Statistical formalism for particle emission

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A simple transparent formalism is presented for following the ensemble averaged evolution of an excited system as it statistically emits particles. The formalism provides spectra and multiplicities for the emitted particles. It permits consideration of large numbers of emitted particle species, including those of medium mass, and a large number of excited states. The time developments of temperature, mass, charge, Coulomb barrier, and recoil velocity are investigated with regard to their influence on multiplicity and spectra. Emission of resonant states is limited to those with decay rates, $\hbar/\Gamma$, slow compared with emission rates which are readily calculable with the formalism.

[NUCLEAR REACTIONS Developed ensemble averaged formalism for statistical decay of excited nuclear system. Time evolution of nuclear temperature, mass, charge, Coulomb barrier, recoil velocity; emission of light, medium mass particle stable, unstable composite fragments; multiplicities, energy spectra calculated.]

I. INTRODUCTION

We consider here the nuclear process in which a succession of particles is emitted from an excited nucleus. To analyze this we develop a formalism for following the averaged evolution of the system as the initial excitation energy is transferred to the emitted particles. The approach permits the types of emitted nuclear particles to have arbitrary mass number, charge, spin, and state of excitation. In order to predict multiplicities and spectra, we integrate the contributions for each type of particle from each stage of the deexcitation process. The formalism that we have constructed is simple in structure and content, yet appears to be sophisticated enough to include those important average properties of nuclei that are necessary to provide reliable estimates of multiplicities and the shapes of emission spectra. The method is much faster at calculating multiplicities and spectra of emitted particles while treating a larger assortment of emission species than currently used procedures[1-3] which successively calculate the individual decay chains; however, some loss of detail does occur especially concerning the target residuals. Although earlier works have attempted to accomplish the goal of integrating the averaged ensemble of systems,[4] the formalism here is specifically chosen to make transparent the physical phenomena influencing the emission process. It uses a new and reasonable criterion to select excited states for inclusion in the emission process.

Let us consider the decay of a compound system labeled C into a daughter B, capable of further decay, and a particle labeled b. We take N_b to be the number of emitted particles of type b. Then $d^2N_b/dt\,dE$ expresses the rate of emission (at kinetic energy E into a given interval dE) of that type of particle.

We build our formalism about a model expression for this double differential quantity and obtain spectra from it by the time integration

$$\frac{dN_b}{dE}=\int_0^\infty\left[\frac{d^2N_b}{dE\,dt}\right]dt\ . \tag{1.1}$$

The emission rate is obtained by an energy integration

$$\frac{dN_b}{dt}=\int_0^\infty\left[\frac{d^2N_b}{dE\,dt}\right]dE\ , \tag{1.2}$$

and the multiplicity by a further time integration

$$N_b=\int_0^\infty\left[\frac{dN_b}{dt}\right]dt\ . \tag{1.3}$$

It is convenient to characterize the instantaneous state of excitation of the residual system by a temperature T, which ranges, during deexcitation, from $T_{\max}$ down to zero. The time integrations indicated in Eqs. (1.1) and (1.3) are performed by converting to the corresponding temperature variable and using

$$\int_0^\infty dt=\int_{T_{\max}}^0\left[\frac{dt}{dT}\right]dT\ . \tag{1.4}$$

Our approach is first to obtain an expression for $d^2N_b/dE\,dt$ which is a function of T, and using that to obtain the emission rate dN_b/dt and subsequently a cooling rate dT/dt which also is a function of T.

The following two sections (II and III) are devoted to developing these expressions. In Sec. IV, we discuss the question of the emission of excited nuclei and in Sec. V we discuss the question of the validity and certain general properties of the formalism.

II. EVALUATION OF $d^2N/dE\,dt$

Let us first consider the emission of neutrons (particles of unit mass and zero charge) and treat the process from two points of view: (a) the evaporation of a Fermi gas from a spherical well of depth U_0 and radius R, and (b) statistical emission from a many-body excited nucleus. We shall compare the results of the two approaches.

For the evaporation, we take k to be the particle momentum measured from the bottom of the potential well and p the particle momentum measured from the top, so that

$$\frac{k^2}{2m}=\frac{p^2}{2m}+U_0 \,. \tag{2.1}$$

The phase-space density for a Fermi gas is given by

$$\frac{d^6n}{d^3r\,d^3k}=\frac{2s+1}{(2\pi\hbar)^3}\frac{1}{(e^{(k^2/2m-\mu)/T}+1)} \,, \tag{2.2}$$

where T is the temperature, μ is the chemical potential, and s is the spin of the particle. ($s=\frac{1}{2}$ for neutrons.)

