PHYSICAL REVIEW C VOLUME 28, NUMBER 3 SEPTEMBER 1983

Reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ and the axial current form factor in the timelike region

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(Received 24 February 1983)

The differential muon-capture rate $d\Gamma/dE_T$ is obtained for the reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ over the allowed range of E_T, the tritium energy, for two assumptions concerning the behavior of F_A, the axial current form factor, in the timelike region; analytic continuation from the spacelike region and mirror behavior, $F_A(q^2, \text{timelike}) = F_A(q^2, \text{spacelike})$. The values of $d\Gamma/dE_T$ under these two assumptions are found to vary substantially in the timelike region as a function of the mass M_A in the dipole fit to F_A. Values of $d\Gamma/dE_T$ are given for $M_A{}^2 = 2m_\pi{}^2$, $4.95m_\pi{}^2$, and $8m_\pi{}^2$.

[NUCLEAR REACTIONS Muon capture ${}^6\text{Li}(\mu^-, \nu_\mu){}^3\text{H}{}^3\text{H}$, Γ, $d\Gamma/dE_T$ calculated for two assumptions concerning the axial current form factor behavior in timelike region.]

There has recently been some interest in the behavior of nuclear form factors in the timelike region.[1] In order to study this question we had undertaken calculations[2] for the capture rate and differential capture rate $d\Gamma/dE_n$ for the reaction $\mu^- + {}^2\text{H} \to \nu_\mu + n + n$. We had found a 10% difference in the extreme timelike region for the value of $d\Gamma/dE_n$ under two different[1] assumptions for the behavior of the axial current form factor F_A in the timelike region.

In this report we undertake a similar calculation for the reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$. This process has not yet been experimentally observed; however, it has been proposed[3] as a possible experiment to reduce the limit on the muon neutrino mass.[4–6] It has a number of potential advantages over the corresponding process in deuterium. The final tritium state being charged should be more easily observed. Furthermore, the axial current mass M_A, which occurs in the dipole fit to the axial current form factor F_A, used here,

$$F_A(q^2) = f_A(0)/(1 - q^2/M_A{}^2)^2 \ , \tag{1a}$$

with

$$F_A(q^2, Q\cdot L, P\cdot L) = F_A(Q\cdot L, P\cdot L) f_A(q^2) \ , \tag{1b}$$

is much smaller than that in the deuterium case. This, as will be noted, induces a much larger difference in the results for $d\Gamma/dE_T$ between the two assumptions made here for the behavior of F_A in the timelike region.

The matrix element for the reaction $\mu^- + {}^6\text{Li} \to {}^3\text{H} + {}^3\text{H} + \nu_\mu$ is kinematically[5,6] equivalent to the deuterium case because ${}^6\text{Li}$ is a 1^+, $I=0$ particle and the tritons are $\frac{1}{2}^+$, $I=\frac{1}{2}$, $I_z = -\frac{1}{2}$ particles analogous to the ${}^2\text{H} \to n + n$ case. Writing the weak hadronic current as $J_\mu = V_\mu - A_\mu$, the matrix element for this reaction to the lowest order in G, the weak coupling constant, is given by

$$\langle {}^3\text{H}, {}^3\text{H}, \nu_\mu | H_W(0) | {}^6\text{Li}, \mu^- \rangle = \frac{G}{\sqrt{2}} \cos\theta_C \bar{u}_\nu \gamma^\lambda (1-\gamma_5) u_\mu \langle {}^3\text{H}{}^3\text{H} | J_\lambda^\dagger(0) | {}^6\text{Li} \rangle \ , \tag{2}$$

with[5,6]

$$\langle {}^3\text{H}{}^3\text{H} | V_\lambda^\dagger(0) | {}^6\text{Li} \rangle = \eta \bar{u}(P_1) \left[\frac{F_1}{M_L{}^2} \epsilon_{\lambda\nu\rho\sigma} \xi^\nu Q^\rho L^\sigma + \frac{F_2}{M_L{}^2} \epsilon_{\nu\rho\sigma\lambda} \gamma^\nu \xi^\rho q^\sigma \right] \gamma_5 v(P_2) \ , \tag{3a}$$

$$\langle {}^3\text{H}{}^3\text{H} | A_\lambda^\dagger(0) | {}^6\text{Li} \rangle = \eta \bar{u}(P_1) \left[F_A \xi_\lambda + \frac{F_P \xi\cdot Q q_\lambda}{M_L{}^2} \right] \gamma_5 v(P_2) \ , \tag{3b}$$

