PHYSICAL REVIEW C VOLUME 28, NUMBER 3 SEPTEMBER 1983

Theory for a hybrid quark-baryon model for the nucleon-nucleon system

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(Received 23 May 1983)

A hybrid quark model for two baryons is developed. The short distance part is represented by six-quark wave functions, while the long distance part is described by conventional wave functions representing the relative motion of three-quark baryons. Formulae which give the probabilities of the six-quark configurations as a function of energy, in terms of the exterior NN interactions and experimental phase shifts, are derived and applied.

NUCLEAR REACTIONS Quark structure of nuclei, hybrid quark-baryon model of two baryons. Derivation of interior quark amplitudes from phase shifts and exterior forces. Numerical values for interior six quark probabilities for $0 < E < 800$ MeV nucleon-nucleon states.

I. INTRODUCTION

In recent years there has been a great surge of interest in models of the nucleon which incorporate the underlying quark structure. Such models have stressed the spectroscopic and static properties of the nucleon and its excited states. They have been extended to the structure of the deuteron as seen in electron scattering at high momentum transfer[1] and to the nucleon-nucleon scattering problem,[2] and to the study of high momentum electron scattering from He.[1,3] In many of these cases bags are used to contain the quarks.

The model we develop and propose herein also contains some of the above features. It makes use of theoretical characteristics which have been shown to be valid experimentally. Thus, there is good evidence that the long range force between nucleons is mediated by single pion exchange; for instance, the scattering amplitude in high partial waves in nucleon-nucleon scattering is totally determined by the one-pion exchange force. On the other hand there is increasing evidence that the short distance hadronic force, such as that between nucleons, is given by perturbative quantum chromodynamics (QCD). It is the intermediate range nucleon-nucleon force, say, between 0.3—1.5 fm which is most difficult to characterize and to handle in a quantitative manner. Meson exchanges and phenomenological models have been used to characterize this region, but additional short range features, e.g., soft or hard cores, are often required. In QCD nonlinear and nonperturbative effects are known to be important; confinement of the quarks must be one of the features of the theory. Models which describe the N-N force in this region generally require many parameters.

In order to develop a model which bridges the gap between the short-distance perturbative QCD region and the long-range pion exchange force without a multitude of parameters, we adopt the point of view that there are two connected regions of space: the first region is an "external" one in which the baryons remain undeformed (unpolarized, in the classical sense), and therefore the dynamics can be determined by conventional nuclear potentials for which the one pion exchange is an important part; the second region is an internal one determined by quark-gluon dynamics. The quark states are represented by a complete set of states confined to the internal region. The connection between these regions is described below. It is akin to the R-matrix description of the compound nucleus in nuclear reactions.

We restrict ourselves to the nonrelativistic energy region for nucleons, although the model is readily generalized. If one wishes to represent the quark states by definite wave functions, this can be done, but it is not required. A model which makes use of such wave functions connected to pions on the outside was introduced by one of us (Kisslinger[4]) and is related to the MIT bag model[5] and to the cloudy bag model.[6] This model of two-baryon systems has been used for the study of nuclear Λ decay,[7] pion absorption,[8] and weak p-p asymmetry.[9]

Our motivation in the model described below is to see how much can be learned about quark wave functions from our knowledge of the asymptotic behavior of scattering amplitudes *without* introducing specific models for the dynamics of the quarks and with minimal assumptions. Of course, specific models, such as the MIT bag wave functions can be introduced for the quarks, but this specification is not necessary for our model.

Our work is related to other approaches, but is different from them. Unlike many authors,[1,3] we do not restrict ourselves to large momentum transfers and asymptotic conditions, but rather consider arbitrary momentum transfers at nonrelativistic nucleon energies. Nor do we develop or attempt to develop nuclear forces from QCD.[2] Rather we attempt to develop information on the (inner) quark region which is adequate to make contact with experiment and for further developments.

In Sec. II, below, we derive and introduce the main features of our hybrid model. In Sec. III we give a simple application of the model. In Sec. IV a realistic application to the two-nucleon system is developed. In Sec. V some examples of generalizations are developed. A brief summary appears in Sec. VI.

II. HYBRID MODEL: THEORY

The hybrid model[4] is based on a coordinate space representation of nuclear systems: there are external regions in which separated baryons are represented as color singlets interaction via forces arising from the exchange of color singlet objects (mainly pions), and internal regions where the quarks associated with two or more baryons interact with full color freedom. In the present work it is the relative motion of the two nucleons, considered as two systems of three quarks, which is our primary concern. When the two nucleons overlap appreciably this relative separation between two color singlet configurations can still be defined.[2] The dynamics of the internal motion for separated nucleons is of secondary concern for us and is not treated specifically.

The objective of the present work is to derive *global* information about the interior NN six-quark wave functions in each channel by using our extensive knowledge of the exterior NN interaction and experimental NN phase shifts. A specific quark effective Hamiltonian is not assumed. The ultimate goal of the model is to provide a complete microscopic description of the hadronic and quark structure of nuclei within the limits of quark models with fixed numbers of degrees of freedom. For this one needs the Hamiltonian in the internal as well as the external region, and wave functions in each of the quark variables must satisfy continuity conditions. Although here we do not attempt such an amibitous goal, it will be seen that we make progress sufficient to give the present theory some predictive power for a number of short range processes involving weak, electromagnetic, and even strong interactions. However, the model is incomplete, and at the present stage should be thought of as a framework which can be developed into a complete theory.