The differential flow from a surface element d^2A is given by

$$\frac{d^3N}{dt\,d^2A}=(\hat{n}\cdot\vec{v})\theta(\hat{n}\cdot\vec{v})\frac{d^6n}{d^3r\,d^3k}\,d^3p \,, \tag{2.3}$$

where $\hat{n}$ is the normal to the surface, $\vec{v}$ is the external velocity of the emitted particle, and θ is the function which equals unity for a positive argument and is zero otherwise. If $v\,d^3p$ is given in terms of the kinetic energy E,

$$v\,d^3p=2mE\,dE\,d\Omega \,, \tag{2.4}$$

and the yield is integrated over the entire surface, one obtains the following differential result:

$$\frac{d^2N}{dE\,dt}=\left[\frac{m\pi R^2}{\pi^2\hbar^3}\right](2s+1)E\theta(E)\frac{1}{(e^{+(E+U_0-\mu)/T}+1)} \,. \tag{2.5}$$

For the case of a proton or heavier charged particle, the emitted particle must surmount a barrier of height V. Then the result of Eq. (2.5) generalized to

$$\frac{d^2N}{dE\,dT}=\left[\frac{m\pi R^2}{\pi^2\hbar^3}\right](2s+1)(E-V)\theta(E-V)\times\frac{1}{(e^{+(E+U_0-\mu)/T}+1)} \,. \tag{2.6}$$

For

$$E+U_0-\mu\gg T$$

we obtain

$$\frac{d^2N}{dE\,dT}=\left[\frac{m\pi R^2}{\pi^2\hbar^3}\right](2s+1)(E-V)\theta(E-V)\times e^{-E/T}e^{(\mu-U_0)/T} \,. \tag{2.7}$$

Let us now consider the statistical emission approach of Weisskopf.[5] For the decay of a system labeled C into $b+B$, we take

$$\frac{d^2N}{dE\,dt}=\frac{2s+1}{\pi^2\hbar^3}EM\sigma_{b+B\to C}[\omega_B(E_B^*)/\omega_c(E_c^*)] \,, \tag{2.8}$$

where M, E, and s refer to the mass, kinetic energy, and spin of the emitted particle b; and where $\omega_B(E_B^*)$ is the density of states for the daughter system B at the excitation energy E_B^*, which remains after particle b is emitted with kinetic energy E from the compound system C, originally having excitation energy E_c^*. For the present, let us neglect the corrections due to recoil of daughter nucleus B. The quantity $\sigma_{b+B\to C}$ is the cross section for the formation of system C which we take to be geometrical. For neutral particles,

$$\sigma_{b+B\to C}=\pi R_b{}^2 \,. \tag{2.9}$$

For charged particles, on the other hand, the cross section is effected by the Coulomb repulsion and is approximately

$$\sigma_{b+B\to C}=\pi R_b{}^2\left[\frac{E-V}{E}\right]\theta(E-V) \,, \tag{2.10}$$

where V is the Coulomb barrier.

With Eq. (2.10) inserted into (2.8),

$$\frac{d^2N_b}{dE\,dt}=(2s+1)\left[\frac{MR^2}{\pi\hbar^3}\right](E-V)\theta(E-V)\times[\omega_B(E_B^*)/\omega_c(E_c^*)] \,. \tag{2.11}$$

In order to compare Eq. (2.11) with the Fermi gas evaporation result of Eq. (2.7) and to generalize this expression, let us express the densities ω_B and ω_c in terms of the corresponding entropy functions $S(E^*)$,

$$\omega(E^*)=e^{S(E^*)} \,, \tag{2.12}$$

so that

$$(\omega_B/\omega_c)=e^{-[S(E_c^*)-S(E_B^*)]} \,. \tag{2.13}$$

The entropy functions depend not only on E but also on volume (V) and particle number (N), with

$$\left.\frac{\partial S}{\partial E}\right|_{N,V}\equiv\frac{1}{T} \,, \tag{2.14a}$$

$$\left.\frac{\partial S}{\partial N}\right|_{E,V}=-\frac{\mu}{T} \,. \tag{2.14b}$$

For a change in entropy at constant volume, Eqs. (2.14) provide

$$\Delta S=\frac{1}{T}\Delta E-\frac{\mu}{T}\Delta N \,. \tag{2.15}$$

To use this expression for changes at constant volume in Eq. (2.13) we take

$$E_c^*=E_B^*+E+\tilde{U} \,,$$

where $\tilde{U}$ reflects the fact that the emitted particle is initially bound. For emission of a nucleon ($\Delta N=-1$) the ratio ω_B/ω_c becomes

$$\omega_B/\omega_c = e^{-E/T} e^{(\mu-\tilde{U})/T} . \tag{2.16}$$

When this is substituted into Eq. (2.11) the result is identical to Eq. (2.7) provided $\tilde{U}$ is associated with the well depth U_0.