where $\eta = [M_T/(E_1E_2)^{1/2}](\pi)^{-1/2}(2L_0)^{-1/2}$, M_T and M_L are the triton and ${}^6\text{Li}$ mass, respectively, L_μ is the ${}^6\text{Li}$ four-momentum, E_1 and E_2 are the triton energies, and P_{1_μ} and P_{2_μ} are the triton four-momenta, respectively. Furthermore, ξ_μ is the ${}^6\text{Li}$ polarization vector and

$$Q_\mu = P_{1_\mu} + P_{2_\mu} \ , \quad q_\mu = P_{1_\mu} + P_{2_\mu} - L_\mu \ , \quad P_\mu = P_{1_\mu} - P_{2_\mu} \ . \tag{4}$$

The form factors F_1, F_2, F_A, and F_P which determine the matrix elements, Eqs. (3a) and (3b), are not as well known as the deuterium form factors. We use the results obtained in Refs. 5 and 6, noting that they produced total muon-capture rates in reasonable agreement with an impulse approximation based calculation.[4]

The results for F_1, F_2, and F_A are[5,6]

$$F_i(Q\cdot L,P\cdot L,q^2)\simeq F(Q\cdot L,P\cdot L)f_i(q^2),\quad i=1,2,A,P\ , \tag{5}$$

$$|F(Q\cdot L,P\cdot L)|^2=\{1-0.33\exp[-9.59\times10^{-2}(q_0-20)^2]\}\times\left\{\frac{20.84+2.01\exp[-1.589\times10^{-3}(q_0-95)^2]-0.357q_0\exp[-6.01\times10^{-5}(q_0-16.5)^2]}{(q_0-16.5)^2+12.04}\right\}\ ,\quad 0\leq q_0\leq 75.659\text{ MeV} \tag{6a}$$

$$|F(Q\cdot L,P\cdot L)|^2=1.15\times10^{-5}(105.659-q_0)\ ,\quad 76.659\text{ MeV}\leq q_0\leq 105.659\text{ MeV}\ , \tag{6b}$$

with

$$f_A(q^2)=f_A(0)/(1-q^2/M_A{}^2)^2\ , \tag{7a}$$

$$M_A=4.95m_\pi{}^2,\quad f_A(0)=0.296\ ,$$

and

$$|f_1(q^2)-f_2(q^2)|=|f_1(0)-f_2(0)|/(1-q^2/M_V{}^2)^2\ , \tag{7b}$$

with $M_V\simeq4.95m_\pi{}^2$ and $|f_1(0)-f_2(0)|\simeq4.49$, and where all parameters are compatible with energy and momentum in units of MeV. These values were obtained from data on the reactions[7] $\pi^-+{}^6\text{Li}\rightarrow{}^3\text{H}+{}^3\text{H}$, $\gamma+{}^6\text{Li}\rightarrow{}^3\text{H}+{}^3\text{H}$, and ${}^3\text{H}+{}^3\text{H}\rightarrow\gamma+{}^6\text{Li}$ by the use of the partially conserved axial vector current (PCAC) and conserved vector current (CVC) as well as arguments[5] based on the structure of ^{6}Li. We use a relationship derived by Nambu,[8]

$$F_P=-M_L{}^2F_A/(q^2-m_\pi{}^2)\ , \tag{8}$$

to obtain a value for the pseudoscalar form factor from the axial form factor.

Using the following expression for the differential muon-capture rate,

$$d\Gamma=\frac{C_{\text{Li}}M_T{}^2m_\nu m_\mu|\psi(0)|^2|M|^2d^3\nu d^3P_1d^3P_2}{2M_L(2\pi)^5\nu E_1E_2}\delta^4[P_{1_\mu}+P_{2_\mu}+\nu_\mu-(L_\mu+\mu_\mu)]\ , \tag{9}$$

we obtain both Γ, the total capture rate, and $d\Gamma/dE_T$, the differential capture rate, as a function of triton energy from the matrix element squared,[9] which is now completely determined. For the process $\mu^-+{}^6\text{Li}\rightarrow{}^3\text{H}+{}^3\text{H}+\nu_\mu$,

$$|M|^2=\frac{2}{6m_\mu M_T{}^2}\left[F_A(m_\mu+M_L)(m_\mu+M_L-\nu)\left\{3m_\mu\nu+\frac{2m_\mu{}^2\nu^2}{m_\mu{}^2-2m_\mu\nu-m_\pi{}^2}+\frac{\nu^3m_\mu{}^3}{(m_\mu{}^2-2\nu m_\mu-m_\pi{}^2)^2}\right\}+(F_1-F_2)^2m_\mu\nu^3\right]\ , \tag{10}$$

and [10] $C_{\text{Li}}=0.928$, which takes into account the spread in charge of the ^{6}Li nucleus.

