Spherical nucleon bag deformations in the two-nucleon system

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It is argued that the pion mediated interaction between two nucleon chiral bags should lead to changes in bag radii, relative to the value for an isolated nucleon if bag shapes are maintained. These changes are calculated as a function of the separation distance for several nucleon spin-isospin states, with the isolated nucleon size and compressibility regarded as parameters. Sizeable radius changes are found for typical values of these parameters, and for separations probed in medium energy ($\gtrsim 70$ MeV) nucleon-nucleon collisions.

[NUCLEAR REACTIONS Chiral bag pair; static approximation; spherical deformation calculated as function of separation.]

I. INTRODUCTION

Whether quark degrees of freedom can at all be made manifest in low-energy nuclear phenomena is a central issue for nuclear physics today. A preliminary advance was made rather recently with the introduction of chirally symmetric bag models of the nucleon.[1-6] In all these models, the spherical bag surface acts as a fixed source, absorbing and emitting pions. This "pion cloud" has important physical consequences, one of which is the relatively small size of the nucleon bag (0.6—1.0 fm) as compared to the MIT bag[7] size of ~ 1.6 fm. Physically, this compression occurs because the pion cloud exerts an inwards directed pressure on the bag surface, thus adding to "vacuum pressure" arising from the phenomenological BV term[7] in the Lagrangian. This additional pressure can be compensated for only by an increase in quark kinetic energy. Hence the bag shrinks.

Given the importance of pion pressure generated by self-interactions (i.e., emission and absorption from the same surface), one may logically ask whether a nucleon in a many-nucleon environment would have its intrinsic properties (e.g., charge radius, magnetic moments, etc.) altered as a result of the pion clouds of neighboring nucleons. Therefore, consider the simplest possible, but nontrivial, many-body system—two static nucleon bags each of radius R with center-to-center separation r. (See Fig. 1.) The bag radius at infinite separation is denoted by R_0.

Consider now the situation as r is continuously decreased from infinity downwards. For $r \gg m_\pi^{-1}$, each bag is separately in equilibrium with quark, pion, and vacuum pressure mutually balancing out at radius R_0. However, as r becomes smaller, pions emitted from one nucleon surface will be absorbed at the other surface and will hence transfer momentum between nucleons. The average momentum transfer (or force) becomes larger with decreasing distance, and increasing numbers of mesons participate. To be sure, the deforming force is not spherically symmetric. Neither is the deforming force arising from self-interactions angle independent. Nevertheless, for reasons of tractability we follow the universal route of dealing only with spherical bags.

To proceed, let us write the total system energy E as a sum of two parts, i.e., the self-energies of the two nucleons $\Sigma^{A,B}$ and the potential V,

$$E(R,r)=\Sigma^A(R)+\Sigma^B(R)+V(R,r) . \tag{1}$$

For small deformations $\Delta R \equiv R-R_0$, and at fixed r, Σ and V can be expanded,

$$\Sigma(R)=m_N+\tfrac{1}{2}\kappa(\Delta R)^2+\cdots \tag{2}$$

and

$$V(R,r)=V(R_0,r)+\Delta R\frac{\partial V}{\partial R}(R,r)\bigg|_{R_0}+\tfrac{1}{2}(\Delta R)^2\frac{\partial^2 V(R,r)}{\partial R^2}\bigg|_{R_0}+\cdots , \tag{3}$$

where

$$\kappa=\frac{\partial^2\Sigma(R)}{\partial R^2}\bigg|_{R_0} \tag{4}$$

is the nucleon "stiffness" or compressibility. Minimizing $E(R,r)$ with respect to R yields the deformation ΔR as a function of separation r:

$$\Delta R(r)=\frac{-\dfrac{\partial V}{\partial R}(R,r)\bigg|_{R_0}}{2\kappa+\dfrac{\partial^2 V(R,r)}{\partial R^2}\bigg|_{R_0}} . \tag{5}$$

We calculated the potential energy $V(R,r)$ using perturbation theory. Whereas the one pion exchange is adequate

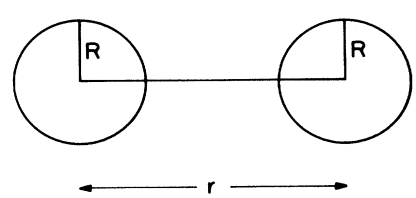

FIG. 1. Static nucleon bags at radius R with center-to-center separation r.

