K^-p bound states with coupling to hyperon channels

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(Received 6 December 1982)

The states of kaonic hydrogen bound by the combined Coulomb plus nuclear potentials are determined exactly in momentum space and a study is made of the dominant physics. Couplings to the $\Sigma\pi$, $\bar{K}^0 n$, and $\Lambda\pi$ channels are included with different models for the strong potentials and different wave equations. The existence of both open and closed channels demands an extension of the Kwon, Tabakin, Landé treatment of the Coulomb singularity. Model studies indicate a high sensitivity to channel coupling and relativistic kinematics. The calculated K^-p atomic shift and width agree in magnitude with recent experiments, but the theoretical shift is towards the less bound whereas the experimental shifts appear more bound.

[NUCLEAR REACTIONS $(K^-,K^-),(K^-,\pi),(K^-,\bar{K}^0)$, $E=-(0.8$ keV—20 MeV); Γ hadronic atom, momentum space, coupled channels, exact Coulomb, nonlocal potentials, relativistic kinematics.]

I. INTRODUCTION

In this paper we study the bound K^-p system, kaonic hydrogen. Since the low energy $\bar{K}N$ scattering data are fit most conveniently with a nonlocal separable potential model, and since we use that same model for the bound states, it is simplest to solve the relevant integral Schrödinger equations in momentum space. Since even zero energy K^-p couples strongly to the stable strangeness -1 baryons, with some in the continuum,

$$K^-p \rightarrow \begin{cases} K^-p + 0 \text{ MeV}, & (1.1a) \\ \bar{K}^0 n - 5 \text{ MeV}, & (1.1b) \\ Y_0^*(1405) + 35 \text{ MeV}, & (1.1c) \\ \quad \rightarrow \Sigma\pi + 100 \text{ MeV}, & (1.1d) \\ \Lambda\pi + 180 \text{ MeV}, & (1.1e) \end{cases}$$

the equations are coupled. And since the K^-p atom is bound predominantly by the Coulomb force, we employ a new procedure[1] to solve in momentum space the combined Coulomb plus nuclear problem for coupled bound and continuum states.

Whereas the variety of physics present in this exotic hydrogen system may make it appear complicated, it also produces much interesting structure with something of interest for everyone. Firstly, it is the simplest kaonic atom and understanding this problem may further the understanding of the more complicated K^--nucleus atoms, where the strong absorption channels (1.1c)—(1.1e) are dominant. Secondly, one of the ways to investigate the subthreshold Y_0^* (1405) resonance, (1.1c), is by studying K^-p bound states. (Since the structure of this Y_0^* is uncertain,[2–5] determining whether it truly is an unstable $\bar{K}N$ bound state embedded in the $\Sigma\pi$ continuum is important.) And finally, the exact calculation of coupled Coulomb, nuclear, and open channel states is a challenge to theoretical and computational physics.

In discussing the strong interactions shift ϵ and width Γ of a Coulomb level, we adopt the Warsaw convention[6]:

$$\Delta E = -\epsilon - i\Gamma/2 , \tag{1.2}$$

so that a positive shift, $\epsilon > 0$, describes a state more bound by the strong interaction and $\epsilon < 0$, less bound. To avoid confusion, we call potentials—but not shifts—repulsive or attractive. This distinction is important since it is related to one of the most interesting phenomena to occur in kaonic atoms, namely, the Krell oscillations.[7] This is a nonmonotonic dependence of ϵ and Γ upon the depth of the strongly absorptive optical potential, U, which occurs when the absorption is too strong for perturbation theory to be correct. The real parts of ΔE and U then can have opposite signs, the imaginary part of the energy, Γ, can decrease as $|\mathrm{Im}U|$ increases, and, as found in kaonic atom fits, the optical potential produces negative shifts even though

its real part is attractive.

Although these interesting effects have been known for some time, they have recently been called into question by the experimental discovery[8–10] that the $1S$ level in kaonic hydrogen does not follow the trend of all other nuclei. In all other nuclei the levels are shifted upwards to less bound ($\epsilon<0$) positions, whereas K^-p appears to be more bound than a pure Coulomb level. Furthermore, the expected proportionality of the shift ΔE and the scattering length for this elementary system lead to Re $a(K^-p)<0$—in disagreement with all analyses of scattering data. Although the extreme difficulty of these experiments suggests some caution in their interpretation,[11] their interest and importance makes further study appealing.

One possible explanation of the hydrogen mystery is that the Trueman or Deser formula[12] which directly relates the level shift to the (Coulomb corrected) complex scattering length and Bohr radius,

$$\frac{\Delta E_n}{E_n}\simeq-\frac{4}{n}\frac{a}{R_B}\ , \tag{1.3}$$

may not be sufficiently accurate. Deloff and Law[13] have argued that the Coulomb corrections may change "a" entirely from its strong interaction value, but the change depends so sensitively upon the strong interaction model that no conclusion can be drawn. Other calculations of this sort, e.g., those of Stepién-Rudzka and Wycech,[13] have not found overwhelming corrections.