The basic theoretical problem being studied in the present work is the matching of the internal to the external wave functions within the confines described in the previous paragraph. The method makes use only of the continuity of the baryonic four-current, so that it can be applied for relativistic as well as nonrelativistic models. There is a long history of nuclear and particle reaction models which involve matching at a boundary. Although we do not attempt a review of this vast subject, some relations between our present model and other theoretical approaches are given in the Appendix.

For simplicity the external and internal regions are delimited by a sphere of radius r_0. Note, however, that deformed many-body systems can be represented by configurations of particles with wave functions defined in a sphere. Using projection operators defined by the relative coordinate r, the wave functions for two (three-quark) nucleons in the c.m. system can be written as

$$\Psi_E^{NN}=\begin{cases}\widetilde{\Psi}_E=\mathscr{A}\phi_a(\vec{r}_1-\vec{r}_2,\vec{r}_2-\vec{r}_3)\phi_b(\vec{r}_4-\vec{r}_5,\vec{r}_5-\vec{r}_6)\psi_E(\vec{r}), & r\geq r_0\\ \Phi_E^{6q}(\vec{r}_1,\ldots,\vec{r}_6), & r\leq r_0\ ,\end{cases} \tag{2.1}$$

where $\mathscr{A}$ is an antisymmetrization operator, ϕ_a (ϕ_b) are the wave functions of the quarks inside nucleon a (b), and ψ_E is the conventional wave function which describes the dynamics of the relative motion of the two nucleons. For $r<r_0$, Φ_E^{6q} describes the complicated dynamics of six (or more) quarks. Here, we neglect all but valence quarks. The coordinates $r_1 \cdots r_6$ should exclude the c.m. coordinate, as for the outside wave function; these coordinates can be chosen so that the separation between two three-quark color singlet configurations is one of the dynamical variables. (If shell model wave functions are used inside the region r_0, then the c.m. is not treated correctly and projection methods or other techniques are required to correct for the defect.) Although we do not specify ϕ_a (ϕ_b) it should be noted that the nucleon confining region need not have a radius r_0. These internal quark wave functions must readjust when the two nucleons overlap appreciably. This readjustment is ignored herein. In Eq. (2.1) color, spin, and isospin labels are not shown explicitly but are assumed to be present.

For the discussion which follows it is convenient to define the projection operators $P_>, P_<$ by

$$P_>\Psi_E^{NN}=\phi_a\phi_b\psi_E(\vec{r})\ ,\qquad P_<\Psi_E^{NN}=\Phi_E^{6q} \tag{2.2}$$

with the notation of Eq. (2.1). In terms of these projection operators one can define an outside and an inside Hamiltonian,

$$H^>=P^>HP^>\ , \tag{2.3a}$$

$$H^<=P^<HP^<\ . \tag{2.3b}$$

The outside Hamiltonian consists of quark interactions within the nucleons and an external NN potential. The inside Hamiltonian in a bag model would typically consist of, say, single-quark Dirac Hamiltonians and qq hyperfine potentials. In some models there can also appear terms in the Hamiltonian which couple the inside and outside sectors,

$$H^{><}=P^>HP^<\ , \tag{2.4a}$$

$$H^{<>}=P^<HP^>\ . \tag{2.4b}$$

In any case, the stationary solutions (2.1) are obtained by

$$H\Psi_E^{NN}=E\Psi_E^{NN}\ . \tag{2.5}$$

The general form of the inside wave function (2.1) is

$$\Phi_E^{6q}=\sum_n c_n(E)\Phi_n(r_1\cdots r_6)\ , \tag{2.6}$$

where the Φ_n is an orthonormal set of energy-independent six-quark wave functions. A detailed microscopic treatment would lead to an evaluation of the spectroscopic amplitudes $c_n(E)$. In previous work on electron scattering[4] and the lifetime of the Λ in nuclei[7] attempts have been made to extract some information about these quantities by comparison with experiment. Note that in a complete microscopic description the wave functions and derivatives of the wave functions would be continuous (or have well-defined discontinuities) in each of the six quark coordinates at the matching point r_0. Here we do not attempt such a complete description, but seek global information about the inside six-quark wave function. This requires continuity requirements for the baryon current, rather than individual quark currents, which makes the present theory less restrictive than the complete treatments.

We make the assumption introduced by Wigner[10] in his treatment of compound states in nuclear reactions that the inner wave function can be written as

$$\Phi_E^{6q}=a(E)\sum_n c_n\phi_n(r_1\cdots r_6) \tag{2.7}$$

with c_n independent of energy and $\sum_n |c_n|^2=1$. Thus, the development we carry out is akin to that of treating the internal region like a compound nucleus with many degrees of freedom. The expansion (2.7) is particularly sensible near an isolated resonance of the internal region. Since resonances of six-quark systems are well separated in energy from the low-energy N-N system, this energy domain can be quite large. The assumption which underlies (2.7) can be relaxed and generalized, but only at the expense of including more parameters, as in Eq. (2.6). The coefficient $a(E)$ is proportional to the lifetime[10] of the "compound" state described by Eq. (2.6). We shall see that the restriction we make is not too limiting. If there is an appreciable "mismatch" between the internal ($r<r_0$) region and the external one, then considerable reflection occurs at $r=r_0$ and this may account for the (hard or soft) repulsive core required in normal descriptions of N-N scattering. Near an isolated resonance, this core should then be absent since there is appreciable penetration into the internal region in that instance. If a single quark configuration dominates a resonance, then an R-matrix (or P-matrix[11] representation is particularly useful, and there is a direct relationship between the quark configuration and the isolated resonance (see the Appendix).