For particle emission from nuclei, we can relate the change in excitation energy to the masses of parent and daughter nuclei by the conservation of energy:

$$E_c^* - E_B^* = E + (M_B + M_b - M_c)c^2 . \tag{2.17}$$

The expression in parentheses is just the ground state separation or binding energy for particle b which we shall call B_b here.

With emission from a nuclear system it is the density of the daughter system which is more likely to be equal to the density of the compound system than the volume of the daughter is to be equal to the volume of the compound. For this reason we consider an entropy change which occurs at constant density (rather than constant volume). Let the intensive quantities ϵ^* and σ be related to the excitation energy and entropy by

$$\epsilon^* = E^*/A , \tag{2.18}$$

$$\sigma = S/A , \tag{2.19}$$

where A is the number of particles. While the entropy is a function of E^*, V, and A, the intensive quantity σ must depend on ϵ^* and ρ, i.e., other intensive variables. A change in entropy with constant density thus becomes

$$\Delta S = A \left[\frac{\partial \sigma}{\partial \epsilon^*} \right]_{A,\rho} \Delta\epsilon^* + \sigma \Delta A \tag{2.20a}$$

$$= \frac{A \Delta \epsilon^*}{T} + \sigma \Delta A . \tag{2.20b}$$

To calculate the change in the reduced energy, $\Delta\epsilon^*$, for the emission process, we use Eqs. (2.17) and (2.18) to obtain

$$A_B \epsilon_B^* = A_c \epsilon_c^* - (E + B_b) , \tag{2.21}$$

and with A_b the mass number for the emitted particle,

$$\epsilon_B^* - \epsilon_c^* = \frac{A_b \epsilon_B^*}{A_c} - \frac{E + B_b}{A_c} . \tag{2.22}$$

Finally, approximating ϵ_B^* by $\epsilon_c^* \equiv \epsilon^*$ we obtain the change in entropy at constant density from Eqs. (2.20) and (2.22),

$$\Delta S = -E/T + \frac{A_b(\epsilon^* - T\sigma) - B_b}{T} . \tag{2.23}$$

It is convenient here to define a reduced free excitation energy f^*,

$$f^* = \epsilon^* - T\sigma , \tag{2.24}$$

which is related to the Helmholtz free energy F by

$$f^* = \frac{F - E_0}{A} , \tag{2.25}$$

where E_0 is the ground state energy. In terms of f^*, the ratio ω_B/ω_C which appears in Eq. (2.11) becomes

$$\omega_B/\omega_c = e^{-E/T} e^{(A_b f^* - B_b)/T} . \tag{2.26}$$

For the corresponding factor in Eq. (2.7) we have

$$e^{-E/T} e^{(\mu - U_0)/T} .$$

The finite temperature chemical potential μ can be expressed as follows;

$$\mu = \mu_0 + \tfrac{2}{3}\epsilon^* + f^* , \tag{2.27}$$

where μ_0 is μ $(T=0)$. Substituting for a we obtain

$$e^{+(\mu - U_0)/T} = e^{(\mu_0 - U_0 + f^*)/T} e^{(2/3)\epsilon^*/T} . \tag{2.28}$$

If $(U_0 - \mu_0)$ is identified with the separation energy B_b, Eq. (2.28) gives

$$e^{(\mu - U_0)/T} = e^{(f^* - B_b)/T} e^{(2/3)\epsilon^*/T} \tag{2.29}$$

for single nucleon evaporation from a fixed potential well. For emission of a particle of mass A_b we may approximate the result by

$$e^{(\mu - U_0)/T} \to e^{(A_b f^* - B_b)/T} e^{(2/3)A_b \epsilon^*/T} . \tag{2.30}$$

Note that the evaporation expression differs from the corresponding constant density statistical emission expression in Eq. (2.26) by a factor of $e^{(2/3)A_b\epsilon^*/T}$. A significant difference is therefore expected in the prediction of the particle yield depending on whether one uses the evaporation approach from a fixed potential well or the constant density statistical emission approach. We will use the latter in the remainder of this work.

To evaluate f^*, which is a function of the instantaneous temperature T, we use the Fermi gas model in which protons and neutrons are treated separately while being confined to the same volume. Thus there are different densities for each type of particle, ρ_π (for protons) and ρ_ν (for neutrons) with

$$N/\rho_\nu = Z/\rho_\pi = A/\rho . \tag{2.31}$$

The quantity f^* is a function of temperature and density,

$$f^*(T,\rho) = \epsilon^*(T,\rho) - T\sigma(T,\rho) . \tag{2.32}$$

While we assume ρ is constant in the emission process the relative number of residual neutrons and protons changes with time, and thus, in general, ρ_π and ρ_ν change and hence, in general, f^* is different for protons and neutrons.