A step towards performing more exact calculations for kaonic hydrogen was recently taken by Barrett,[14] who included Coulomb corrections by using potentials to calculate the $1S$ level. For the strong interaction he used some of the coupled channel separable potentials fit by Henley, Alberg, and Wilets[15] (HAW). To simulate the Coulomb interaction, Barrett added in another nonlocal separable potential with range and strength adjusted so that by itself this potential gives the correct pure Coulomb binding energy of the $1S$ state. Again, in disagreement with experiment, the $1S$ state is less bound than a pure Coulomb state. No further conclusions were drawn.

A possible cause of discrepancy between the K^-p experiment and the pseudopotential formula (1.3) is the uncertainty in the value of the scattering length. Firstly, the number of low energy data is low. Secondly, the scattering amplitude, which is what should be used in (1.3), is highly energy dependent,[4,5,15] and must be extrapolated from fits to medium energy scattering data (200–300 MeV) to a subthreshold value near -8613 eV. Typical values[5,16] of the scattering lengths at threshold are

$$a(K^-p)=-\lim_{k\to0}(k\cot\delta)^{-1} \tag{1.4}$$

$$=(a_0+a_1)/2 \tag{1.5}$$

$$=\begin{cases}0.89-i\,0.62\ \text{fm (von Hippel and Kim)} & \text{(1.6a)}\\ 0.66-i\,0.71\ \text{fm (Martin)}\ . & \text{(1.6b)}\end{cases}$$

At present these experimental uncertainties do not appear large enough to support a complete sign reversal for Re $a(K^-p)$, and the subthreshold extrapolation for hydrogen is so small that there is little change in the amplitude.[15,16] (Because of binding and Fermi motion effects, the extrapolation for a heavy nucleus is much larger, ≈25 MeV, and a sign reversal occurs.)

The calculation we report upon here is our first in a study of coupled bound and continuum "eigenstates." It is similar in spirit to the HAW[15] and Barrett[14] calculations. It differs in being a general momentum space calculation with no restriction to separable potentials, and in its exact treatment of the Coulomb force for open and closed channels via an extension[1] of the Kwon, Tabakin, and Landé procedure.[17] We thus can calculate all the states for hydrogen (not just $1S$) and extend the formalism to heavy nuclei where the strong potentials are no longer separable. In Sec. II we describe the theoretical formulation, in Sec. III we present some results of a model study, and in Sec. IV we give our conclusions. Although we are keenly interested in understanding the experimental results, our goal at this stage is weighted more towards understanding the dominant physics and developing the needed theoretical tools.

II. THEORY

A. Strong interaction channels and potentials

We wish to apply a potential model to describe the bound and scattering states for the $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ channels, (1.1). In an isospin basis these states are

$$|K^-p\rangle=-\sqrt{1/2}\,|00\rangle+\sqrt{1/2}\,|10\rangle \quad : 1, \tag{2.1}$$

$$|\Sigma^+\pi^-\rangle=\sqrt{1/6}\,|20\rangle'+\sqrt{1/2}\,|10\rangle'+\sqrt{1/3}\,|00\rangle' : 2a, \tag{2.2}$$

$$|\Sigma^-\pi^+\rangle=\sqrt{1/6}\,|20\rangle'-\sqrt{1/2}\,|10\rangle'+\sqrt{1/3}\,|00\rangle': \text{ 2b}, \tag{2.3}$$

$$|\Sigma^0\pi^0\rangle=\sqrt{2/3}\,|20\rangle'+0\qquad-\sqrt{1/3}\,|00\rangle': \text{ 2c}, \tag{2.4}$$

$$|\bar{K}^0 n\rangle=\sqrt{1/2}\,|00\rangle+\sqrt{1/2}\,|10\rangle \quad : 3, \tag{2.5}$$

$$|\Lambda^0\pi^0\rangle=|10\rangle'' \quad : 4, \tag{2.6}$$

where 1–4 refer to the channel numbers. The channel numbers will be used as subscripts later. Since we always start in the K^-p channel, isospin conservation forbids transitions to the $I=2$ pieces of the $\Sigma\pi$ state, and so we drop them. To keep the number of channels (and computing) relatively low in this initial study, and to make use of potentials already in the literature,[15,18] we truncate our space in two ways. First, we completely eliminate the $\Lambda\pi$ channel, 4, and account for transitions to this pure $I=1$ state by making the $I=1$ piece of our resulting potentials complex (since we must deal with complex matrices anyway to describe coupling to open channels this is a savings). Second, we follow the approach of Ref. 15 in considering the three $\Sigma\pi$ channels as one effective, pure $I=0$ channel,

$$|\Sigma^\pm\pi^\mp\rangle\simeq|\Sigma^0\pi^0\rangle=|00\rangle': \text{ 2}. \tag{2.7}$$

The three channels are thus

$$K^-p\rightarrow\begin{cases}K^-p\text{:1},\\ \Sigma\pi\ \text{:2},\\ \bar{K}^0 n\ \text{:3}.\end{cases} \tag{2.8}$$