If phase shifts are known, then it is feasible to use standard NN potentials to obtain $\psi(r)$ for all radii $\geq r_0$; if r_0 is sufficiently large then the external potential is dominated by one pion exchange. We now show that the knowledge of $\psi(r)$, the external wave function, at $r=r_0$ allows us to determine $a(E)$ or $\sum_n |c_n(E)|^2$, without knowledge of the internal (quark) dynamics. The derivation makes use of an extension of the method introduced by Wigner[10] based on continuity relations at the boundary. We use two approaches, the first is based on current conservation and continuity of current, and the second makes use of the Hamiltonian. These are discussed separately in the next two subsections.

A. Current conservation and current continuity

We define a four current for the NN system by

$$\hat{j}_\mu=\begin{cases}\hat{j}_\mu(\text{in})=\sum_{i=1}^{6}\Phi_{E_2}^{6q\,+}\alpha^{(i)}\Phi_{E_1}^{6q},\ \Phi_{E_2}^{+}\Phi_{E_1}, & r<r_0\\[2mm] \hat{j}_\mu(\text{out})=\dfrac{1}{2mi}(\tilde{\psi}^*_{E_2}\vec{\nabla}_r\tilde{\psi}_{E_1}-\tilde{\psi}^*_{E_1}\vec{\nabla}_r\tilde{\psi}_{E_2}),\ \tilde{\psi}^*_{E_2}\tilde{\psi}_{E_1}, & r>r_0\end{cases} \tag{2.8}$$

with use of the notation of Eq. (2.1), and separation of the space-time components of the four-vector current [i.e., $j_\mu=(\vec{\hat{j}},j_0)$]. Note that $\hat{j}$ is a six-quark operator. Here we assume that the quarks within $r<r_0$ satisfy a Dirac equation, while for the external region, $r>r_0$, the quark motion is given by the relative motion of the two nonrelativistic nucleons, as discussed above. The current $\hat{j}_\mu$ can be obtained[10] as the time-dependent part of the probability four-current by using a wave function

$$\psi=\psi_{E_1}e^{-iE_1t}+\psi_{E_2}e^{-iE_2t}\ .$$

Note that, in the limit $E_2\to E_1$,

$$\hat{j}_\mu \underset{E_2\to E_1}{\longrightarrow} j_\mu\ , \tag{2.9}$$

the usual four-current for Dirac particles ($r<r_0$) or for the Schrödinger equation ($r>r_0$).

Our basic assumptions are the following:

(i) current conservation:

$$\sum_\mu\sum_{i=1}^{6}\partial_i^\mu\hat{j}_\mu=0\ , \tag{2.10a}$$

and (ii) current continuity:

$$\hat{j}_\mu\,|_{r_0-\epsilon}=\hat{j}_\mu\,|_{r_0+\epsilon}\ . \tag{2.10b}$$

Integrating Eq. (2.10a) over the volume V_B of a sphere within r_0, one finds

$$\int dV_B\sum_i\vec{\nabla}^{(i)}\cdot\hat{j}(\text{in})=i(E_1-E_2)\int dV_B\hat{j}_0(\text{in})\ . \tag{2.11}$$

Use of the divergence theorem and assumption (ii), Eq. (2.10b), leads to

$$\int dV_B\sum_i\vec{\nabla}^{(i)}\cdot\hat{j}(\text{in}) =\int_{r_0}d\vec{S}_B\cdot\frac{\hbar}{2mi}(\tilde{\psi}^+_{E_2}\vec{\nabla}_r\psi_{E_1}-\tilde{\psi}^+_{E_1}\vec{\nabla}_r\tilde{\psi}_{E_2})\ , \tag{2.12}$$

where

$$d\vec{S}_B = d\xi\, r_0^2 d\hat{r} ,$$

with $d\xi$ the volume element for the four internal coordinates orthogonal to $\vec{r}$. The form of the right-hand side of Eq. (2.12) follows from the assumption that the baryon current for $r > r_0$ arises from interbaryon motion [Eq. (2.8)].

From Eqs. (2.12), (2.11), and (2.8), and using the ansatz (2.7), it follows that

$$\frac{1}{2m}\int d^2\hat{r}\cdot(\psi_2^*\vec{\nabla}\psi_1 - \psi_1\vec{\nabla}\psi_2^*) = (E_2 - E_1)a^*(E_2)a(E_1) . \tag{2.13}$$

Our central result, Eq. (2.13), is that knowledge of the wave function for infinite separation of the two nucleons and of the external NN potential is sufficient to determine $a(E_2)$ at an energy E_2 if it is known at an energy E_1. The magnitude of $a(E)$ can be determined at any and all energies from Eq. (2.13) by considering $E_2 = E_1 + dE$. For this situation Eq. (2.13) reduces to

$$|a(E)|^2 = \frac{1}{2m}\int \left[\frac{\partial \psi_E^*}{\partial E}\vec{\nabla}\psi_E - \psi_E\frac{\partial}{\partial E}\vec{\nabla}\psi_E^*\right]\cdot d^2\hat{r} . \tag{2.14}$$

Note that if (2.6) were used instead of (2.7) the left-hand side of (2.14) would simply be $\sum_n |c_n(E)|^2$. The combination of (2.13) and (2.14) determines $a(E)$ up to a phase. However, this phase is determined by the continuity of the wave function ψ. As $\vec{r}\to\infty$, the phase of each partial wave is given by the appropriate phase shift. As long as the potential for $r > r_0$ is purely real, this phase is unaltered and determines the phase of $a(E)$. See Sec. II C for further discussion.