The change in residual mass number A_c and change Z_c introduces a time (temperature) dependence for the separation energy B_b. We use the liquid drop formula for the masses of the instantaneous compound and residual nuclei and the tabulated masses for the emitted particles. For the former we have used[6]

$$M(A,Z) = 14.1A - 13A^{2/3} - 0.595Z^2/A^{1/3} - 19(A-2Z)^2/A . \tag{2.33}$$

The pairing term is omitted because one is at high excitation energy and is averaging over neighboring nuclei in the evolution.

In treating the Coulomb barrier for $A_b \geq 2$ we use a touching-sphere approximation,

$$V_b=\begin{cases}\dfrac{Z_b(Z_c-Z_b)e^2}{r_c(A_b^{1/3}+(A_c-A_b)^{1/3})}\ , & \text{for } A_b\geq 2\ ,\\ \dfrac{(Z_c-1)e^2}{r_cA_c^{1/3}}\ , & \text{for protons,}\end{cases} \tag{2.34}$$

where r_c is taken as 1.44 fm. The barrier V_b is implicitly a function of time (temperature) through the corresponding dependence of Z_c and A_c.

Combining all of the ingredients above we have the following expression for the rate of emission of particles of type b:

$$\frac{d^2N_B}{dE\,dt}=(2S_b+1)\left[\frac{m_b\pi R_b^{\,2}}{\pi^2\hbar^3}\right](E-V_b)\exp^{-E/T}\theta(E-V_b)\exp[(Z_bf^*(T,\rho_\pi)+n_bf^*(T,\rho_\nu)-B_b)/T]\ , \tag{2.35}$$

where $Z_b+n_b=A_b$, with Z_b the number of protons and n_b the number of neutrons for the emitted nucleus. $\pi R_b^{\,2}$ is the geometrical cross section associated with the formation of the compound nucleus from particles b and B. We take

$$R_b=\begin{cases}[(A_c-A_b)^{1/3}+A_b^{1/3}]r_0\ , & \text{for } A_b\geq 2\ ,\\ r_0(A_c-1)^{1/3}\ , & \text{for } A_b=1\ ,\end{cases} \tag{2.36}$$

where $r_0=1.2$ fm.

We have to this point neglected the recoil of the daughter nucleus B which attends the emission of nucleus b. This feature is easily included by replacing the kinetic energy E above with the total kinetic energy of both b and B in the rest frame of the emitting nucleus c, E_{cm}, and by replacing m_b by the reduced mass,

$$\hat{m}_b=m_bM_B/(M_b+M_B)\ .$$

This provides

$$\frac{d^2N_b}{dE_{cm}dt}=\gamma\cdot\Sigma(E_{cm},T,b)F(T,b)\ , \tag{2.37}$$

with

$$\gamma=\frac{m_pr_0^{\,2}}{\pi\hbar^3}\ , \tag{2.38a}$$

$$\Sigma(E,T,b)=[E-V_b(A_c(T),Z_c(T))]e^{-E/T}\theta[E-V_b(A_c(T),Z_c(T))]\ , \tag{2.38b}$$

$$F(T,b)=\frac{A_b(A_c-A_b)}{A_c}(2S_b+1)[(A_c(T)-A_b)^{1/3}+A_b^{1/3}]^2$$

$$\times\exp\{[Z_bf^*(T,\rho_\pi)+n_bf^*(T,\rho_\nu)-B_b(A_c(T),Z_c(T))]/T\}\ . \tag{2.38c}$$

The first factor γ in Eq. (2.37) is a constant, while the second factor Σ contains all of the energy dependence. The factors Σ and F depend on the type of particle emitted and the temperature both explicitly and implicitly through the time dependence of A_c and Z_c. The energy spectrum of particle b in the rest frame of the emitting nucleus is therefore given by

$$\frac{d^2N_b}{dE_{cm}dt}=\frac{d^2N_b}{dE_{cm}dt}\frac{dE_{cm}}{dE_{1cm}}=\frac{d^2N_b}{dE_{cm}dt}\frac{A_c}{A_c-A_b}\ , \tag{2.39}$$

where E_{1cm} is the energy of particle b alone in this frame.

III. THE COOLING CURVE $T(t)$

In the preceding section we established an expression for the rate of emission of particles as a function of temperature. In this section we relate the temperature to time so that the spectra and multiplicities arising from the entire evolution may be calculated via Eqs. (1.1)—(1.4).

The variation of the temperature with time for our ensemble averaged system is given by the ensemble averaged expression of the conservation of energy,

$$\frac{dE_c^*}{dt}+\sum_b\left[\frac{dN_b}{dt}\right]B_b+\sum_b\frac{d\,(E_K)_b}{dt}=0\ , \tag{3.1}$$

where

$$\frac{d\,(E_K)_b}{dt}$$

is the average rate of change of kinetic energy carried by the emitted particles of type b and the recoiling nucleus B.