If we assume isospin conservation, the potentials in and between the $\bar{K}N$ and $\Sigma\pi$ multiplets are

$$\langle K^-p\,|\,V\,|\,K^-p\rangle=(V^1+V^0)/2\,, \tag{2.9}$$

$$\langle\bar{K}^0 n\,|\,V\,|\,\bar{K}^0 n\rangle=(V^1+V^0)/2 \tag{2.10a}$$

$$=V_{33}\,, \tag{2.10b}$$

$$\langle\bar{K}^0 n\,|\,V\,|\,K^-p\rangle=(V^1-V^0)/2 \tag{2.11a}$$

$$=V_{13}=V_{31}\,, \tag{2.11b}$$

$$\langle\Sigma\pi\,|\,V\,|\,\Sigma\pi\rangle=V^{0\prime}_{22}=V_{22}\,, \tag{2.12}$$

$$\langle K^-p\,|\,V\,|\,\Sigma\pi\rangle=-\sqrt{1/2}V^{0\prime} \tag{2.13a}$$

$$=V_{12}=V_{21}\,, \tag{2.13b}$$

$$\langle\bar{K}^0 n\,|\,V\,|\,\Sigma\pi\rangle=\sqrt{1/2}V^{0\prime} \tag{2.14a}$$

$$=-V_{12}\,, \tag{2.14b}$$

$$=V_{32}=V_{23}\,. \tag{2.14c}$$

We assume the potentials have the Legendre polynomial expansion

$$\langle\vec{p}\,|\,V\,|\,\vec{p}\,'\rangle=\int d^3r\,d^3r'\frac{e^{-i\vec{p}\cdot\vec{r}}}{(2\pi)^{3/2}}\langle\vec{r}\,|\,V\,|\,\vec{r}\,'\rangle \times\frac{e^{i\vec{p}\,'\cdot\vec{r}\,'}}{(2\pi)^{3/2}} \tag{2.15}$$

$$=\frac{1}{2\pi^2}\sum_{L=0}^{\infty}(2L+1)V_L(p\,|\,p')\times P_L(\cos\theta_{pp'})\,, \tag{2.16}$$

TABLE I. $\bar{K}N$, $\Sigma\pi$ separable potential parameters ($L=0$). Subscripts: $1=K^-p$, $2=\Sigma\pi$, and $3=\bar{K}^0 n$.

Set	$I=0$[a] λ^0 (10^4 MeV^2)[b]	β^0 (MeV)	$I=1$ ($\bar{K}N$) λ^1 (10^4 MeV^2)	β^1 (MeV)
HAW4B	$\lambda_{11}=-35.8$ $(-326\beta^0)$ $\lambda_{12}=-12.3$ $(-112\beta^0)$	1097 (0.18 fm)$^{-1}$	$-7.66-i3.45$ $(-388-i175$ MeV fm$^{-1})$	789 (0.25 fm)$^{-1}$
HAW2C	$\lambda_{22}=-45.2$ $(-412\beta^0)$	1097	$-0.608-i0.422$ $(-30.8-i22.4$ MeV fm$^{-1})$	395 (0.5 fm)$^{-1}$
SGK[b]	$-39.5-i3.62$ $(-\pi/2(6.45+i0.5928)$ fm$^{-2})$	1096 (0.18 fm)$^{-1}$	$-23.58-i18.97$ $(-\pi/2(3.855+i3.101)$ fm$^{-2})$	1096 (0.18 fm)$^{-1}$
SGB	$-476-i20.1$ $(\pi/2$ $(77.89$ $+i3.289)$ fm$^{-2})$	2582 (0.076 fm)$^{-1}$	$-336-i101$ $(-\pi/2(54.94+i16.55)$ fm$^{-2})$	2582 (13.08 fm^{-1})
V^{Coul}(RCB)	-1.2×10^{-6}	2.4 (84 fm)$^{-1}$		

[a]The $I=0$ parameters are identical for both HAW sets, $I=1$ differ.

[b]$\lambda=\frac{\pi}{2}\lambda' sg/(2\pi)^3$.

and restrict ourselves here only to $L=0$. For the potential coupling channels i and j $(=1,2,3)$ we assume the simple separable form:

$$V_{ij}^I(p\,|\,p')=\frac{4\pi\lambda_{ij}^I}{(\beta_I{}^2+p^2)(\beta_I{}^2+p'^2)}\ . \tag{2.17}$$

In the future we plan both to use the $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ potential fits of Toker *et al.*,[2] and fit the reaction data with more general forms than (2.17).

The strengths and ranges for the potential models used are given in Table I [converted to the conventions of (2.17)]. The HAW (Ref. 15) potentials used are from their recent fit to the actual scattering data for $p_k<250$ MeV/c, and to the scattering length of Martin[16] (which has the opposite sign to Kim's[16] for $I=1$). The HAW4B and HAW2C potentials include K^-p, $\bar{K}^0n$, and $\Sigma\pi$, but differ in their value for the inverse range β (4 and 2 fm^{-1}) and in their complex λ for the $I=1$ potentials. We do not consider their fit A since its complex range can lead to unphysical results.

The other two potentials are Schick and Gibson's[18] "single channel" fit to the K^- matrix analyses of Kim (SGK) and Berley *et al.* (SGB).[16] (In our charge basis, the single channel means two, K^-p and $\bar{K}^0n$.) Although the SG potentials are one step further removed from the data than HAW's fits, and do not reproduce all the effective ranges, they permit us to see the effect of truncating the $\Sigma\pi$ channel and to compare the Kim and Berley *et al.* analyses (Kim's produces a larger scattering cross section). In any case, the potential has large uncertainties arising from the poor quality and quantity of the experimental data, and it seems valuable to gauge the model dependence of our answers by trying several models.

