Relativistic nuclear optical models

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Recently it has been shown that the use of a relativistic impulse approximation allows one to calculate a nucleon optical potential for use in the Dirac equation. Excellent fits have been obtained for differential cross sections and spin-dependent observables for energies greater than 400 MeV. In this work we provide a general description of the form of the momentum-space optical potential for use in the Dirac equation. It is shown that there are eight invariant functions that determine the potential. Procedures for calculating these functions are given. (In the case of nuclear matter only three of these functions are required to specify the nucleon self-energy.)

[NUCLEAR REACTIONS Nucleon optical potentials for the Dirac equation; general analysis of scattering of relativistic nucleons from nuclei.]

I. INTRODUCTION

By now there have been a large number of publications dealing with relativistic models of nuclear structure. These works may be grouped into two categories. The first may be termed Dirac phenomenology. Here a number of free parameters are introduced in order to construct relativistic models either of nuclear bound states[1] or of the nuclear optical potential.[2,3] The second category is represented by a body of work which does not require the introduction of free parameters.[4,5] In the latter case one uses interactions derived from the study of free-space nucleon-nucleon scattering to study nuclear saturation, the effective interactions of nucleons in nuclei and the nucleon self-energy for bound and continuum nucleons.[3]

Recently, there have been several calculations of the relativistic nucleon optical potential which are also parameter free.[6–9] These calculations are based on the relativistic impulse approximation and form a natural extension of methods which have been used extensively to study the optical potential appropriate to the Schrödinger equation. *The improvement in the fit to the data for spin-dependent observables is quite remarkable when one passes from the Schrödinger to the Dirac formalism.*[6,9] These recent results provide strong support for relativistic models of nuclear structure under investigation at this time.

It is our purpose in this work to provide a theoretical foundation for calculations of the relativistic optical model. In particular, we wish to formulate a general theory since the phenomenological analysis[1,2] and the analysis based upon the relativistic impulse approximation[6–9] use optical potentials which have only (Lorentz) scalar and vector terms. The fits to the data achieved with these forms is very good. However, as we will see, the *general form* of the potential is more complex. We believe it will be of interest to calculate the amplitudes which specify the general form and attempt to determine why the simplified form mentioned above does as well as it does in fitting the data.

The plan of our work is as follows. In Sec. II we write the general form of the relativistic optical potential for the case of an off-mass-shell nucleon. (The massive nucleus is placed on mass shell throughout our analysis.) We present the form the potential takes in the center of mass of the nucleon-nucleus system. In Sec. III we introduce a specific scheme for analyzing the Dirac equation. The matrix elements of the nucleon self-energy taken between positive- and negative-energy spinor solutions of the free Dirac equation are presented. Knowledge of these matrix elements allows one to compare formalisms which either do or do not allow the nucleon to propagate in negative-energy states. In this section we also discuss procedures for numerical calculations of the self-energy or optical potential. In Sec. IV we discuss the reduction of the formalism to describe propagation in nuclear matter. At this point we make contact with previous calculations of the self-energy of continuum nucleons in nuclear matter. Finally, Sec. V contains a short summary of our results.

II. THE RELATIVISTIC OPTICAL MODEL

The construction of a covariant optical potential has been discussed in great detail in the study of pion-nucleus scattering.[10] That problem is particularly simple since the pion has zero intrinsic angular momentum. We follow a similar path in discussing nucleon-nucleus scattering, taking into account the complications due to the spin of the nucleon. Part of this development has already been presented in Ref. 11.

We can summarize the more formal aspects of the analysis as follows. One considers a Bethe-Salpeter equation for the scattering of a nucleon from a nucleus. The nucleus is restricted to its ground state configuration and therefore the "potential term" of the Bethe-Salpeter equation plays the role of an optical potential. All reference to excited states of the target resides in this optical potential. We may write

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$$M=K+KG_FM\ , \tag{2.1}$$

where K is the generalized optical potential and M is a relativistic scattering amplitude. There G_F is the product of Feynman propagator for the nucleon and the nucleus which is in its ground state.

