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Coulomb plus separable potential in coupled channels

W. Schweiger and W. Plessas
Institute for Theoretical Physics, University of Graz, A-8010 Graz, Austria

L. P. Kok
Institute for Theoretical Physics, University of Groningen, 9700-AV Groningen, The Netherlands

H. van Haeringen
Department of Mathematics, Delft University of Technology, 2628-BL Delft, The Netherlands

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The two-potential formalism for long-range forces is generalized to coupled channels. Assuming the short-range potential to be separable we treat the proton-proton interaction in the coupled partial waves $^3P_2$-$^3F_2$. We derive the corresponding Coulomb-modified nuclear transition matrices in closed analytic form and calculate the Coulomb distortion of the phase shifts and of the mixing parameter. The separable parametrization proposed for the p-p interaction in $^3P_2$-$^3F_2$ yields an accurate fit to the experimental data available for these channels. The potential completes the description of the N-N interaction by the Graz-II separable model.

[NUCLEAR REACTIONS Coupled-channel problem with long-range forces; p-p interaction in $^3P_2$-$^3F_2$; separable potential proposed; rigorous treatment of Coulomb distortion; closed analytic formulas for transition matrices.]

In a recent paper[1] we proposed a separable representation of the nuclear part of the proton-proton (p-p) interaction in angular momentum states $l=0$, 1, and 2. The separable form factors were chosen as rational functions in momentum space. Thereby it was possible to treat the Coulomb distortion of the nuclear interaction exactly and to derive the corresponding p-p T matrix in closed analytic form. With our choice of form factors we could precisely reproduce all experimental data currently accepted for elastic p-p scattering. In addition, the off-shell behavior of the purely nuclear T matrix was modeled after the Graz-II n-p potential.[2] For simplicity, however, we then treated the $^3P_2$ state as uncoupled.

In this article we provide the generalization of our approach to the problem of coupled partial waves. We derive analytic formulas for the Coulomb-modified nuclear T matrices and phase shifts as well as mixing parameters. For the $^3P_2$-$^3F_2$ coupled states we construct a separable representation of the nuclear p-p interaction, which yields an accurate fit to all experimental data available for this channel. Thereby we complete the description of the N-N interaction by the Graz-II separable potential.

We follow the notation used in Ref. 1 but allow for spin-dependent short-range forces. Let us begin with the appropriate partial-wave decomposition of the relevant T operators. For coupled partial-wave states with angular momenta $l_< = J-1$ and $l_> = J+1$ ($J>0$ being the total angular momentum of this channel) the partial-wave T operators consist of

$$T_{ll'} = T_{cl}\delta_{ll'} + T_{csll'} \quad (l,l' = l_<, l_>) \ . \tag{1}$$

T_{cl} is the Coulomb T operator in the lth partial wave. The Coulomb-modified nuclear T operators are defined by

$$T_{csll'} = (1 + T_{cl}G_{0l})\, t_{csll'} (1 + G_{0l'}T_{cl'}) \ , \tag{2}$$

where the $t_{csll'}$ operators satisfy the set of Lippmann-Schwinger equations

$$t_{csll'} = V_{sll'} + \sum_{L=l_<,l_>} V_{slL}\, G_{cL}\, t_{csLl'} \ . \tag{3}$$

The resolvents G_{0l} and G_{cl} correspond to the free and Coulomb Hamiltonians, respectively.

Now we assume the short-range potentials $V_{sll'}$ to be separable of the form

$$V_{sll'} = \sum_{i=1}^{n_l} \sum_{j=1}^{n_{l'}} |g_{li}\rangle \lambda_{ij}^{ll'} \langle g_{l'j}| \quad (\lambda_{ij}^{ll'} = \lambda_{ji}^{l'l}) \tag{4}$$

so that Eq. (3) can be solved algebraically. It is convenient to introduce a matrix notation.[3] After solution of Eq. (3) the operator $T_{csll'}$ can then be represented in the form

$$T_{csll'} = |\vec{g}^c\rangle \Delta_l (\underline{1} - \Lambda \underline{G}_c)^{-1} \Lambda \Delta_{l'} \langle \vec{g}^c| \ . \tag{5}$$