We can see from Table I that the $\bar{K}N$ potential is attractive in all channels (Re$\lambda<0$) and stronger in $I=0$ than $I=1$. As Dover and Walker[5] indicate, this is due to the coherent exchange of ϵ and ω mesons reinforcing the isovector ρ exchange. In fact, the HAW $I=0$ potential, V_{11}^0, produces a strong bound state at -4.4 MeV, although the linear combination (2.9) is not strong enough until the coupled channels are included. It is also interesting to note that in Table I the fits with a longer range $1/\beta$ compensate by having a decreased strength λ. The $1S$ equivalent Coulomb potential of Barrett,[14] $V^{\rm Coul}$(RCB) in turn, is some 500 times longer in range than the strong potentials—but $\sim 10^5$ times weaker.

B. Coulomb interaction and coupled channels

To include the Coulomb force we use charge basis states with experimental particle masses and add the Coulomb potential V^c in channel 1 (Ref. 1):

$$V_{11}=\langle K^-p\,|\,V\,|\,K^-p\rangle+V^c\ , \tag{2.18}$$

$$\langle\vec{\rm p}\,|\,V^c\,|\,\vec{\rm p}'\rangle=-\frac{Ze^2}{2\pi^2}\frac{1}{|\,\vec{\rm p}-\vec{\rm p}'\,|^2}\ , \tag{2.19}$$

$$V_{L=0}^c(p\,|\,p')=-\frac{Ze^2}{2pp'}Q_0(z_{pp'})$$

$$=-\frac{Ze^2}{2pp'}\ln\left|\frac{p+p'}{p-p'}\right|\ , \tag{2.20}$$

$$z_{pp'}=(p^2+p'^2)/2pp'\ . \tag{2.21}$$

Although both the use of experimental masses and the inclusion of V^c break isospin symmetry, our approach is dictated by the presence of Coulomb bound states in channel 1 (we ignore Coulomb effects in other channels).

The logarithmic singularity of the Coulomb potential (2.20) at $p=p'$ makes a direct numerical solution of the Schrödinger equation difficult. Kwon and Tabakin[17] showed how to obtain a nonsingular equation for the pure bound state problem, whereas Landau[1] extended this technique to the coupled continuum plus bound state problem. In the latter work a Green's function technique was used to include the pure outgoing wave boundary condition in the open channel. Since there is no incident wave in either the open or closed channel, the Lippmann-Schwinger equation takes the form

$$[\Psi]=[F][\Psi]\ , \tag{2.22}$$

$$[\Psi]=\begin{bmatrix}\Psi_1\\ \Psi_2\\ \Psi_3\end{bmatrix}\ ,$$

$$[F]=\begin{bmatrix}G_1V_{11} & G_1V_{12} & G_1V_{13}\\ G_2V_{21} & G_2V_{22} & G_2V_{23}\\ G_3V_{31} & G_3V_{32} & G_3V_{33}\end{bmatrix}\ . \tag{2.23}$$

The technique requires Green's functions defined for real E by

$$\langle p\,|\,G_i\,|\,p\rangle=(E+\Delta M_{1i}+i\epsilon-p^2/2\mu_i)^{-1}\ , \tag{2.24}$$

which are continued into the complex plane after the ϵ limit has been carried out. μ_i is the reduced mass in channel i, e.g.,

$$\mu_1=m_{K^-}m_p/(m_{K^-}+m_p)\ , \tag{2.25}$$

and ΔM_{1i} the difference in total mass, e.g.,

$$\Delta M_{12}=(m_{K^-}+m_p)-(m_\Sigma+m_\pi)\ . \tag{2.26}$$

The "eigenenergies" (since the bound states are in a continuum, the energy is really continuous) follow from the condition for a nontrivial solution of the discretized form of (2.22),

$$\det[1-F]=0\ . \tag{2.27}$$

The Kwon, Tabakin, and Landé procedure[17] requires that the integrals be replaced by sums over a discrete grid and that the Coulomb part of V_{11} contains a "correction" term along the diagonal which removes the singularity, i.e.,

$$V^c_{11}(p_m\,|\,p_m)=\frac{-1}{W_M}\left[\frac{Ze^2\pi^2}{4}+\sum_{j\neq m}^{N}\frac{V^c_0(p_m\,|\,p_j)}{P_0(z_{jm})}\right], \tag{2.28}$$

where W_M is an integration weight.

Since G_2, (2.24), is a complex function of p if channel 2 is open, (2.27) produces complex eigenenergies even for real potentials.

In addition, as we search to obtain our complex E we also produce a complex channel momentum k_2, the on-shell momentum for the strong potential in channel 2, (2.26). If we then evaluate the on-shell potential term in channel 2 (the $i\epsilon$ part) at this complex momentum, we are analytically continuing our solution into the complex plane and simultaneously requiring a self-consistent complex solution. Since the Coulomb binding energy, ~ 9 keV, is very small compared to the mass difference, ~ 100 MeV, this is a small continuation for our problem and causes no noticeable change in the "eigenvalue." The effect on strongly bound states, however, may be more significant.