Let us consider protons and neutrons separately then,

$$E_c^*=Z_c\epsilon^*(T,\rho_\pi)+N_c\epsilon^*(T,\rho_\nu)\ . \tag{3.2}$$

Differentiating with respect to time,

$$\frac{dE_c^*}{dt}=\frac{dZ_c}{dt}\epsilon^*(T,\rho_\pi)+\frac{dN_c}{dt}\epsilon^*(T,\rho_\nu)+N_c\frac{\partial\epsilon^*(T,\rho\nu)}{\partial T}\frac{\partial T}{\partial t}+Z_c\frac{\partial\epsilon^*(T,\rho_\pi)}{\partial T}\frac{\partial T}{\partial t}\ . \tag{3.3}$$

Conservation of particle number relates the loss of particles from the compound nucleus to the number of emitted particles by

$$\frac{dZ_c}{dt}=-\sum_b \frac{dN_b}{dt}Z_b \ , \tag{3.4a}$$

$$\frac{dN_c}{dt}=-\sum_b \frac{dN_b}{dt}n_b \ . \tag{3.4b}$$

Combining Eqs. (3.3) and (3.4) in (3.1), and using the specific heat $C_v \equiv \partial\epsilon^*/\partial T$ we obtain the desired cooling curve

$$-\frac{dT}{dt}=[N_c C_v(\nu)+Z_c C_v(\pi)]^{-1}\left[\sum_b \left[\frac{dN_b}{dt}\right][B_b-Z_b\epsilon^*(\pi)-n_b\epsilon^*(\nu)]+\frac{d(E_K)_b}{dt}\right] , \tag{3.5}$$

where

$$\epsilon^*(\pi)=\epsilon(T,\rho_\pi) \ , \quad \epsilon^*(\nu)=\epsilon(T,\rho_\nu) \ , \quad C_v(\pi)=\frac{d\epsilon^*(\pi)}{dT} \ ,$$

and

$$C_v(\nu)=\frac{d\epsilon^*(\nu)}{dT} \ .$$

For $d(E_K)_b/dt$ we have

$$\frac{d(E_K)_b}{dt}=\int_{V_b}^{\infty} E_{cm}\left[\frac{d^2N_b}{dt\,dE_{cm}}\right]dE_{cm}=2T^3(1+0.5V_b/T)e^{-V_b/T}\gamma F(T,b) \ , \tag{3.6}$$

and for dN_b/dt,

$$\frac{dN_b}{dt}=\int_{V_b}^{\infty}\frac{d^2N_b}{dt\,dE_{cm}}dE_{cm}=T^2e^{-V_b/T}\gamma F(T,b) \ . \tag{3.7}$$

Integrating Eq. (3.5) enables us to uniquely relate temperature to time as shown in the example displayed in Fig. 1. We can obtain the history of the ensemble averaged mass number and charge by

$$A_c(T)=A_c(T_{\max})-\sum_b \int_{T_{\max}}^{T}\frac{dN_b}{dt}A_b\frac{dt}{DT}\,dT \ , \tag{3.8a}$$

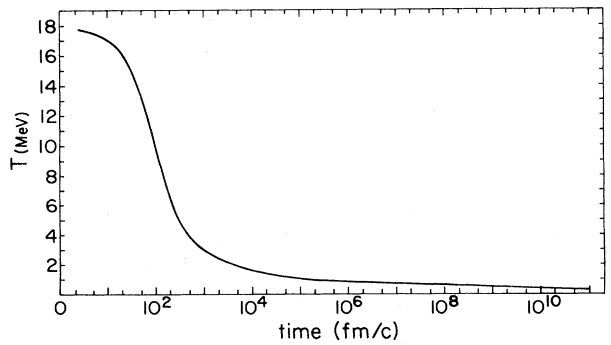

FIG. 1. The monotonically decreasing dependence of nuclear temperature on elapsed time for evaporation is shown for a nucleus with initial parameters $Z_c(T_{\max})=55$, $A_c(T_{\max})=133$, $T_{\max}=18$, and $\epsilon_F=38$.

$$Z_c(T)=Z_c(T_{\max})-\sum_b \int_{T_{\max}}^{T}\frac{dN_b}{dt}Z_b\frac{dt}{dT}\,dT \ . \tag{3.8b}$$

The Coulomb barrier for each type of emitted particle evolves with T according to Eqs. (3.8) and (2.34). The binding energy also evolves with time,

$$B_b(T)=B_b(A_c(T),Z_c(T)) \ , \tag{3.9}$$

where

$$B_b(T)=M(A_c(T),Z_c(T)) -M(A(T)-A_b,Z_c(T)-Z_b)-M(A_b,Z_b) \ , \tag{3.10}$$

and Eq. (2.33) is used for $M(A,Z)$ of the compound and the residual systems. Figure 2 illustrates the decrease in quantities $A_c(T)$, $Z_c(T)$, $B_b(T)$, and $V_b(T)$, and Fig. 3 shows typical variations in decay rate $dN_b(T)/dT$ as the system cools.