The next step involves the reduction of Eq. (2.1) to a (covariant) three-dimensional equation. One may write

$$M=\Sigma+\Sigma g_0^{(+)}M\ , \tag{2.2}$$

$$\Sigma=K+K(G_F-g_0^{(+)})\Sigma\ . \tag{2.3}$$

In these two equations $g_0^{(+)}$ is a member of an infinite class of propagators which have the same right-hand cut as G_F. (This feature preserves the unitarity relations satisfied by M.) In a previous work we chose a form for $g_0^{(+)}$ that led to a (covariant) Lippmann-Schwinger equation for M.[12] Clearly, for the considerations of this work a natural choice for $g_0^{(+)}$ is one in which the massive nucleus is kept on mass shell and the nucleon is allowed to propagate in *both* negative- and positive-energy states. This choice is analogous to that made by Gross in his studies of nucleon-nucleon scattering where one of the nucleons was kept on mass shell.[13]

In Eq. (2.2) we introduced a nucleon self-energy Σ. The (covariant) optical potential V is related to the self-energy operator by the Dirac matrix γ^0, $V=\gamma^0\Sigma$. The relativistic wave equation corresponding to Eq. (2.2) has been given previously.[14] In the nucleon-nucleus center-of-mass system, we have (see Appendix A)

$$[\gamma^0(W-E_A(\vec{k}))-\vec{\gamma}\cdot\vec{k}-m_N]\psi^{(+)}_{\vec{k}_i}(\vec{k})=\int\frac{d\vec{k}\,'}{(2\pi)^3}\langle\vec{k}\,|\,\bar{\Sigma}(W)\,|\,\vec{k}'\rangle\psi^{(+)}_{\vec{k}_i}(\vec{k}\,')\ . \tag{2.4}$$

The quantity $\bar{\Sigma}$ is defined in Appendix A and is simply related to Σ. In Eq. (2.4), $\vec{k}_i$ denotes the momentum of the incident nucleon and

$$E_N(\vec{k}_i)=(\vec{k}_i^{\,2}+m_N^2)^{1/2}\ .$$

Here W denotes the total energy in the center-of-mass frame. The corresponding equation involving V may be obtained by multiplying Eq. (2.4) by γ^0. Thus,

$$[W-E_A(\vec{k})-\vec{\alpha}\cdot\vec{k}-\gamma^0m_N]\psi^{(+)}_{\vec{k}_i}(\vec{k})=\int d\vec{k}\,'\langle\vec{k}\,|\,\bar{V}\,|\,\vec{k}\,'\rangle\psi^{(+)}_{\vec{k}_i}(\vec{k}\,')\ . \tag{2.5}$$

[These equations have been given previously in Ref. 11. There a further decomposition of Eq. (2.4) was presented which made use of off-shell spinors for the nucleon.]

Although we will perform our calculations in the center-of-mass frame, it is useful to consider the form of Σ in a general frame. In such a frame the nucleon will have four momentum p or p', while the (on-mass-shell) nucleus will have the four-momenta P and P'. [In the center-of-mass frame we will put $p=\{p^0,\vec{k}\}$ and $P=\{E_A(\vec{k}),-\vec{k}\}$, etc. Here $p^0=W-E_A(\vec{k})$.] We are here interested in specifying the self-energy operator $\langle p',P'\,|\,\Sigma\,|\,p,P\rangle$. To this end it is useful to introduce a series of four-vectors:

$$W_\mu=(p+P)_\mu=(p'+P')_\mu\ , \tag{2.6}$$

$$s=(p+P)^2=(p'+P')^2=W_\mu W_\mu\ , \tag{2.7}$$

$$\hat{W}_\mu=W_\mu/\sqrt{s}\ , \tag{2.8}$$

$$\hat{\pi}_\mu=(P'+P)_\mu/(2M_A)\ , \tag{2.9}$$

$$q_\mu=(p-p')_\mu=(P'-P)_\mu\ , \tag{2.10}$$

and

$$\hat{q}_\mu=q_\mu/m_N\ . \tag{2.11}$$

Here m_N is the nucleon mass and M_A is the mass of the target. In terms of these four-vectors, we may write

$$\langle p',P'\,|\,\Sigma\,|\,p,P\rangle=a+b(\gamma\cdot\hat{W})+c(\gamma\cdot\hat{\pi})+d(i\gamma\cdot\hat{q})+e(i\sigma_{\mu\nu}\hat{q}_\mu\hat{\pi}_\nu)+f(i\sigma_{\mu\nu}\hat{q}_\mu\hat{W}_\nu)+g(\sigma_{\mu\nu}\hat{\pi}_\mu\hat{W}_\nu)+h(i\gamma_5\hat{W}_\mu\gamma_\nu\hat{q}_\rho\hat{\pi}_\sigma\epsilon_{\mu\nu\rho\sigma})\ , \tag{2.12}$$

where we have introduced eight scalar functions a, b, c, d, e, f, g, and h. (We recall that if the nucleon were on mass shell, this problem would be analogous to the study of pion-nucleon scattering and only two scalar functions would be required to specify the relativistic scattering amplitude.[15]) Some relations satisfied by the a, b, ..., etc., which follow from symmetry considerations are given in Appendix B.