III. RESULTS

A. Accuracy: Single channel bound states

Good accuracy with momentum space techniques requires an appropriate choice of grid points.[17,1] We determined our best grid by considering the single channel problem where analytic solutions make comparisons easy. First, we found the grid which produced the most accurate $1S$ binding energies for the pure Coulomb K^-p potential (-8613 eV), and then found the grid which produced the most accurate binding energies for the pure strong potential $\mathrm{Re}V\,(I=0)$ (-4.40, -19.8, and -38.0 MeV for the HAW, SGK, and SGB potentials, respectively). The two grids were then combined for the Coulomb plus nuclear problem. Note that since only $V(I=0)/2$ contributes to $\langle K^-p\,|\,V\,|\,K^-p\rangle$, (2.9), there is *no* strong bound state in the single channel problem (there is for coupled channels).

B. Model study

Since two of the goals of this work are to understand the physics of the K^-p system and to test our technique, we describe some theoretical experiments in which we varied the parameters at our disposal. To start, we performed a computer experiment in which we took the potentials and varied the strength λ of the coupling to the $\bar{K}^0n$, $\Sigma\pi$, and $\Lambda\pi$ channels ($\Lambda\pi$ indirectly by varying $\mathrm{Im}V^{I=1}$). The result is shown in Fig. 1. $\lambda=0$ is the pure single channel with real potentials, whereas $\lambda=1$ is the full coupling which reproduces scattering data. We see that for $\lambda\simeq 0$ there are large positive (more bound) shifts ϵ, but very small widths Γ for the K^-p $1S$ and $2S$ states. This is as expected for an attractive real potential. However, as the coupling increases to $\lambda\simeq 0.65$, the shifts pass through zero and the widths simultaneously attain large maxima. Further increase in the coupling actually decreases the width, and beyond $\lambda\simeq 0.8$ the state becomes *more* bound as the coupling increases. As λ increases still further no additional sign changes occur, and ϵ remains negative. The behavior shown in Fig. 1 is both an indication of a high sensitivity to the channel coupling, and a version of the Krell oscillations[7] for coupled channels.

For couplings below the crossover point our numerical search procedure found no strongly bound

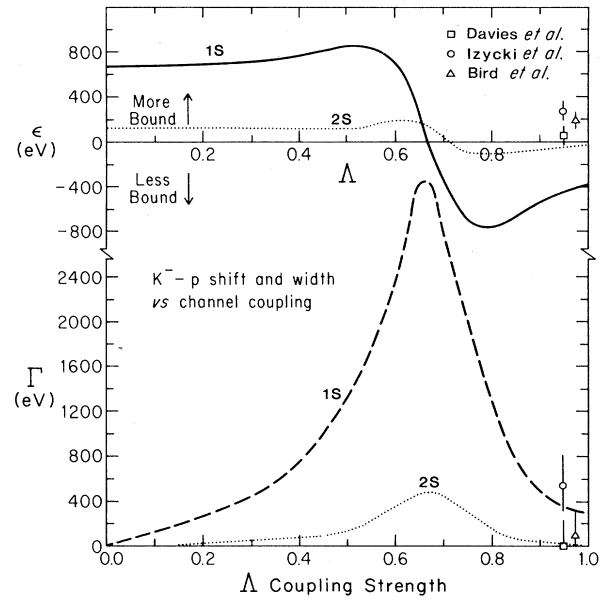

FIG. 1. The shift ϵ and width Γ of the $1S$ and $2S$ K^--hydrogen levels as a function of λ, the coupling strength to the $\Sigma\pi$, $\bar{K}^0n$, and $\Lambda\pi$ channels. $\lambda=1$ is the HAW2C potential. The data are from Davies *et al.* (Ref. 8) (boxes), Izycki *et al.* (Ref. 9) (circles), and Bird *et al.* (Ref. 10) (triangles).

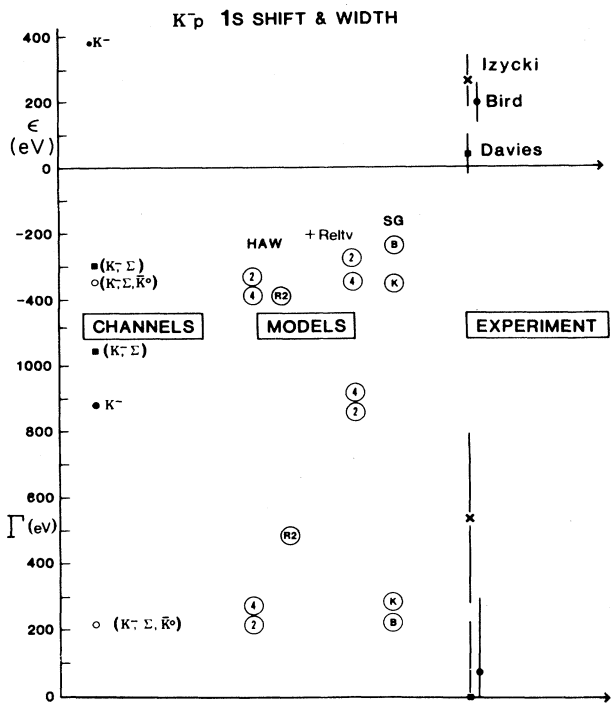

FIG. 2. The shift ϵ and width Γ of the K^-p $1S$ level. The effect of coupling different channels (with otherwise the same potentials) is shown by the filled circles, filled squares, and open circles. Different potential models (see text) are indicated by the 2 HAW2C, 4 HAW4B, *B* SGB, and *K* SGK. *R*2 represents the results obtained with the HAW2C potential if the pseudo-Coulomb approximation of Ref. 14 is used. Reltv indicates how the computed shifts change when relativistic kinematics are used. The data are from Refs. 8—10.