Finally, we consider the evolution of the mean recoil momentum for the emitting system. Conservation of momentum provides

$$\vec{P}_c=\sum_b \vec{p}_b \ . \tag{3.11}$$

Then

$$\langle P_c^2\rangle=\left\langle \sum_{bb'}\vec{P}_b\cdot\vec{P}_{b'}\right\rangle \ . \tag{3.12}$$

For random orientation of $\vec{p}_b$,

$$\langle P_c^2(T)\rangle=\sum_b\langle P_b^2\rangle \ . \tag{3.13}$$

To obtain $\langle p_b^2\rangle$ we use Eq. (3.6) and perform the follow-

ing integration:

$$\langle p_b{}^2\rangle = \int_{T_{\max}}^{T} 2\hat{m}_b \frac{d(E_K)_b}{dt}\frac{dt}{dT}\,dT\,, \tag{3.14}$$

where $\hat{m}_b(T)$ is the reduced mass

$$\frac{A_b[A_c(T)-A_b]}{A_c(T)}m_p\,.$$

The velocity of the emitting source is then taken to have a Gaussian distribution about its mean with a distribution $e^{-v^2/2\Delta^2(T)}$ where

$$\Delta^2(T)=\frac{1}{3}\frac{1}{m_p{}^2A_c{}^2}\langle P_c{}^2(T)\rangle\,. \tag{3.15}$$

The recoil correction in Eq. (3.15) modifies $d^2N_b/dE_{1cm}dt$ in Eq. (2.39),

$$\frac{d^2N_b}{dE_{1cm}dt}=\gamma\sum{}'(E_{1cm},T,b)F(T,b)\cdot A_c/(A_c-A_b)\,, \tag{3.16}$$

where

$$\sum{}'(E_{1cm},T,b)=\int\sum\left[\frac{1}{2}\left[\frac{m_b{}^2}{\hat{m}_b}\right](v-v_s)^2,T,b\right]e^{-v_s{}^2/2\Delta^2(T)}\frac{d^3v_s}{(2\pi\Delta^2)^{3/2}} \tag{3.17}$$

and $\frac{1}{2}m_bv^2=E_{1cm}$. This recoil correction modifies significantly the energy spectra for heavy particles which primarily are emitted in later stages of the cooling process, but the light particles tend to be unaffected as indicated by Fig. 4. The rate of particle emission dN_b/dt given by Eq. (1.2) is of course unmodified.

In summary we have shown in this section how the ultimate observables, multiplicities, and energy spectra are influenced by the evolution of the cooling process.

IV. EMISSION OF EXCITED NUCLEI

Nuclei may be emitted not only in their ground states but also in any of their excited particle stable states and, in certain circumstances, in particle unstable states. These nuclei must be counted along with the ground state in determining the yield of a given isotope. If emitted, the particle-unstable excited states should be included in the experimental data as part of the yields of neighboring nuclei. Some criterion is necessary to determine which particle-unstable resonances must be included. This criterion is an important consideration in any model of particle emission and has a significant influence on the predicted yields and spectra and thus can be tested. The formalism given in this paper provides a time scale, namely, the mean time until emission of particle species b, $(dN_b/dt)^{-1}$. We include resonances with lifetime, $\hbar/\Gamma_b$, long with respect to the emission time

$$\hbar/\Gamma_b \gg \left[\frac{dN_b}{dt}\right]^{-1}\,.$$

Conversely, resonances with

$$\hbar/\Gamma_b \ll \left[\frac{dN_b}{dt}\right]^{-1}$$

are not included. We find most resonances satisfy one of these criteria.

For $T_{\max}$ 10—20 MeV the emission rate is sufficiently fast to require inclusion of particle unstable isotopes of hydrogen and helium, both of which experience little Coulomb inhibition and are light masses and hence are easily emitted. For the heavier elements $\hbar dN/dt$ corresponds to widths of the order of 1 keV or less and hence we exclude the particle unstable states of these nuclei. The emission of unstable states of few nucleon systems influences the detected ratio of yields of p, d, t, ^{3}He, and α particles. The emission of particle stable states of the heavier elements greatly enhances the detected yield of these nuclides.