Here $|\vec{g}^c\rangle$ represents the $(n_{l_<} + n_{l_>})$-dimensional row matrix

$$|\vec{g}^c\rangle = (|g^c_{l_<1}\rangle, \ldots, |g^c_{l_<n_{l_<}}\rangle, |g^c_{l_>1}\rangle, \ldots, |g^c_{l_>n_{l_>}}\rangle) \ , \tag{6}$$

containing the Coulomb-modified form factors

$$|g^c_{li}\rangle = (1 + T_{cl}G_{0l})|g_{li}\rangle \quad (i = 1, \ldots, n_l) \ ; \tag{7}$$

$\langle \vec{g}^c|$ is the transpose of $|\vec{g}^c\rangle$. The $(n_{l_<} + n_{l_>})(n_{l_<} + n_{l_>})$ matrices Δ_l, Λ, and $\underline{G}^c$ are defined by

$$\Delta_l = \begin{pmatrix} \underline{1}_{n_{l_<}}\delta_{ll_<} & 0 \\ 0 & \underline{1}_{n_{l_>}}\delta_{ll_>} \end{pmatrix} \ , \tag{8}$$

with $\underline{1}_n$ being the n-dimensional unit matrix

$$\Lambda = \begin{pmatrix} \Lambda_{l_< l_<} & \Lambda_{l_< l_>} \\ \Lambda_{l_> l_<} & \Lambda_{l_> l_>} \end{pmatrix} , \tag{9}$$

with

$$(\Lambda_{ll'})_{ij} = (\Lambda_{l'l})_{ji} = \lambda_{ij}^{ll'} \quad (i=1, \ldots, n_l;\ j=1, \ldots, n_{l'}) , \tag{10}$$

and

$$\underline{G}^c = \begin{pmatrix} \underline{G}f_< & 0 \\ 0 & \underline{G}f_> \end{pmatrix} , \tag{11}$$

with

$$(\underline{G}f)_{ij} = \langle g_{li} | G_{cl} | g_{lj} \rangle \quad (i,j=1, \ldots, n_l) . \tag{12}$$

In the above equations we suppressed the dependence on the (generally complex) energy variable E in the T operators, the resolvents, and the Coulomb-modified form factors.

In order to calculate Coulomb-modified nuclear phase shifts $\delta^J_{csl_<}$ and $\delta^J_{csl_>}$ as well as mixing parameters ϵ_{csJ}, one has to know the physical on-shell matrix elements of $T^J_{csll'}$ $(l,l'=l_<,l_>)$. They can be obtained from Eq. (5) by sandwiching between Coulombian asymptotic states[4] $|k\infty\pm\rangle$. In the Stapp parametrization[5] the relation between the S-matrix elements and the (bar) phase parameters for fixed J reads

$$\begin{aligned} S_{ll} &= \exp(2i\delta_l)\cos 2\epsilon_{cs} \quad (l=l_<,l_>) , \\ S_{l_< l_>} &= S_{l_> l_<} = i\exp(i\delta_{l_<} + i\delta_{l_>})\sin 2\epsilon_{cs} , \end{aligned} \tag{13}$$

where the total phase shift $\delta_l = \sigma_l + \delta_{csl}$ (σ_l is the pure Coulomb phase). The desired phase parameters can therefore be obtained from the Coulomb-modified nuclear scattering amplitudes via the relations

$$\begin{aligned} \delta_{csl} &= \tfrac{1}{2}\arctan\frac{\mathrm{Re}\,\tilde{T}_{csll}}{1-\mathrm{Im}\,\tilde{T}_{csll}} \quad (l=l_<,l_>) , \\ \epsilon_{cs} &= \tfrac{1}{2}\arcsin\frac{\mathrm{Re}\,\tilde{T}_{csl_<l_>}}{\cos(\delta_{csl_<}+\delta_{csl_>})} , \end{aligned} \tag{14}$$

where

$$\begin{aligned} \tilde{T}_{csll'}(k) = -2\pi\frac{\mu k}{\hbar^2}\exp(-i\sigma_l - i\sigma_{l'}) \\ \times \langle k\infty - | T_{csll'}(k^2+i0) | k\infty + \rangle . \end{aligned} \tag{15}$$

In the argument of the T operator on the right-hand side the abbreviation k^2 has to be understood as the energy $\hbar^2k^2/2\mu$ (μ being the reduced mass).