state (Y_0^*) below the K^-p threshold which also satisfied our imposed boundary conditions. Above the crossover we find a deeply bound, strong (inner) state and many Coulomb (outer) states. At $\lambda \simeq 0.65$ both have the same energy and we have a level splitting or crossing effect.[20] Since our simple K^-p separable potential (2.17) produces only one bound state, there are no further oscillations. This leaves the sign of ϵ negative and, like the scattering length, with a value which can also be obtained with a weakly "repulsive" potential. In future studies we plan to study simultaneously the above threshold scattering solutions.

The hadronic bound state we calculate with HAW2C has binding energy $E_h = -(16.4,\ 11.1i)$ MeV for nonrelativistic kinematics or $E_h = -(14.1,\ 9.4i)$ MeV for relativistic kinematics

$$\bar{K} = (m^2 + k^2)^{1/2} .$$

Although our strong K^-p state is not pure $I=0$, it is hard to keep from identifying this state with the Y_0^* which has binding energy[4,5]

$$E(Y_0^*) = -(10-30, 10i - i27)\ \text{MeV} .$$

Our state is a K^-p bound state in the $\Sigma\pi$ continuum[21] and is thus unstable.[2-4]

In Fig. 2 and Table II we display some of the model dependence of our calculated ϵ and Γ. The experimental findings of Davies *et al.*,[8] Izycki *et al.*,[9] and Bird *et al.*[10] that ϵ_{1S} is more bound is in disagreement with other K^--atom shifts,[6,7] and our calculations. In the left-hand portion of Fig. 2 we present another "theoretical" experiment in which the HAW2C potential is used with different combinations of channels turned on and off (again we note that only the results with all three channels included are realistic). When only the K^- channel is used (filled circles), the shift is positive (more bound) and the width large. As the other channels are switched on (filled squares and open circles), ϵ enters the less bound region ($\epsilon < 0$), whereas Γ ultimately assumes $\sim \frac{1}{4}$ of its former value. If we turn

TABLE II. K^-p level shifts in eV. $E^{\text{Coul}}(1S,2S) = -(8613,2153)$.

Potential	Channels	ϵ_{1S}	Γ_{1S}	ϵ_{2S}	Γ_{2S}	ϵ_{1S}	Γ_{1S}	ϵ_{2S}	Γ_{2S}
		Nonrelativistic				Relativistic			
HAW4B	$K^-,\bar{K}^0,\Sigma$	−383	270	−41	5	−341	924	−38	107
HAW2C	$K^-,\bar{K}^0,\Sigma$	−328	223	−34	0	−269	873	−28	98
SG1	$\bar{K},\bar{K}^0$	−350	258	−33	3	−147	126	−19	10
SG2	$\bar{K},\bar{K}^0$	−228	222	−13	3	−58	37	−6	4
Expt. (Ref. 8)	K^-p	40 (±60)	0 (+230)						
Expt. (Ref. 9)	K^-p	264 (±76)	544 (±256)						
Expt. (Ref. 10)	K^-p	200 (±60)	80 (+220 −80)						

off the absorption into the $I=1$, $\Lambda\pi$ channel by making the potentials pure real, the $1S$ level becomes more bound but its width larger, another example of strong absorption/coupling acting like a repulsion. Note, that although the potentials are all real in this case, the eigenenergies are complex because of channel coupling.

C. Potential model and wave equation dependence

In Table II and Fig. 2 the 2's, 4's, B's and K's represent the range of values scanned for (ϵ,Γ) using the four potential models of Table I. (Table II also presents results for the $2S$ state.) Whereas ϵ_{1S} varies significantly with these models, it always remains less bound; our calculated Γ's, on the other hand, fall within the range of the data, not a great achievement considering the range of the data. It is interesting to note that the Schick, Gibson-Kim potential with only the K^- and $\bar{K}^0$ channels gives results similar to the three channel (K^-, K^0, $\bar{\Sigma}$) HAW potentials. The SG-BEA two channel potential, which has a much smaller $I=0$ scattering length, produces smaller $(|\epsilon|,\Gamma)$.

We also note in Fig. 2 (point $R2$) and Table III, the equivalent Coulomb calculation of Ref. 14. The differences between our calculations are larger in the coupled channels case than in the single channel case; this is probably caused by the high sensitivity of these results to the channel coupling and the different treatments of the $i\epsilon$ prescription.