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for large r, at shorter distances one must supplement this with additional terms. In Fig. 2 we show the diagrams which were evaluated. The value of $f^2_{\pi NN}$ has some dependence on R_0 and was always chosen such that it resulted in a Yukawa potential with the correct asymptotic strength. For $f^2_{\pi N\Delta}$, the SU(6) value was used, i.e., $f^2_{\pi N\Delta}/f^2_{\pi NN}=72/25$. We note that each vertex in Fig. 2 carries a bag form factor $u(kR)$. This quantity is a fundamental consequence of the extended nature of the pion source and is the cause of the R dependence of V. Evaluation of the diagrams is straightforward, but entails tedious algebra and a numerical integration for two-dimensional integrals in k space.

Because we also wish to explore the region where the bags have overlapped to some extent, we shall simply continue to use the potential generated by pion exchanges even when $r<2R$. This, of course, can only be a valid approximation if the quark contribution to the potential is small compared to the pionic contribution. The calculations of De Tar,[8] as well as the conclusions of Brown,[9] support this view for even moderately large overlaps. In any case, given the exploratory nature of this calculation, we shall simply assume the approximation as valid.

A reliable first principle calculation of the nucleon compressibility κ appears difficult, even though the chiral bag models do predict its value. Théberge *et al.*[3] and Myhrer *et al.*[10] calculated $\Sigma(R)$ in the one pion approximation. With their $\Sigma(R)$, the bag turned out to be stable ($\kappa>0$), but only barely so. A mean field calculation[11] incorporating higher order pionic self-energy effects showed the one pion approximation (for the self-energy) to be invalid for $R_0 \lesssim 1$ fm and the bag turned out to be unstable. The reason for bag instability is well known—the pion self-energy (actually its negative) grows much more rapidly as $R_0 \to 0$ than the compensating quark kinetic energy. Thus, a mechanism must be found which allows for either an additional outward force (e.g., through ω mesons coupled to the bag[12]), or a means to slow the decrease of pion self-energy as $R_0 \to 0$. We experimented with the latter in the mean field approximation[11] by weighting all self-energy integrals with a pion form factor. Unfortunately, the results for κ depend rather strongly on the (essentially unknown) high momentum behavior of this form factor and it seems that κ cannot be calculated with assurance at the present time. We shall, therefore, simply parametrize our results with representative values of κ.

II. RESULTS AND DISCUSSION

The essential physical input required in our calculation, other than the value of κ, is the hadron form factor $u(kR)$. The cloudy bag model,[3–5] which permits pions both inside and outside the bag, predicts $u(kR)$ to be

$$u_{\rm CBM}(kR)=3j_1(kR)/kR \approx e^{-k^2R^2/10} . \qquad (6)$$

In Eq. (6), the oscillatory Bessel function has been replaced by a Gaussian for both theoretical and computational reasons. First, the oscillations of $u_{\rm CBM}$ are the direct consequence of a sharp bag boundary assumption, and are hence unphysical. Second, the Gaussian is easier to treat in the numerical integrations for evaluating V. One could, of course, also impose a cutoff at the first zero of $j_1(kR)$ as in Ref. 2. The changes are of the order of 10%.

The results of the numerical calculations are shown in Figs. 3–5 where we plot $\Delta R/R_0$, i.e., the shift in radius relative to the value at infinity, versus the center-to-center distance r. We have considered two typical bag sizes $R_0=0.6$ and 0.8 fm, together with two values for κ. These values (2000 and 3000 MeV fm^{-2}) are considerably more conservative than the MIT bag value ($\kappa \sim 1500$ MeV fm^{-2}). Given ΔR, the shift in nucleon properties may be directly obtained from the curves of Refs. 5 or 11. The shift in neutron charge radius $\Delta\langle r^2\rangle^{1/2}$, for example, is $\Delta\langle r^2\rangle^{1/2} \approx 0.5\Delta R$ (fm).