For our purposes the emission of an excited state involves a process similar to the emission of the ground state but differing in spin and in binding energy by an amount equal to the energy of excitation. In principle, one should include each excited state as a statistically independent particle in the analysis of the previous sections. We find that the emission of the heavier nuclides occurs in a relatively narrow temperature window during the cooling process. We make use of this feature in approximating the emission of heavy nuclei in particle stable excited states. These are included simply by multiplying the ground state yield by the factor Y_b,

$$Y_b=\sum_{i=0} e^{-(E_i^*)/\langle T_b\rangle}(2S_i+1)/(2S_0+1)\,, \tag{4.1}$$

where the sum on i runs over the ground state and all the particle-stable states with excitation E_i^* and spin S_i, and where b labels the isotope, and $\langle T_b\rangle$ is the mean temperature for emission of the ground state of that nuclide. The exponential cutoff reduces the influence of discrete high lying excited states since $\langle T_b\rangle$ is typically 5—7 MeV for $T_{\max}\cong 20$ MeV.

V. DISCUSSION

The method for calculating the particle emission described above assumes the existence of statistical (equilibrated) distributions of states during the process. The method, however, also provides a means of calculating a definite time scale associated with the emission. Thus for any given system this calculated scale can be compared with other characteristic periods in order to evaluate the validity of these underlying assumptions which rely on time. We used this procedure in the previous section for the case of the emission of resonances. The availability of this time scale is one of the attractive features of our approach.

It is explicitly clear that the calculated particle spectra

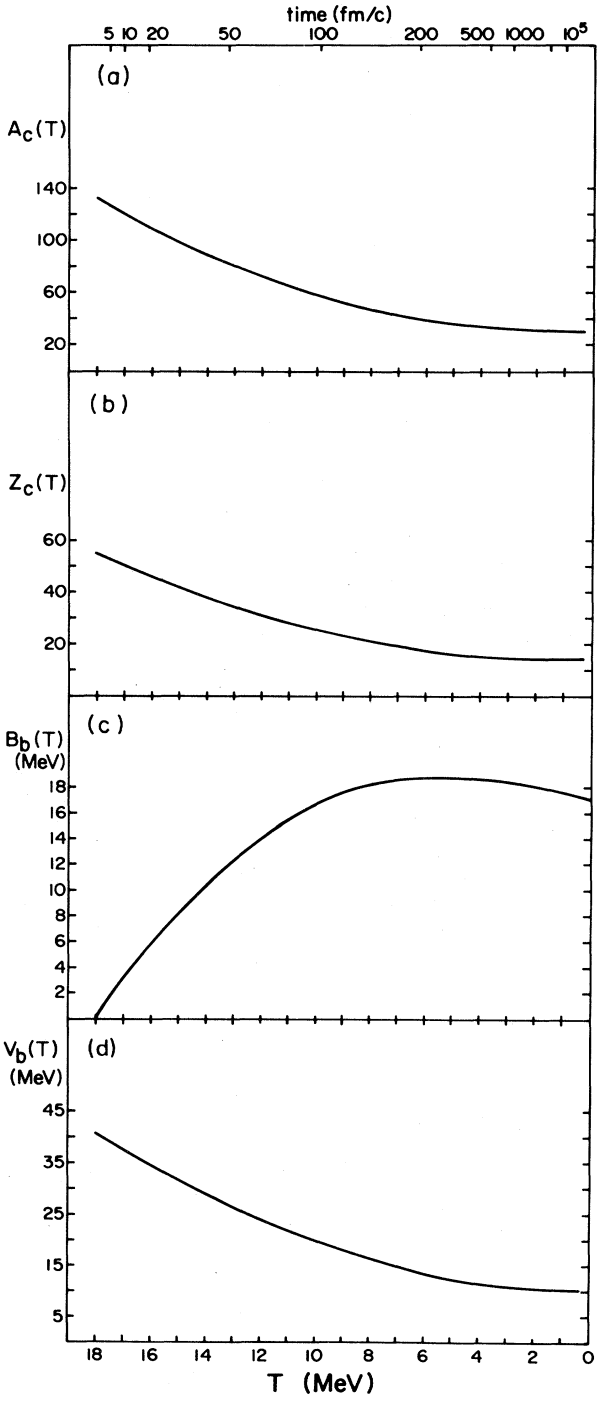

FIG. 2. Evolution of the instantaneous parameters $A_c(T)$, $Z_c(T)$, $B_b(T)$, and $V_b(T)$ as a function of temperature for a nucleus with initial parameters $A_c(T_{\max})=133$, $Z_c(T_{\max})=55$, $T_{\max}=18$, and $\epsilon_F=38$. (a) and (b) indicate the decrease with temperature of the parent nucleus mass and charge. (c) and (d) show the variation with temperature [and implicitly with $A_c(T)$ and $Z_c(T)$] of the separation energy $B_b(T)$ and Coulomb barrier $V_b(T)$ when the emitted particle is a ^{12}C nucleus in its ground state.