In the calculations described up until now we have used the Schrödinger equation with nonrelativistic kinematics. However, since we are in momentum space we can change the definition of our Green's function (2.24) to

$$\langle p\,|\,G_i\,|\,p\rangle=-1m_i+m_i'+E-i\epsilon -(m_i^2+p^2)^{1/2}-(m_i'^2+p^2)^{1/2}\,, \tag{3.1}$$

and obtain a Schrödinger equation with relativistic kinematics. Since there is an energy release of ~ 100 MeV in the $\Sigma\pi$ channel, the pion is always relativistic and (3.1) is preferred; yet since it is somewhat inconsistent to use this equation with potentials from a nonrelativistic Schrödinger equation, we do it only to gauge the size of the effect. As we can see in Fig. 2 (+ Reltv) and Table II, the width is greatly increased by relativistic corrections. This is a reflection of the significant changes (25%) in the channel 2 momentum, $k_2\simeq 177$ MeV/c, and the importance of this momentum in determining the strength of the coupling to channel 2. In fact, we have found that just using relativistic[15] or nonrelativistic definitions of the reduced mass in channel 2,

$$\mu_2(\mathrm{NR})=m_\pi m_\Sigma/M_2\simeq 125\ \mathrm{MeV}\,, \tag{3.2}$$

$$\mu(\mathrm{REL})=(M_1+M_2)[M_1^2-(m_\Sigma-m_\pi)^2]/8M_1^2 \approx 158\ \mathrm{MeV}\,, \tag{3.3}$$

$$M_1=m_K+m_p,\quad M_2=m_\Sigma+m_\pi\,, \tag{3.4}$$

in an otherwise nonrelativistic calculation, can produce similar changes.

D. Comparison with the pseudopotential formula

Since the energy level shifts are small compared to the binding energy, it seems obvious that some type of perturbation theory should be valid (although since the potentials are large, $V/E\gg 1$). Indeed, for pionic atoms the pseudopotential Deser[12] formula (here for S states)

$$\frac{\Delta E_n}{E_n}\simeq-\frac{4}{n}\frac{a}{R_B} \tag{3.5}$$

has proven to be quite accurate.[12] In (3.5), ΔE_n $(=-\epsilon-i\Gamma/2)$ is the shift of the S state, E_n is the energy of the Coulomb bound state (principal quantum number n), R_B is the Born radius, and a is the hadron-nucleus scattering length. For the K^-p system we take

$$R_B=P_B^{-1}=\hbar^2/Ze^2\mu=83.6\ \mathrm{fm}\,, \tag{3.6}$$

$$E_n=-\mu Z^2e^4/2\hbar^2n^2=-8613\ \mathrm{eV}/n^2\,, \tag{3.7}$$

$$\mu=m_{K^-}m_p/(m_{K^-}+m_p)=323.48\ \mathrm{MeV}\,, \tag{3.8}$$

TABLE III. Shift of $1S$ level. Comparison with pseudo-Coulomb potential.

	V_c(RCB)[a]		V_c(RCB) via RHL		V_c^{exact}	
	ϵ	Γ	ϵ	Γ	ϵ	Γ
Strong potential	(eV)		(eV)		(eV)	
HAW4B: $K^-,\bar{K}^0,\Sigma$	−417	381	−416	350	−383	270
HAW2C: $K^-,\bar{K}^0,\Sigma$	−386	488	−364	301	−328	223

[a]Reference 14. A recent corrigendum (personal communication) changes shifts and widths to (−393,659) eV.

$$a(K^-p)\equiv(a_0+a_1)/2 \tag{3.9}$$

$$=(0.892-0.582i)\text{ fm: SGK}, \tag{3.10a}$$

$$=(0.483-0.389i)\text{ fm: SGB}, \tag{3.10b}$$

where values of a_0 and a_1 in (3.10) are from the pure strong SGB/SGK potentials of Table I. Values of $a(K^-p)$ obtained directly from fitting scattering data are given by (1.6).

If the scattering length in (3.5) has Re $a>0$, then ReΔE is >0 and ϵ is <0. Since all K^-p models give Re $a>0$ we would expect $\epsilon<0$, in agreement with our calculations but not with experiment. Conversely, since K^--nucleus levels have $\Delta E>0$, (3.5) implies Re $a(K^-A)>0$. Yet since K^--N and K^--A potentials are attractive, this sign reversal from that of a weak attraction implies strong K^--A bound states (if there were no absorption). While this cannot as yet be ruled out, it is also known that (3.5) requires some corrections.

Trueman,[12] Stepién-Rudzka and Wycech,[13] and others have indicated that the scattering length in (3.5) should be corrected for the presence of the Coulomb field and for the difference in energy between the bound state E_B (K) and the zero energy scattering.[22] These corrections have the form[13]

$$\frac{\Delta E_n}{E_n}\simeq\frac{-4a_c/nR_B}{1+3.154a_cR_B}, \tag{3.11}$$

$$a_c^{-1}\simeq a^{-1}-\frac{2}{R_B}\left[\frac{1}{aK}+\ln\frac{4}{KR_B}+0.5777\right], \tag{3.12}$$

$$E_n=-K^2/2\mu, \tag{3.13}$$

and amount to only a few percent for K^-p.