Our result is a relativsitic generalization of Wigner's nonrelativistic derivation,[10] obtained using Schrödinger currents. In fact, Eq. (2.13), our basic equation, is to be found in Ref. 10. However, our result is more than a relativistic extension, for it involves a particular interpretation of bag models. This will be brought out in Sec. II B where we rederive Eq. (2.13) using the Hamiltonian.

B. Hamiltonian formulation

Consider the internal wave function at two different energies E_1 and E_2. These wave functions are assumed to be solutions of the stationary Hamilton (2.3b),

$$H\Phi_1 = E_1\Phi_1, \tag{2.15a}$$

$$H\Phi_2 = E_2\Phi_2 , \tag{2.15b}$$

where $\Phi_i = \Phi(E_i)$. Multiply (2.15a) by Φ_2^* and the complex conjugate equation to (2.15b) by Φ_i. Integrate the difference between the resulting equations over the volume inside the sphere of radius r_0. We obtain

$$\int dV_B(\Phi_2^* H\Phi_1 - \Phi_1 H\Phi_2^*) = (E_1 - E_2)a^*(E_2)a(E_1) . \tag{2.16}$$

Let us consider the nonrelativistic and relativistic versions for the internal wave functions.

(i) Nonrelativistic cluster model. In this model the quarks in the interior region are treated as color zero clusters, with a Schrödinger wave function prescribing the relative motion of the clusters. This model is implied by the nonrelativistic quark bag treatments of the NN interaction.[2] The interior wave function of Eq. (2.2) becomes (with c_i and d_i indicating three-quark clusters)

$$\Psi_E^{6q} \approx \sum_i \phi_{3q}^{c_i}\phi_{3q}^{d_i}\psi^{(i)}(\vec{r}) , \tag{2.17}$$

which satisfies the Schrödinger equation in the relative coordinate. Equation (2.16) immediately leads to the Wigner result (2.13), for this is nothing but the problem considered by Wigner. Strictly speaking, the ansatz (2.2) is not suitable for this model, and the inner quark wave function would vanish. This was discussed in Ref. 10, and is treated in the Appendix, where some limitations of the allowed forms of (2.17) are discussed. The R-matrix theory[11] was developed for this model of nuclear physics.

(ii) Relativistic cluster model. In relativistic models the inner Hamiltonian is of the form

$$H = \sum_i \vec{\alpha}_i\cdot\vec{p}_i + \sum_{i<j} v_{ij} .$$

One can define a momentum operator $\vec{p}$ conjugate to $\vec{r}$, the relative separation of two three-quark clusters, and a relativistic kinetic energy operator $\tilde{\alpha}\cdot\vec{p}$ with $\tilde{\alpha}$ a linear combination of single-particle Dirac spinors.[12] If the interaction terms are momentum independent, then integration over all variables in the left-hand side of (2.16), except r, with the divergence theorem gives

$$\frac{1}{i}\int (\Phi_2^*\tilde{\alpha}\Phi_1 + \Phi_1\tilde{\alpha}\phi_2^*)\cdot d\vec{S} = (E_1 - E_2)a^*(E_2)a(E_1) . \tag{2.18}$$

Recognizing that the left-hand side of (2.18) is the relative current, and using the continuity of current, one once again obtains Eq. (2.13).

Note that this derivation implicitly implies that $H^{><} = H^{<>} = 0$ [Eq. (2.4)]. Thus there are no surface terms in the Hamiltonian.

C. Six-quark amplitudes and probabilities

The determination of the six-quark amplitudes $a(E)$ clearly does not completely specify the six-quark wave function in the inner region. Indeed, any complete set of ϕ_n which satisfies appropriate boundary conditions can be used in Eq. (2.7). For instance, one could use shell model states as in the MIT bag, but it is then necessary to resolve the center-of-mass problem. To complete the model one either attempts to compare the theory to experiment, as in Refs. 4 and 5, or introduces the $H^{<}$ and possibly the $H^{><}$ Hamiltonians. However, since the quark excitation energies are large, it is expected that in a suitable representation only a few ϕ_n will be important for each channel, and that the ansatz (2.7) is applicable. Therefore, the $a(E)$ are important quantities. We devote the remainder of the paper to their study.