We have actually determined the form factors only in the spacelike region. We consider two different assumptions for the behavior of F_A in the timelike region. The first, which we shall call assumption I, is ordinary analytic continuation of Eq. (7a), which is most commonly done. The second, which was suggested by the work of Ref. 1, and which we shall call assumption II, is that $F_A(q'^2,q'^2\text{ timelike})=F_A(q^2,q^2\text{ spacelike})$, where $q'^2=-q^2$.

The total muon-capture rates, Γ, obtained are

$$\Gamma=104.9\text{ sec}^{-1},\quad\text{assumption I}\ , \tag{11a}$$

$$\Gamma=104.8\text{ sec}^{-1},\quad\text{assumption II}\ . \tag{11b}$$

It is clear that such a difference is well beyond present experimental distinction for muon-capture measurements in nuclei.

The situation for the differential muon-capture rate is substantially more interesting. If we use the value for $M_A{}^2=4.95m_\pi{}^2$ obtained in Ref. 5 and set, for example, $E_T=2850$ MeV, which is dominated by timelike q^2, we find that assumption I yields a value for $d\Gamma/dE_T$, a factor of 2 greater than that yielded using assumption I.

Calling

$$\alpha=(d\Gamma/dE_T,\text{ assumption I})/(d\Gamma/dE_T,\text{ assumption II})\ , \tag{12}$$

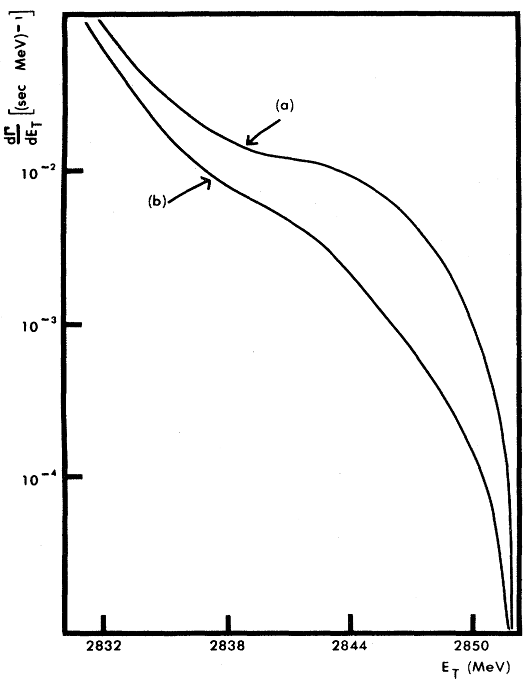

FIG. 1. Plot of the differential muon-capture rate as a function of triton energy for large triton energy and $M_A{}^2 = 2m_\pi{}^2$. Curve (a) is the rate under the assumption of analytic continuation for F_A (assumption I of the text). Curve (b) is the rate assuming $F_A(q^2$, timelike$) = F_A(q'^2$, spacelike) with $q'^2 = -q^2$ (assumption II of the text).

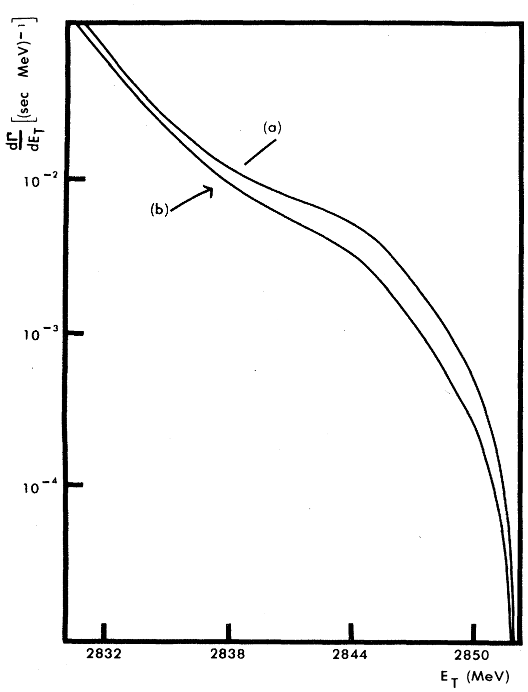

FIG. 2. Plot of the differential muon-capture rate as a function of triton energy for large triton energy and $M_A{}^2 = 4.95m_\pi{}^2$. Curve (a) is the rate, under the assumption of analytic continuation for F_A (assumption I of the text). Curve (b) is the rate assuming $F_A(q^2$, timelike$) = F_A(q'^2$, spacelike) with $q'^2 = -q^2$ (assumption II of the text).