In order to avoid effects arising from the mixing of different angular states by the tensor force, we limit spin-isospin configurations to $^1P_1$, $^3P_1$, $^1D_2$, and $^3D_2$. The $^1S_0$ state is omitted because, in the absence of a centrifugal barrier, nucleons may come arbitrarily close and break down our physical assumptions.

From Figs. 3–5 some interesting general conclusions

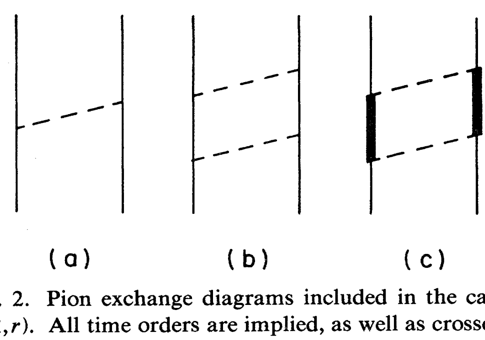

FIG. 2. Pion exchange diagrams included in the calculation of $V(R,r)$. All time orders are implied, as well as crossed meson lines. However, in (b) the two pion part corresponding to an iteration of (a) has been specifically excluded. Also, in (c) the crossed meson contribution has been neglected since this has a considerably larger energy denominator.

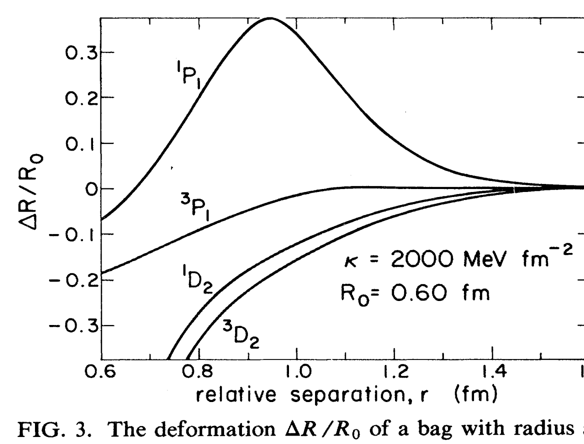

FIG. 3. The deformation $\Delta R/R_0$ of a bag with radius at infinity $R_0=0.60$ plotted as a function of nucleon-nucleon separation r in four different spin-isospin states.

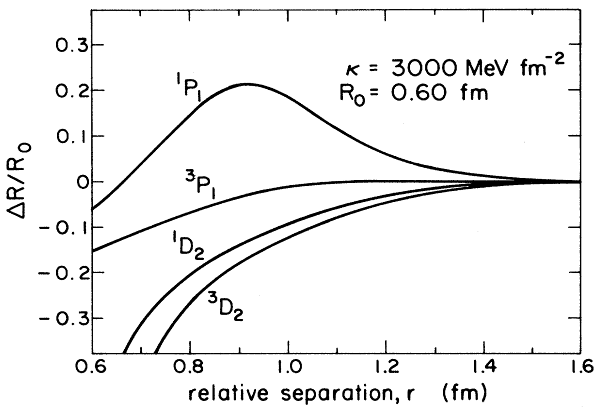

FIG. 4. Same as Fig. 3 except $\kappa=3000$ MeV fm^{-2}.