Now it is evident from Eqs. (5) and (15) that the on-shell, as well as off-shell, matrix elements of $T_{csll'}(k^2+i0)$ can be calculated via the quantities $\langle k\infty - | g^c_{li}\rangle$, $\langle p | g^c_{li}\rangle$, and $\langle g_{li} | G_{cl}(k^2+i0) | g_{lj}\rangle$. For arbitrary l the corresponding analytical formulas for the separable form factors we are going to employ can be found in our earlier work.[1,6,7]

Let us now turn to the special case of the $^3P_2$-$^3F_2$ coupled partial-wave state. In accordance with the Graz-II potential,[1,2] we adopted the form

$$V_{sll'} = |g_l\rangle \lambda^{ll'} \langle g_{l'}| \quad (l,l'=l_<,l_>) , \tag{16}$$

with

$$\langle p | g_l \rangle = \frac{p^l}{(p^2+\beta_{l1}^2)^{l+1}} + \gamma_l \frac{p^{l+2}}{(p^2+\beta_{l2}^2)^{l+2}} . \tag{17}$$

TABLE I. Numerical values of potential parameters as obtained by fitting the phenomenological phase shifts of Ref. 9 (energy-dependent solution). In our system of units the dimensions are $(\beta)=\mathrm{fm}^{-1}$, $(\gamma)=\mathrm{fm}^0$, and $(\lambda_{ll'})=\mathrm{MeV\,fm}^{-(l+l'+1)}$.

$\beta_{11}=1.833\,890$	$\gamma_1 = 8.177\,706$
$\beta_{12}=3.087\,726$	$\gamma_3 = -25.679\,56$
$\beta_{31}=1.826\,304$	$\lambda^{11} = -830.4213$
$\beta_{32}=3.514\,753$	$\lambda^{13} = 3481.064$
	$\lambda^{33} = -42\,065.41$

Using the closed formulas for the on-shell matrix elements of $T_{csll'}$ the phase parameters $\delta_{cs}(^3P_2)$, $\delta_{cs}(^3F_2)$, and $\epsilon_{csJ=2}$ could be calculated exactly in a coupled-channel approach including the Coulomb-distortion effect to all orders in the fine-structure constant. To our knowledge this has not been done before; only an approximate description of the Coulomb distortion for such phase parameters was proposed by Fröhlich *et al.*[8] The parameters of the potential were obtained by a least-squares fit to the data of Arndt *et al.*[9]; their numerical values are given in Table I.

Our results are shown in Figs. 1 and 2 in comparison with the latest phenomenological data.[9,10] With regard to the phase shift $\delta(^3P_2)$ there is a marked improvement as compared with our previous work,[1] where this state was treated as uncoupled. The phase shift $\delta(^3F_2)$ and the mixing parameter ϵ_2 are also reproduced satisfactorily. In order to improve the fit even further we would have needed an addi-

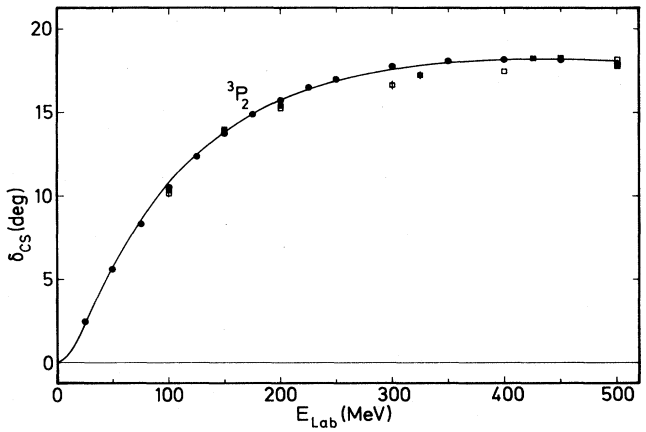

FIG. 1. Separable-potential results for $^3P_2$ Coulomb-modified nuclear phase shifts compared with predictions of phase-shift analyses by Arndt and VerWest (Ref. 9) (● energy-dependent, ■ energy-independent solution) and Arndt *et al.* (Ref. 10) (□ energy-independent solution).