In actual calculations, the wave functions $\psi(r)$ (and ϕ_n) are decomposed into partial waves,

$$\psi(\vec{r})=\sum_{\substack{LSJ\\M_LM_S}}\frac{4\pi i^L}{(2\pi)^{3/2}}\frac{u_{LSJ}(kr)}{kr}\mathscr{Y}_{LSJM}(\vec{r})\langle LM_LSM_S\,|\,JM\rangle Y^*_{LM_L(\hat{k})}\ . \tag{2.19}$$

Here L, M_L, S, M_S, J, and M are orbital, spin and total angular momentum quantum numbers and magnetic quantum numbers, and u is a radial wave function, which is a spherical Bessel function in the absence of interactions. As usual, we can write[13]

$$\mathscr{Y}_{LSJM}(\hat{r})=\sum_{M_LM_S}Y_{LM_L}(\hat{r})\chi_{SM_S}(\vec{\sigma})\langle LM_LSM_S\,|\,JM\rangle\ , \tag{2.19a}$$

where χ is a spin wave function.

Asymptotically, we have, for u_{LSJ},

$$\lim_{r\to\infty}u_{LSJ}(kr)=\sin(kr-\tfrac{1}{2}L\pi+\delta_{LSJ})e^{i\delta_{LSJ}} \tag{2.20}$$

if outgoing boundary conditions are used. Thus the phase of u_{LSJ} and of each partial wave of $\psi(\vec{r})$ is fixed for infinite separation of the two nucleons. This phase remains unchanged for all radii $r\geq r_0$ as long as the potential interaction between the nucleons is real. A similar decomposition of the internal wave functions Φ into the complete set

$$\Phi=\sum_{L,SJ,n}a_{LSJ}(E)c_nR^n_{LSJ}i^L\mathscr{Y}_{LMSJ}(\hat{r})(LM_LSM_S\,|\,JM)Y^{J*}_{LM_L}(\hat{k})\ , \tag{2.21}$$

where R is a radial function, leads to

$$-\frac{1}{2\mu}\frac{4\pi}{(2\pi)^3}\frac{1}{k_1k_2}\frac{1}{r_0^2}\left[u^*_{LSJ}(k_2r)\frac{\partial}{\partial r}u_{LSJ}(r)-u_{LSJ}(k_1r)\frac{\partial}{\partial r}u_{LSJ}(r)\right]=(E_1-E_2)a^*_{LSJ}(E_2)a_{LSJ}(E_1) \tag{2.22}$$

and

$$\frac{1}{2\mu}\frac{4\pi}{(2\pi)^3}\frac{1}{k^2}\frac{1}{r_0^2}\left[\frac{\partial}{\partial E}u^*_{LSJ}\frac{\partial}{\partial r}u_{LSJ}-u_{LSJ}\frac{\partial^2}{\partial E\,\partial r}u^*_{LSJ}\right]=|\,a_{LSJ}(E)\,|^2\ . \tag{2.23}$$

These are our final equations. The phase of $a_{LSJ}(E)$ is determined and given by $\exp(i\delta_{LSJ})$ as long as the internucleon potential is real. It is straightforward to show, by use of the continuity equation or by means of the Schrödinger equation, that, for scattering states,

$$|\,a(E)\,|^2=\int_0^{r_0}|\,\psi_E\,|^2d^3r\ , \tag{2.24}$$

where $\psi(E)$ is the wave function of the two nucleons in the external potential, treated as though the potential $V(r)$ held everywhere in space. Thus, $|\,a(E)\,|^2$ is the missing probability or that probability excluded from the normal (external or potential) description by the presence of the inner baglike region. In using (2.14) or (2.23) one is thereby replacing a nucleon-nucleon probability, defined by the potential $V(r)$ used to fit the phase shifts, by a six quark probability. For bound states the asymptotic normalization is not determined, and probability can be shifted from the inside to the outside region with different choices of $V(r)$ inside r_0, with the same binding energy. However, Eq. (2.24) can still be used.

This result also demonstrates the incompleteness of our model and the essential phenomenological aspects at the present stage. Since there are potentials which are phase shift equivalent within errors in the data, and which predict quite different values for $\int^{r_0}|\,\psi\,|^2d^3r$, one must now rely on a systematic comparison between theory and experiment, with guidance from our present knowledge of NN potentials. It is our expectation that this program will be completed with an effective quark Hamiltonian for the inner region, obtained in part by these studies.

III. A SIMPLE EXAMPLE

In this section the quantities $a_{LSJ}(E)$ are computed for a simple model problem. Assume that r_0 is sufficiently large that the NN interaction is approximately zero for $r>r_0$. Further assume that $\delta(E)=0$ at all energies. This example is used for two reasons: (1) to gain experience in determining $a_{LSJ}(E)$; and (2) to better understand a calculation (Sec. IV) which employs the Reid soft-core potential for $r>r_0$. In the absence of the external NN interaction and with $\delta(E)=0$ one has

$$u_{LSJ}(kr)=krj_L(kr)\ , \tag{3.1}$$

so that (2.23) leads to

$$\frac{1}{4\pi^2}\frac{1}{k^3r_0^2}\left\{\frac{\partial}{\partial k}[krj_L(kr)]\frac{\partial}{\partial r}krj_L(kr)-krj_L(kr)\frac{\partial}{\partial k}\frac{\partial}{\partial r}krj_L(kr)\right\}=|\,a^B_{LSJ}(E)\,|^2\ . \tag{3.2}$$

The superscript B on a^B_{LSJ} indicates that the first Born approximation is used to obtain $u_{LSJ}(kr)$.

