|v_4|\xi_a\rangle \; . \tag{15}$$

The spin-isospin matrix element in expression (12) can therefore be finally evaluated, and one gets

$$\begin{aligned} &\langle g_\alpha q\alpha \,|\, G_0\bar{V}_4(1+P)G_0 \,|\, g_{\alpha'}q'\alpha'\rangle \\ &= \mathcal{T}^{(4)}_{\alpha\alpha'} \int_0^\infty dp\, p^2 g_\alpha(p) \int_0^\infty dp' p'^2 g_{\alpha'}(p') \\ &\qquad \times \langle pq(00)L=0 \,|\, G_0(1+P_{\rm sp})v_4^{\rm sp}(1+P_{\rm sp})G_0 \,|\, p'q'(00)L=0\rangle \; , \end{aligned} \tag{16}$$

with

$$\mathcal{T}^{(4)}_{\alpha\alpha'} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} . \tag{17}$$

Also the restriction to $\bar{V}_4$ brings up the scalar $v_4^{\rm sp}$ which excludes a coupling between $L=0$ and $L=2$ and we end up again with two coupled equations.

It is an easy exercise to evaluate the effects of the permutation operators. One finds

$$
\begin{aligned}
&\langle g_\alpha q\alpha \,|\, G_0 \bar{V}_4(1+P)G_0 \,|\, g_{\alpha'}q'\alpha'\rangle \\
&\quad=\mathscr{T}^{(4)}_{\alpha\alpha'}\Bigg[\int_0^\infty dp\,p^2\int_0^\infty dp'p'^2\frac{g_\alpha(p)}{E-p^2-\frac{3}{4}q^2}\langle pq(00)0\,|\,v_4^{\rm sp}\,|\,p'q'(00)0\rangle\frac{g_{\alpha'}(p')}{E-p'^2-\frac{3}{4}q'^2} \\
&\qquad+\int_{-1}^1 dx\int_0^\infty dq_1q_1{}^2\int_0^\infty dp\,p^2\frac{g_\alpha(p)}{E-p^2-\frac{3}{4}q^2} \\
&\qquad\times\langle pq(00)0\,|\,v_4^{\rm sp}\,|\,|\tfrac{1}{2}\vec{q}_1+\vec{q}\,'\,|\,q_1(00)0\rangle\frac{g_{\alpha'}(\,|\vec{q}_1+\frac{1}{2}\vec{q}\,'\,|\,)}{E-q_1{}^2-q'^2-q_1q'x} \\
&\qquad+\int_{-1}^1 dx\int_0^\infty dq_1q_1{}^2\int_0^\infty dp'p'^2\frac{g_\alpha(\,|\frac{1}{2}\vec{q}+\vec{q}_1|\,)}{E-q^2-q_1{}^2-qq_1x} \\
&\qquad\times\langle\,|\vec{q}+\tfrac{1}{2}\vec{q}_1\,|\,q_1(00)0\,|\,v_4^{\rm sp}\,|\,p'q'(00)0\rangle\frac{g_{\alpha'}(p')}{E-p'^2-\frac{3}{4}q'^2} \\
&\qquad+\int_{-1}^1 dx\int_0^\infty dq_1q_1{}^2\int_{-1}^1 dx'\int_0^\infty dq_1'q_1'^2\frac{g_\alpha(\,|\frac{1}{2}\vec{q}+\vec{q}_1|\,)}{E-q^2-q_1^2-qq_1x} \\
&\qquad\times\langle\,|\vec{q}+\tfrac{1}{2}\vec{q}_1\,|\,q_1(00)0\,|\,v_4^{\rm sp}\,|\,|\tfrac{1}{2}\vec{q}_1'+\vec{q}\,'\,|\,q_1'(00)0\rangle\frac{g_{\alpha'}(\,|\vec{q}_1'+\frac{1}{2}\vec{q}\,'\,|\,)}{E-q_1'^2-q'^2-q_1'q'x'}\Bigg]\,. \qquad (18)
\end{aligned}
$$

As a further simplification in expression (18) only the $v_4^{\rm sp}$ matrix elements to $l=\lambda=0$ have been kept different from zero. The calculation of these matrix elements can be reduced to a twofold angular integration, as has been described in the second listing in Ref. 2. The necessary interpolations were conveniently carried through by the spline method in the manner worked out in Ref. 5.

We used the two-pion-exchange three-nucleon force of the second listing in Ref. 2 with the strong πNN form factor parameter $\Lambda^2=25$ fm^{-2}.

The eigenvalue problem (5) has been solved by standard routines.

Let us first regard the Yamaguchi form factors which have often been and are still being used.[6] Since they are purely attractive, they will not keep two nucleons apart at short distances. On the other hand, the TPTNF, averaged over the spin-isospin state $|\xi_\alpha\rangle$, Eq. (15), is strongly attractive, especially at short distances, as has been exhibited in Ref. 4. So we have to expect that the unrealistic purely attractive nature of the Yamaguchi form factors together with the TPTNF will lead to an unrealistic increase in binding energy. This is indeed the case, as is shown in the first row of Table I. The obvious reason for that overbinding (with and without V_4) is the missing repulsion, which leads to positive g_α's and thus to too attractive transition potentials (8) and (18).

