

Effect of core polarization on the charge distribution in ²⁰⁸Pb

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The coefficients of Fourier-Bessel expansion of charge distribution in ²⁰⁸Pb are determined from the single proton wave functions derived from two sets of optimized Woods-Saxon potentials. The set of potentials optimized with respect to the observed single particle states in ²⁰⁸Pb give excellent agreement with the model independent analysis of the charge distribution. But the potentials obtained from core-polarization corrected single particle states give very poor charge distribution.

[NUCLEAR STRUCTURE ²⁰⁸Pb charge distribution. Deduced effect of core polarization.]

From electron scattering data detailed model independent analyses of the charge distribution in ²⁰⁸Pb are currently available.¹ The charge distribution is expressed in terms of the coefficients for a Fourier-Bessel expansion of charge:

$$\rho(r) = Z \sum_{\nu=1}^N a_{\nu} j_0(q_{\nu} r) \text{ for } r \leq R$$

$$= 0 \text{ for } r > R$$

with $q_{\nu} = \nu\pi/R$. For ²⁰⁸Pb the cutoff radius R is 12.0 fm. Extremely accurate values of the first 11 coefficients a_{ν} are given in Ref. 1. Any single particle model of the nucleus should try to reproduce these coefficients. This will be the severest test of the model. Simultaneously, one should also reproduce the observed single particle spectra near the Fermi surface as obtained from the stripping reaction. Such work has been done recently by Batty and Friedman² for the charge distribution in ²⁰⁸Pb. They conclude that the local Woods-Saxon potential of Rost³ gives excellent fit to the charge distribution in ²⁰⁸Pb. They also showed that the non-local potential gives a very poor charge distribution. This apparent failure of the nonlocal potential to give the correct charge distribution in ²⁰⁸Pb has been ascribed by Batty and Friedman as due to the core-polarization effect.

Recent reaction data^{4,5} have indicated the extent of core-polarization effect on the single particle states of ²⁰⁸Pb. From the analysis of the observed distribution of the single particle strengths in terms of the weak coupling of the particle motion with the collective vibrational states of ²⁰⁸Pb, we can estimate the extent of the core-polarization effect in the single particle spectra. The resulting core-polarization corrected single particle spectra can be utilized to optimize

the parameters of the Woods-Saxon potential. This has been recently done by Mukherjee, Krishan, and Banerjee.⁶ It is the purpose of the present work to examine the correctness of the proton wave functions as given by the revised potential. As already indicated this is best done by calculating the 11 coefficients a_{ν} of the Fourier-Bessel expansion of the charge distribution using the proton wave functions.

The local potential taken by us is

$$V(r) = V_c(r) - V_0 f_0(r) + V_s \frac{1}{r} \frac{d}{dr} f_{s0}(r) \bar{L} \cdot \bar{S} ,$$

where

$$f(r) = \left[1 + \exp \left(\frac{r - r_0(A-1)^{1/3}}{a} \right) \right]^{-1}$$

and V_c is the Coulomb potential.

Apart from the potential obtained by Mukherjee, Krishan, and Banerjee in Ref. 6, which we call Pot. A, we give here two more optimized potentials—Pot. B and Pot. C, the parameters of which are listed in Table I. In Pot. B $f_0(r) = f_{s0}(r)$, while in Pot. C $f_0(r)$ and $f_{s0}(r)$ are taken from the analysis of the 65 MeV polarized proton elastic scattering data of the RCNP group at Osaka.⁷ Also listed in Table I are the potential parameters of Rost³ and those of Batty and Greenless.⁸

The bound state wave functions of the protons as given by the above potentials are added up to get the point proton charge distribution

$$\rho_{\text{point}}(r) = \sum_j (2j+1) \psi_j^2(r) ,$$

where the sum is over all occupied j states in ²⁰⁸Pb. The ac-

TABLE I. Woods-Saxon potential parameters for ²⁰⁸Pb.

	V_0	r_0	a_0	V_s	r_s	a_s	r_c
Pot. A	64.619	1.1843	0.6400	32.510	1.1363	0.7850	1.1843
Pot. B	64.230	1.1930	0.6518	39.727	1.1930	0.6518	1.1930
Pot. C	62.928	1.2226	0.7461	31.33	1.1755	0.6103	1.2226
Rost ^a	58.70	1.275	0.700	23.08	0.932	0.700	1.275
Batty and Greenless ^b	59.67	1.277	0.754	23.480	1.055	0.700	1.200

^aReference 3.

^bReference 8.

TABLE II. The coefficient $a_\nu (\times 10^{-3})$ of the Fourier-Bessel expansion of charge distribution in ^{208}Pb .

$\times 10^{-3}$	Expt. ^a	Pot. A	Pot. B	Pot. C	Rost ^b	Batty and Greenless ^c
a_1	0.633 31	0.658	0.655	0.646	0.629	0.635
a_2	0.619 07	0.794	0.771	0.714	0.598	0.653
a_3	-0.483 44	-0.357	-0.369	-0.394	-0.480	-0.387
a_4	-0.344 09	-0.525	-0.489	-0.416	-0.305	-0.285
a_5	0.352 60	0.268	0.284	0.300	0.370	0.284
a_6	0.120 61	0.296	0.264	0.195	0.110	0.019
a_7	-0.175 45	-0.148	-0.161	-0.166	-0.195	-0.210
a_8	-0.011 98	-0.138	-0.118	-0.073	-0.019	0.034
a_9	0.089 85	0.071	0.079	0.083	0.098	0.155
a_{10}	0.010 04	0.082	0.073	0.052	0.028	0.045
a_{11}	-0.018 07	-0.085	-0.011	-0.010	-0.015	-0.031

^aReference 1.^bReference 3.^cReference 8.

tual charge distribution $\rho(r)$ is obtained by folding the proton charge distribution with this point charge distribution. The 11 coefficients a_1 to a_{11} are then determined, which are listed in Table II along with the coefficients obtained from electron scattering data.¹

From Table II it is evident that only the Rost potential gives the correct estimate of the Fourier-Bessel coefficients, whereas the set of potentials A, B, and C, derived from the core-polarization corrected single particle states in ^{208}Pb , fail to give these coefficients correctly. Out of the three potentials obtained by us, Pot. C, corresponding to the polarized proton data, gives a somewhat better result. We conclude that the Rost potential is still the best local potential so far

as the charge distribution of ^{208}Pb is concerned, but the potential does not give the correct proton spectra as obtained after the core-polarization correction. On the other hand, the potentials obtained by us to reproduce the single particle spectra fail to give the proper charge distribution in ^{208}Pb . It is now important to understand why the Rost potential, which has been derived by optimizing the binding energies with respect to the observed proton binding energies in ^{208}Pb , gives such an excellent charge distribution.

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