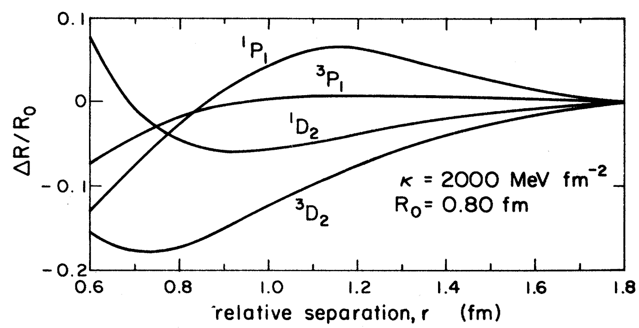

FIG. 5. Same as Fig. 3 except $R_0=0.8$ fm.

can be drawn. First, small bags generally experience bigger deformations than larger bags at the same separation distance. On an intuitive level, one can understand this as follows: A pion emitted from one hadron surface and absorbed at the other is rapidly attenuated with a characteristic falloff $e^{-m_\pi x}$, where x is the distance between the points of emission and absorption. It is then obvious from geometrical reasoning that, for a fixed separation, there should be less attenuation for small R_0.

Second, unlike the case of an isolated nucleon which is always compressed by its pion cloud, nucleon pair interactions lead to either sign for ΔR. Even at a fixed separation, ΔR depends strongly on the spin-isospin configuration. Of all configurations investigated, the $^1P_1$ yields the largest effect for relatively large values of r. Classically, the impact parameter for relative angular momentum l is given by $b^2 \sim l(l+1)/2ME$. For $E=100$ MeV and $l=1$, one has a distance of closest approach $b \sim 0.7$ fm. Thus, the range of values considered for r is relevant to intermediate energy nucleon-nucleon scattering.

Although the relative separation extends from about 2 fm down to 0.6 fm in Figs. 3—5, it should be kept in mind that the calculations become progressively less reliable below $r \approx 1$ fm. There are two reasons for this. First, many meson exchange becomes increasingly important at short distances. Second, the (neglected) component of the potential which derives from quark exchange also becomes more important.

While we are presently unable to comment on quantitative changes to existing medium energy phenomenology, the bag deformations predicted in this paper may well be significant for N-N scattering above 100 MeV and for the $pp \rightarrow d\pi^+$ process, where the uncertainty principle yields a distance scale ~ 0.5 fm. High momentum transfer processes of this type need to be studied with a view toward answering the question of whether fixed nucleon form factors at the vertices are adequate, or whether the experimental data in fact require dependence of these form factors on the relative distance (energy) and angular momentum. The extent of this dependence would be a direct measure of bag deformation.

Finally, it would be interesting to calculate bag deformations in an average nuclear environment. There is already some evidence[13] that the quark distribution in complex nuclei is not the same as for A isolated nucleons, even after correcting for Fermi motion. It is conceivable that one could partially attribute this effect to the same mechanism as explored here for two nucleons.

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[1]R. L. Jaffe, in *Pointlike Structure Inside and Outside Hadrons*, Proceedings of the Seventeenth International School of Subnuclear Physics, Erice, Italy, 1979, edited by A. Zichichi (Plenum, New York, 1982); A Chodos and C. B. Thorn, Phys. Rev. D 12, 2733 (1975).
[2]C. E. De Tar, Phys. Rev. D 24, 752 (1980); 24, 762 (1980).
[3]S. Théberge, A. W. Thomas, and G. A. Miller, Phys. Rev. D 22, 2838 (1980).
[4]A. W. Thomas, S. Théberge, and G. A. Miller, Phys. Rev. D 24, 216 (1981).
[5]S. Théberge, G. A. Miller, and A. W. Thomas, Can. J. Phys. 60, 59 (1982).
[6]G. E. Brown and M. Rho, Phys. Lett. B82, 177 (1979).
[7]A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 10, 2599 (1974); 9, 3471 (1974).
[8]C. De Tar, Phys. Rev. D 17, 302 (1978).
[9]G. E. Brown, private communication.
[10]F. Myhrer, G. E. Brown, and Z. Xu, Nucl. Phys. A362, 317 (1981).
[11]P. Hoodbhoy, Phys. Rev. D 26, 3235 (1982).
[12]V. Vento (unpublished).
[13]G. Berlad, A. Dar, and G. Eilam, Phys. Rev. D 22, 1547 (1980); H. Pirner and J. Vary, Phys. Rev. Lett. 46, 1376 (1981); R. L. Jaffe, *ibid.* 50, 228 (1983).