It is therefore obvious that one should consider form factors based on two-nucleon interactions which have a repulsion built in. To that aim we considered the unitary pole approximation (UPA) to

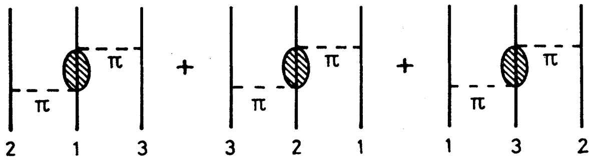

FIG. 1. The two-pion-exchange three-nucleon force.

TABLE I. Triton binding energies (in MeV) for various two-nucleon forces with and without V_4.

	Without V_4	With V_4
Y	-11.0	-24.1
UPA for MT II, IV	-10.4	-21.6
UPA for MT I, III	-8.50	-8.64

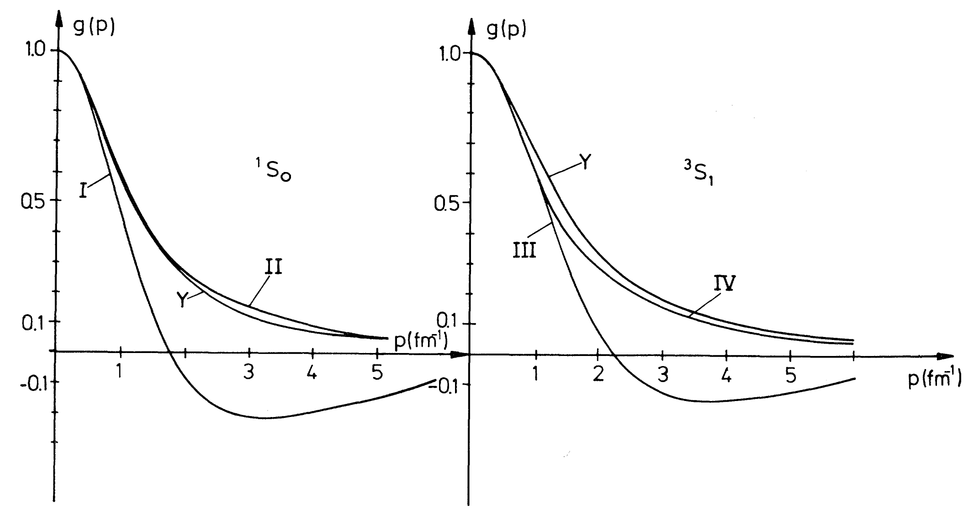

FIG. 2. The form factors for UPA approximations to the Malfliet-Tjon potentials I—IV in comparison to the Yamaguchi ones.

the local Malfliet-Tjon potentials I and III,[7] which fit the $^1S_0$ and $^3S_1$ phases fairly well. They have the form

$$V(r)=(V_R e^{-\mu_R r}-V_A e^{-\mu_A r})\frac{1}{r}\ , \tag{19}$$

where the parameters for the various models are summarized[8] in Table II. It is known from thorough studies[9] that UPA is quite good for interactions which are both repulsive and attractive. The corresponding form factors are shown in Fig. 2 in comparison to the Yamaguchi ones. The node is due to the repulsion. The resulting triton binding energies with and without V_4 are shown in the third row of Table I. The increase in binding energy due to that *restricted* $V_4=\overline{V}_4$ is now of the expected reasonable order of magnitude.

The unrealistic overbinding in the case of the Yamaguchi form factors does not depend on that specific separable force. One can use, for instance, the UPA approximation to the purely attractive Malfliet-Tjon potentials II and IV, which leads also

TABLE II. Potential parameters for the Malfliet-Tjon models.

	V_A (MeV fm)	μ_A (fm^{-1})	V_R (MeV fm)	μ_R (fm^{-1})
I	513.968	1.550	1438.720	3.110
II	52.490	0.809		
III	626.885	1.550	1438.720	3.110
IV	65.120	0.633		

to purely positive form factors as shown in Fig. 2. The results for the triton (with and without V_4) are given in the second row of Table I.

This numerical study is not meant to provide a good estimate for the additional binding energy resulting from a three-nucleon force, but only to demonstrate the relative ease with which the TPTNF can be included numerically into the well established framework of solving the Faddeev equations with separable two-nucleon forces. At the same time, however, this study shows the strong interplay of two- and three-nucleon forces. The TPTNF used is strongly attractive for certain short-distance configurations and therefore necessarily requires two-nucleon forces which have the short range repulsion built in as enforced by the two-nucleon system.

Though purely attractive two-nucleon forces, when used alone, may be useful for certain trend studies, their unrealistic feature of no repulsion is dramatically exhibited when combined with the TPTNF as it is presently understood. The TPTNF may be modified with further insight in the future, and certainly at shorter distances additional processes become important. Thus the *total* three-nucleon force may behave differently, at least at short distances (for instance, it may behave repulsively), and the interplay with certain types of two-nucleon forces may be less disharmonious.

An improvement for the evaluation of the permutation operator in Eq. (18) and the application to the $n+d\rightarrow n+n+p$ process is underway.

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