Kumar *et al.*,[23] in turn, have examined Deloff and Law's[13] suggestion that an anomalously large Coulomb effect may exist in K^- hydrogen. They conclude, however, that such an effect would cause a serious conflict with the scattering data above threshold.

The question still remains as to how valid this pseudopotential formula is for the K^-p problem, where there are coupled channels and nonlocal potentials. [The obvious answer is that (3.5) is perfectly valid if the right a is used.] We have tried to answer this question by comparing the results of Schrödinger equation solutions with (3.5) for the different potential models and wave equations. Since other uncertainties are much larger than the corrections given in Eqs. (3.11)—(3.13), we examine only the simpler (3.5) here. The difficulty is that a meaningful comparison requires "a" to be calculated in a coupled channels formalism consistent with the one used for the bound state. Unfortunately, since we used a charge basis with different channel masses, it is no longer true that

$$a(K^-p)=(a_0+a_1)/2,$$

and we need a coupled channels scattering code to determine the correct $a(K^-p)$ to use in (3.5). Although we are still working on a scattering analysis, for present purposes we must content ourselves with Table IV, where we calculate the shift (3.5) with $(a_0+a_1)/2$ and compare with computed eigenenergies. We do, however, give in row 3 the value of (ϵ,Γ) calculated with a value of $a(k^-p)$ determined in a preliminary coupled channels calculation by Barrett.[19]

We find $\gtrsim 85\%$ agreement with our eigenenergy calculation for the shift ϵ, but the computed Γ's are $\sim 50\%$ too small. Since the width is directly related to the strength of the $K^-p\to\Sigma\pi$ coupling, and this in turn is a sensitive function of the channel masses, channel energies, and relativistic kinematics, the disagreement is somewhat understandable. However, firm conclusions must await further calculations.

IV. CONCLUSION

We have solved the Coulomb plus nuclear potential problem exactly in momentum space for the coupled K^-p, $\Sigma\pi$, $\bar{K}^0n$, and $\Lambda\pi$ systems. Since

TABLE IV. Comparison of scattering length formula and computed eigenenergies.

Model	Channels	$\frac{a_0+a_1}{2}$ (fm)	$(\epsilon,\Gamma)_{1S}$ Eq. (3.11) (eV)	$(\epsilon,\Gamma)_{1S}$ Computed (eV)	Method
SGK	$K^-,\bar{K}^0$,	$0.892-i0.582$	$-368{,}480$	$-350{,}258$	NR
SGB	$K^-,\bar{K}^0$	$0.483-i0.389$	$-199{,}320$	$-288{,}222$	NR
HAW2C	$K^-,\bar{K}^0,\Sigma$	$0.995-i0.801$[a]	$-394{,}660$	$-328{,}223$	NR
				$-269{,}873$	REL

[a] K^-p scattering length calculated by Barrett (Ref. 19) for coupled channels with experimental masses.

there are both open and closed channels, we extended the Kwon, Tabakin, and Landé procedure to the Lippmann-Schwinger equation. The unbalanced ranges and strengths of the Coulomb and nuclear potentials make this a difficult problem, but our coupled channels results do display the behavior expected for strong absorption.

We have found that an accurate description of the K^-p system requires a careful treatment of channel coupling, relativistic kinematics, and very general forms of the potentials, requirements most easily met in momentum space. On the technical side, the development of this exact procedure represents progress in the reliable extraction of two body interaction information from exotic hydrogens.

Although it may sound like a ritualistic chant by now,[2,4,5,24] our understanding of the $\bar{K}N$ and $\bar{K}A$ systems is still severely limited by the poor quality and limited quantity of $\bar{K}N$ data. In particular, a different approach to experimental measurements of kaonic hydrogen seems worthwhile.[11] If, however, the value of the $1S$ shift in present experiments[8–10] remains, then a major change in the low energy $\bar{K}N$ analysis seems needed; e.g., a simultaneous fit of $\bar{K}N$, $\Sigma\pi$, and $\Lambda\pi$ reactions and atomic bound state data. Our present work still indicates that the positive ϵ_{1S} for K^-p (more bound shift) is incompatible with the existing potential fits. In order for other potentials to produce a positive shift, however, they need to produce a second strong state (or none).

Some of the questions in our K^-p study may be answered by a future study of the K^-He atom where the experimental identification of the K^- line is more certain. We are presently working at applying some of the techniques developed for the π-He problem to the K-He problem.

ACKNOWLEDGMENTS

It is a pleasure to thank Dr. F. Tabakin, Dr. V. Mandelzweig, Dr. R. C. Johnson, Dr. R. C. Barrett, Dr. A. Gal, and Dr. E. Schmid for penetrating and helpful discussions. I particularly wish to thank Dr. Barrett for his cooperation in running his computer program to generate intermediate results with which to check, and Prof. E. Schmid and Prof. V. Madsen for a critical reading of the manuscript. Some of this research was conducted while I was on sabbatical leave at the University of Surrey, and I would like to thank the Physics Department there for their hospitality and assistance. Shorter visits at the Weizmann Institute of Science and the Hebrew University of Jerusalem also provided rare and valuable opportunities. Financial support from the Science and Engineering Research Council (U.K.), Oregon State University, and the National Science Foundation (U.S.A.) has been essential and is gratefully acknowledged.

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