It is useful to consider various aspects of (3.2). First, take $L=0$. Then,

$$\frac{1}{4\pi^2}\frac{1}{k^3}[kr_0-\tfrac{1}{2}\sin 2kr_0]=|a^B_{0SS}(E)|^2 . \tag{3.3}$$

At low energies we have

$$\lim_{E\to 0}|a^B_{0SS}(E)|^2=\frac{1}{6\pi^2}r_0^3 , \tag{3.4}$$

whereas, for very high energies,

$$\lim_{E\to\infty}|a^B_{0SS}(E)|^2=\frac{1}{4\pi^2}\frac{r_0 2\mu}{\hbar^2 E} . \tag{3.5}$$

For the simple example the zero energy "probability" $[\,|a^B_{0SS}(E)|^2]$ of quark formation is proportional to the volume of the confinement. At high energies $|a^B_{0SS}(E)|^2$ falls as $1/E$ so that at sufficiently high energies it is negligible. The interpretation of this is that if two nucleons approach each other with high relative velocities, the time available for forming a six-quark state is too small for significant interactions to take place.

It is also useful to consider the high and low energy limits of (3.2) for arbitrary L. It is easy to show that

$$\lim_{E\to 0}|a^B_{LSJ}(E)|^2\propto(kr_0)^{2L} \tag{3.6}$$

and

$$\lim_{E\to\infty}|a^B_{LSJ}(E)|^2\propto 1/E . \tag{3.7}$$

In (3.6) the effects of the centrifugal barrier are exhibited. The high energy falloff of $|a^B_{LSJ}(E)|^2$ is achieved for all values of L.

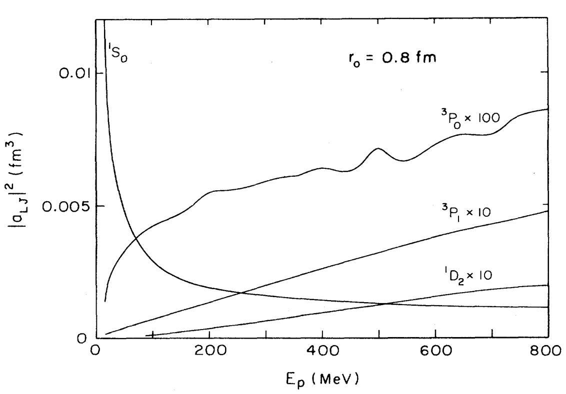

FIG. 1. Probabilities $|a_{LJ}|^2$, defined in text, for pp scattering in various partial waves as a function of energy. The Reid soft core potential is used for the extrapolating potential, with a matching radius $r_0=0.8$ fm. For comparison with Refs. 8 and 9 in that work, a dimensionless probability $P_{LJ}=|a_{LJ}|^2/V_\pi/(2\pi)^3$ was used, where $V_\pi=4\pi/3m_\pi^3$. $P_{LJ}\approx$probability of the six-quark components in the strong interaction region.

IV. NUMERICAL RESULTS

As a first meaningful example of the information which can be obtained from our model, we use the Reid soft-core potential to determine $u_{LSJ}(kr)$ for $r\geq r_0$, and therefore $|a_{LSJ}(E)|^2$. The application of (2.22)—(2.24) is straightforward.

The results for $|a_{LSJ}(E)|^2$ are shown in Fig. 1. The value of r_0 is chosen to be 0.8 fm. The most dramatic feature is the large value of $|a_{001}(E)|^2$ at low energies, and its rapid falloff with increasing energy. This is in sharp contrast with the Born approximation results of (3.3) and (3.4). The cause of this behavior is therefore due to the NN reaction. It can be interpreted as a quasizero energy "compound nuclearlike" resonance. Close to zero energy there is appreciable penetration of the confinement region by the two nucleons. There is thus a large (relative to other states and higher energies) probability of forming the six-quark configuration at low energies in the $^1S_0$ state.

Thereby, our model provides a useful definition and criterion for a "dibaryon resonance": a dibaryon resonance occurs when there is a resonance in a two-baryon channel with the six-quark probability in the interaction region becoming large [see Eq. (2.23)]. This removes the vagueness in interpreting bumps in the energy distribution of, say, polarizations as dibaryons.[14] We expect $a_{LSJ}(E)$ to be large for "other" dibaryon resonances, such as the $^1D_2$ and $^3F_3$ at 2.17 and 2.22 GeV, respectively, if this interpretation is correct. It suggests that such regions may be appropriate for studies of the underlying quark configurations in the NN system. A particular example may be charge asymmetry at low energies.

The results for the $^3P_0$, $^3P_1$, and $^1D_2$ partial waves are also shown in Fig. 1. The centrifugal barrier effect of (3.6) seems to be in evidence, but interaction effects play a significant role also. This is manifest from the difference between $|a(^3P_0)|^2$ and $|a(^3P_1)|^2$. Results for $r_0=0.7$ and 0.9 fm are shown in Figs. 2 and 3. The values of $|a_{LSJ}(E)|^2$ increase as r_0 increases. Our interpretation is that this is because it is easier to form a six-quark system if the allowed "confinement" volume is increased. Also, centrifugal barrier effects are reduced.