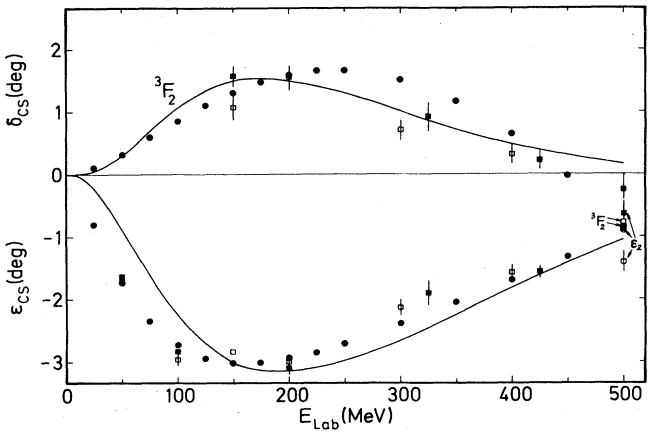

FIG. 2. Same as Fig. 1 for $\delta_{cs}(^3F_2)$ and ϵ_{cs2}.

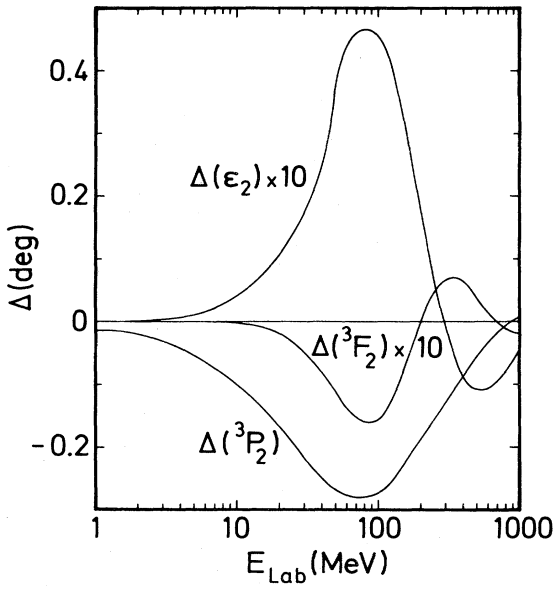

FIG. 3. Coulomb-distortion effects $\Delta=\delta_{cs}-\delta_s$ or $\epsilon_{cs}-\epsilon_s$.

tional term in $^3F_2$ to take into account the short-range repulsion, which is claimed by theoretical[11] and phenomenological[9,10] analyses. However, in view of the discrepancies that still exist in the phenomenological data we rather preferred to stay with the simple form of Eqs. (16) and (17). This allows a better applicability of the potential in few-particle calculations. Our fit is certainly sufficient in view of the relative insignificance of higher partial waves in such applications.

We stress that the mixing parameter ϵ_2 is appreciably different from zero and must not be disregarded. Notice that at least at low energies ($E_{\rm lab}\lesssim 200$ MeV) it is comparable in size with ϵ_1, the mixing parameter in the coupled n-p state $^3S_1$-$^3D_1$. Thus spin-dependent p-p forces turn out to be rather strong and should be included in the $^3P_2$-$^3F_2$ state, even if one only considers $^3P_2$ and neglects, say, $l>2$ partial waves.[12] For the case of separable potentials our formalism provides a rigorous and practical approach to this problem: We have constructed a separable parametrization for the $^3P_2$ state, which takes into account both the Coulomb-distortion effect and the coupling of this partial wave to $^3F_2$.

In Fig. 3 the Coulomb-distortion effect for the various phase parameters is demonstrated. Plotted are the quantities $\Delta=\delta_{csl}-\delta_{sl}$ and $\epsilon_{cs}-\epsilon_s$. In accordance with earlier findings for $l\geq 2$ partial waves[1,6] this effect turns out to be negligible for $\delta(^3F_2)$ and ϵ_2. It is still small for $\delta(^3P_2)$, but should probably be taken into account in connection with few-body applications.

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