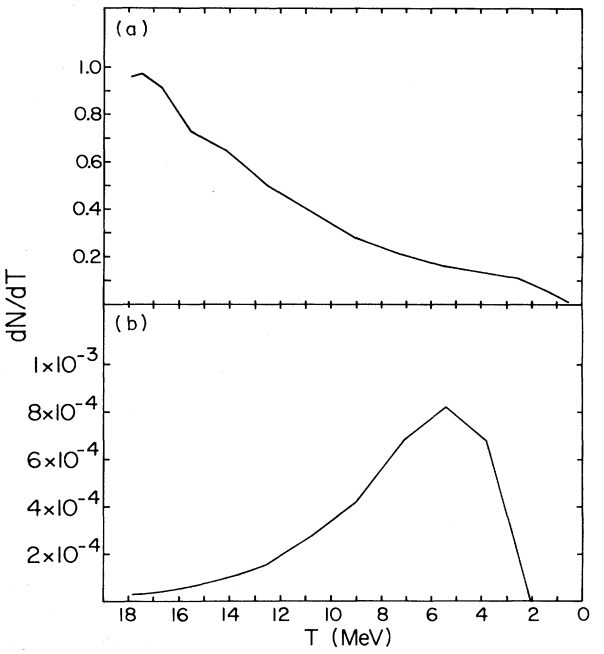

FIG. 3. Derivatives of the emitted particle yields with respect to temperature dN/dT for light and heavy emitted particle species displayed for a parent nucleus with initial parameters $A_c(T_{\max})=133$, $Z_c(T_{\max})=55$, $T_{\max}=18$, and $\epsilon_F=38$. (a) shows dN/dT when the emitted particle is a proton. (b) is the corresponding prediction for ^{12}C nuclei emitted in their ground states.

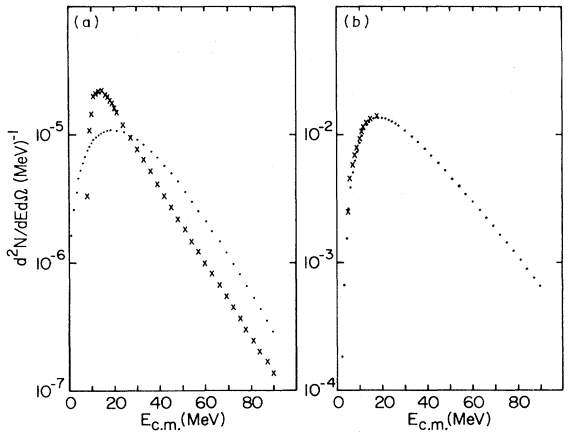

FIG. 4. Energy spectra with and without the random source velocity correction of Eqs. (3.15)—(3.17). Crosses indicate spectra calculated without the velocity corrections and dots indicate spectra calculated with the velocity correction. (a) demonstrates the importance of the random source velocity to a typical heavy particle spectrum for a ^{12}C nucleus emitted in its ground state. (b) shows the comparative insensitivity of lighter mass fragments to the random source velocity. Above 20 MeV the two calculations merge and only the dots are plotted.

involves an integration over the evolution of the cooling system and hence are shaped by more complicated factors than the initial temperature energy distribution. We find that among these features are the isospin of the excited system, the recoil momentum of the target, the reduction of the Coulomb barrier which accompanies charge emission as well as the range of temperatures traversed. The heavier particles are much more sensitive to the latter effects than are the light particles.

The effects due to the charge and isospin of both the emitted species and the compound nucleus are quite subtle. The two competing features here include the Coulomb barrier which favors emission of particles with no, or low, charge and the binding energy which favors a path for emission along the valley of nuclear stability. The results of these two features can affect both the spectra and multiplicities. In this regard, the model shows sensitivity to the liquid-drop parameters.

The excited states of nuclear species can play a large role with regard to multiplicities. As we indicated in the previous section the observed yields are especially sensitive to the level densities of the emitted nuclei at low excitation energy. There are sensitivities of this model to other properties of the emitting system which have not been illustrated in the paper but are worthy of note. One property is the mass density of the emitting system. This enters the formalism through the choice of a mean Fermi energy for the excited decaying nucleus. The value of the Fermi energy especially seems to affect the relative yield of the heavy particles.

The results are also sensitive to the initial mass and temperature of the decaying system. This sensitivity may be useful in providing information about processes in which only a portion of the nucleons of a colliding system are participants.

In this paper we have presented the essential and the detailed features for our model formalism for statistical particle emission. We have also mentioned in very general terms some of the features which determine the calculated results. In succeeding papers we will examine the results of applications of this formalism to particle emission from specific systems.

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