It is generally true that the values of $|a_{LSJ}(E)|^2$ are

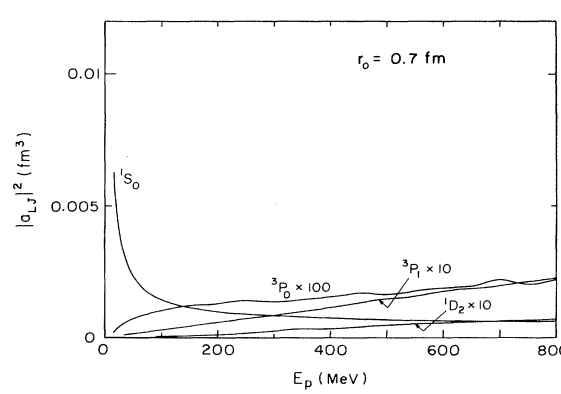

FIG. 2. Same as Fig. 1, with $r_0=0.7$.

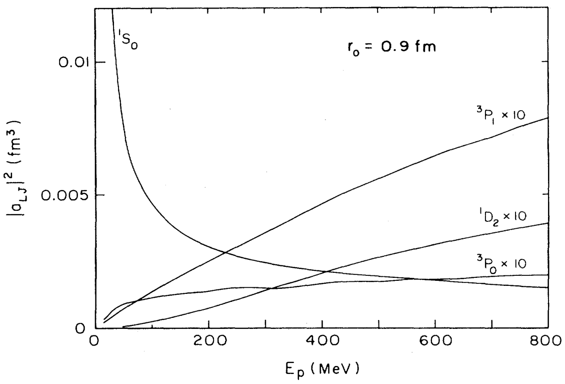

FIG. 3. Same as Fig. 1, with $r_0=0.9$.

very small, e.g., much smaller than the volume of a nucleon. This is because of the strongly repulsive nature of the Reid soft core potential. At low energies the Paris potential gives approximately the same $|a_{^1S_0}|^2$.

The discussion of the N-N bound state is straightforward in our formulation. For a bound state ($^3S_1+{}^3D_1$)—the deuteron—the wave function is normalized to unity in the standard treatment,

$$\int_0^\infty [u^2(r)+w^2(r)]dr=1 \; . \tag{4.1}$$

In Eq. (4.1) u is the $^3S_1$ and w the $^3D_1$ wave function of the deuteron. The coefficient $a_{011}(E)$ is dimensionless in this (bound state) case and the probability

$$|a_{011}(E)|^2=P_{^3S_1}=1-\int_0^\infty (u^2+w^2)dr \; . \tag{4.2}$$

For the Reid soft-core potential, we obtain $P_{^3S_1}=0.029$ for $r_0=0.9$ fm and 0.016 for $r_0=0.8$ fm. Equation (4.2) is the bound state version of (2.24).

V. EXTENSIONS OF FORMALISM

It is possible to generalize Eqs. (2.22) and (2.23) for a number of situations. The general formalism for an N-channel compound nucleus and for overlapping resonances and continuum cases are all described in a number of texts. See, e.g., Blatt and Weisskopf.[13]

The case of coupled channels such as, for instance, that due to tensor forces, is also readily carried out. The standard manner of generalizing the case of spin-orbit forces which we have included is also described in a number of texts.[13]

Perhaps the most interesting generalization is that which includes external nonoverlapping baryon resonances. If the $\Delta(1232)$ is considered as a pure pion-nucleon scattering resonance, it is a resonance which occurs purely in the external region and does not affect the three-quark wave functions inside the nucleon. We can then have ΔN and $\Delta\Delta$ channels coupled to the NN channel in the external region. If we assume this to be the case and further assume that the internal wave function is not affected by the external resonances, we can generalize our formalism also. Our results are obtained with the assumption that the external interactions which couple the various channels and with each channel are Hermitian and independent of energy (momentum). In order to be specific, we write the Hamiltonian in the external ($r\geq r_0$) region as

$$H=\begin{bmatrix} H_{\rm NN} & H_{\rm NN\to\Delta N} \\ H_{\Delta\rm N\to NN} & H_{\Delta\rm N}\end{bmatrix}, \quad r\geq r_0 \; . \tag{5.1}$$

It is easy to see that the derivation of (5.1) and (5.2) is valid for this coupled channel problem, but the replacement of C by C,α where α tells whether the channel is an NN or ΔN, and C specifies the angular momentum quantum numbers, is necessary. Thus $u_{LS}^{C,\alpha}(kr)$ could be NN, or a ΔN relative wave function. Then one has

$$\frac{\hbar^2}{4\pi^2\mu k_1 k_2}\sum_{\alpha,L=\text{allowed value}} u_{LSJ}^{C,\alpha}(k_2r)\frac{\partial}{\partial r}u_{LSJ}^{C,\alpha}(k_1r)-u_{LSJ}^{C,\alpha}(k_1r)\frac{\partial}{\partial r}u_{LSJ}^{C,\alpha *}(kr)=(E_1-E_2)a^{C^*}(E_2)a^C(E_1) \; . \tag{5.2}$$

We note Eqs. (5.1) and (5.2) can be used to provide a somewhat more fundamental treatment of the nuclear force than is usually considered. For $r\geq r_0$ one has one-boson and two-pion exchanges (included via coupling to baryon resonances). For $r\leq r_0$, (5.2) can be used to determine the six-quark amplitudes.