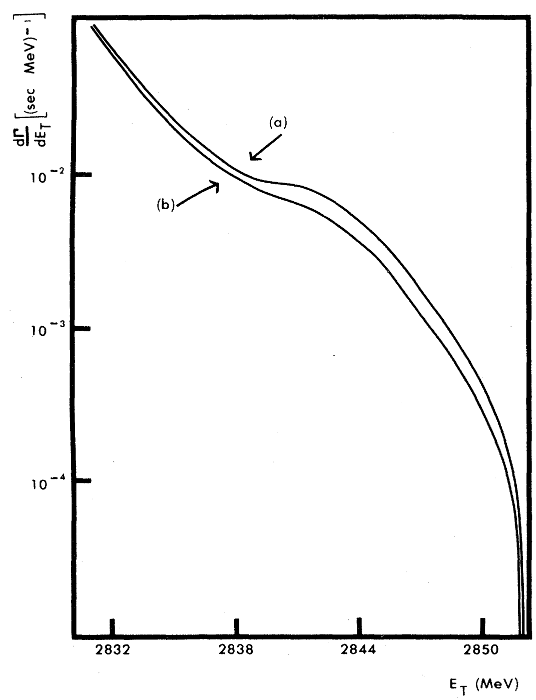

FIG. 3. Plot of the differential muon-capture rate as a function of the triton energy for large triton energy and $M_A{}^2 = 8m_\pi{}^2$. Curve (a) is the rate under the assumption of analytic continuation for F_A (assumption I of the text). Curve (b) is the rate assuming $F_A(q^2$, timelike$) = F_A(q'^2$, spacelike) with $q'^2 = -q^2$ (assumption II of the text).

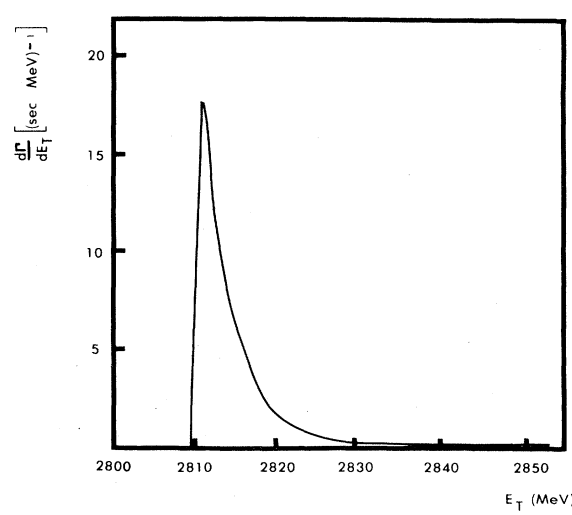

FIG. 4. Plot of the differential muon-capture rate as a function of triton energy. The two assumptions for the behavior of F_A in the timelike region are not distinguishable here.

we find

$$\alpha = 6.1, \quad M_A{}^2 = 2m_\pi{}^2 \ ,$$

$$\alpha = 2.0, \quad M_A{}^2 = 4.95m_\pi{}^2 \ ,$$

$$\alpha = 1.44, \quad M_A = 8m_\pi{}^2 \ ,$$

for $E_T = 2850$ MeV. Other values for $d\Gamma/dE_T$ for these three different values of $M_A{}^2$ are given in Figs. 1, 2, and 3. In Fig. 4 we show $d\Gamma/dE_T$, $M_A{}^2 = 4.95m_\pi{}^2$ over its entire range. In this figure, assumptions I and II cannot be distinguished.

The reaction $\mu^- + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^3\text{H} + \nu_\mu$ thus offers the possibility of distinguishing between assumptions I and II in the timelike region, the values of α being substantially greater than in the deuterium case. As in the case of deuterium it would be very useful to determine experimentally $d\Gamma/dE_T$ over its entire range. The form factors could then be fit from the low E_T region data which is dominated by spacelike q^2. This result could then be analytically continued to the timelike region and the predicted $d\Gamma/dE_T$ checked from the high E_T region data dominated by timelike q^2. In any case, as expected, a small value for M_A induces large changes in $d\Gamma/dE$ between the two assumptions, and thus it is an important asset in an experiment to study form factor behavior in the timelike region.

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