VI. CONCLUSION

A hybrid description of a two-baryon system in terms of an inner six-quark region and an external region with a Schrödinger description of the relative motion of the two baryons is a natural consequence of quark models of hadrons. In the hybrid model introduced in this paper the interior region is modeled by an energy-dependent quark description of each channel, with an energy-dependent probability amplitude detailing the presence of that channel in the system at each energy. These probability amplitudes have been determined in this paper using very general considerations of continuity and conservation of probability. This theory turns an earlier purely phenomenological description into a model with considerable predictive power.

However, the model is still under development. The uncertainty in the short-distance potential—the effective quark Hamiltonian—leads to limits on the accuracy of the theoretical quark probabilities which we derive. In this developmental stage, the parameters of the model are being determined both from the theoretical methods described in this paper and from comparison with experiment. It is our expectation that an effective quark Hamiltonian will be obtained, which will produce a quantitative description of short-range nuclear structure and processes.

The model, although incomplete, can now be applied to a variety of processes. Particularly for weak or electromagnetic interactions, where the quark interactions are well known, it is possible to explore directly the difference between a quark description and a hadron description of short-range nuclear processes. This can enable us to examine the quark structure of nuclear systems and its experimental consequences.

ACKNOWLEDGMENTS

The authors would like to thank Michel Betz, Rudolf Goldflam, Pervez Hoodbhoy, and Larry Wilets for helpful discussions. One of us (L.S.K.) would like to thank the University of Washington Institute for Nuclear Theory for its hospitality during his 1981—82 leave from Carnegie-Mellon University. This work was supported in part by the U.S. Department of Energy and in part by National Science Foundation Grant PHY 81-08380.

APPENDIX: NONRELATIVISTIC QUARK CLUSTER MODEL

In this appendix the model of Sec. II B, in which the relative wave function for the three-quark clusters in both the internal and external regions, is explored. This is equivalent to the original model of Wigner[10] for nuclear collisions, and is obtained in either a nonrelativistic quark model of baryon-baryon interactions or in a relativistic quark model of baryon-baryon interactions if one assumes

$$H = H_{\rm rel} + H_{\rm internal} \;, \tag{A1}$$

with

$$H_{\rm rel} = -\frac{\nabla_r^2}{2\mu^*} + V(\vec{r}) \;, \tag{A2}$$

where $V^\dagger(r) = V(r)$. Equations (A1) and (A2) apply for both the interior and external regions. The relative coordinate r is the usual internucleon coordinate in the external region $r > r_0$, and the relative coordinate between the mass centers of two color-singlet three-quark clusters in the interior region $r < r_0$. In both regions one can write

$$\Psi^\alpha = \Psi^\alpha_{C_1} \Psi^\alpha_{C_2} \Psi^\alpha_{C_1 C_2}(r) \;, \tag{A3}$$

where C_1, C_2 represents two nucleons (two clusters) in the exterior (interior) region. As described in Sec. II B, one obtains the Wigner equation (2.13). With the ansatz (2.2),

$$\Psi^\alpha_E = \mathscr{O}^\alpha(E) \Psi^\alpha_{C_1} \Psi^\alpha_{C_2} \Psi^\alpha_{C_1 C_2}(r) \;, \quad r < r_0 \;, \tag{A4}$$

where the probability amplitude $a^2(E)$ carries the entire energy dependence for channel α, one obtains, from Eq. (2.14) of the text,

$$| \mathscr{O}^\alpha(E) |^2 = \frac{1}{2\mu} \frac{\partial}{\partial E} \int dS \, | \Psi^\alpha_E |^2 [\lambda^\alpha_E - \lambda^{\alpha *}_E] \;, \tag{A5}$$

where

$$\lambda = \frac{\partial}{\partial r} \ln \psi \;.$$

From (A4) one observes that

$$\frac{d}{dr} \ln \Psi^\alpha_E = \lambda^\alpha = \text{const} \;. \tag{A6}$$

If one assumes that ordinary nonrelativistic quantum mechanics pertains, the wave function and its derivative must match at the boundary r_0. From Eq. (2.5) it then follows that $| a(E) |^2$ vanishes. From this we conclude the following: (i) the ansatz (A4) is not possible, and/or (ii) there are surface terms in the Lagrangian, or (iii) no go; there is no six-quark interior.

This situation was addressed by Wigner,[10] who was considering a standard nonrelativistic quantum mechanical system. He concluded that (2.7) and (2.14) were consistent in the sense that (2.7) is used on the left-hand side, while the right-hand side is a first order correction. This difficulty does not arise in our formulation.

However, this theorem must be considered by those formulating quark models of nuclei. It is also interesting to note that it would be natural to treat the exterior region with a version of the Feshbach-Lomon boundary condition model,[15] for as seen in (2.17) and (A5), the boundary parameters λ^α can determine the overall quark probabilities in each channel. However, the version used in the detailed fits to the NN phase shifts, in which the λ^α are constants, is not acceptable without the assumption of (ii) of (A6). Alternatively, a detailed microscopic model for the interior would provide the λ^α for a boundary condition treatment. This is the circumstance in which R-matrix theory is most useful. If a single multiquark configuration dominates at some energy, the P matrix[16] or R matrix is a most useful representation.

It should be noted, however, that we feel that quark model physics for the multibaryon systems is in a somewhat earlier stage. For this reason we have proceeded to introduce only external dynamics, and what we feel is a reasonable framework for the internal quark wave functions, and we use external information to (incompletely) determine these internal wave functions.

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