The functions a, b, ..., may each be taken to be functions of the Lorentz scalars s, $t=q^2$, $\hat{W}\cdot\hat{\pi}$ and $\hat{W}\cdot\hat{q}$. We now specialize to the center-of-mass frame; using the relations given in Appendix C, we have

$$\langle p',P'\,|\,\Sigma\,|\,p,P\rangle\to\langle\vec{k}\,'\,|\,\Sigma\,|\,\vec{k}\rangle$$

with

$$\langle\vec{k}\,'\,|\,\Sigma(W)\,|\,\vec{k}\rangle=A+\gamma^0B+\vec{\gamma}\cdot\frac{(\vec{k}+\vec{k}\,')}{2m_N}C+\frac{iD}{m_N}\vec{\gamma}\cdot(\vec{k}\,'-\vec{k})-\frac{E}{2m_N}\gamma^0\vec{\gamma}\cdot(\vec{k}\,'-\vec{k})-\frac{iF}{2m_N}[\gamma^0\vec{\gamma}\cdot(\vec{k}\,'+\vec{k})]+\frac{iG}{m_N^2}\vec{\Sigma}\cdot(\vec{k}\,'\times\vec{k})-\frac{iH}{m_N^2}\gamma^0\vec{\Sigma}\cdot(\vec{k}\,'\times\vec{k})\ . \tag{2.13}$$

The functions $A, B, \ldots,$ etc., are linear functions of $a, b, \ldots,$ etc. The specific form of these relations need not concern us here. However, for completeness these relations are noted in Appendix D. (We remark that $V=\gamma^0\Sigma$ is Hermitian if A, B, C, D, E, F, G, and H are real.)

We can consider Eq. (2.13) in the limit appropriate to the study of nuclear matter. There $\vec{k}=\vec{k}'$. We should, in general, write $\Sigma(k^0,\vec{k})$. However, we will write $\Sigma(\vec{k})$ in order to keep the notation simple. (More precisely, we can put

$$\Sigma(\vec{k})=\Sigma(k^0(\vec{k}),\vec{k}) ,$$

where $k^0(\vec{k})$ is the solution of the dispersion relation relating the energy variable to the three-momentum—see Ref. 5). We now write

$$\Sigma(\vec{k})=A(\vec{k})+\gamma^0B(\vec{k})+\frac{\vec{\gamma}\cdot\vec{k}}{m_N}C(\vec{k})+\frac{iF}{m_N}\gamma^0\vec{\gamma}\cdot\vec{k} , \tag{2.14}$$

and correspondingly,

$$V(\vec{k})=\gamma^0A(\vec{k})+B(\vec{k})+\gamma^0\frac{\vec{\gamma}\cdot\vec{k}}{m_N}C(\vec{k})+\frac{iF(\vec{k})}{m_N}\vec{\gamma}\cdot\vec{k} . \tag{2.15}$$

This form is Hermitian if A, B, C, and F are real. However, time reversal invariance, as expressed by Eq. (B10), combined with Eq. (D5), requires that we put $F=0$ in the case of nuclear matter. Thus we have

$$\Sigma(\vec{k})=A(\vec{k})+\gamma^0B(\vec{k})+\frac{\vec{\gamma}\cdot\vec{k}}{m_N}C(\vec{k}) , \tag{2.16}$$

which is the form that has been considered in our earlier work which dealt with nuclear matter.[5]

III. CALCULATION OF THE SELF-ENERGY OPERATOR

The solution of the Dirac equation in momentum space is facilitated by expanding the wave function in terms of the positive- and negative-energy solutions of the Dirac equation without interaction. An appropriate basis is provided by the spinors

$$u(\vec{p},s)=\left[\frac{E_N(\vec{p})+m_N}{2m_N}\right]^{1/2}\begin{bmatrix}\chi_s\\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E_N(\vec{p})+m_N}\chi_s\end{bmatrix} \tag{3.1}$$

and

$$w(\vec{p},s)=\gamma_5\gamma^0u(\vec{p},s) .$$

Specifically,

$$w(\vec{p},s)=\left[\frac{E_N(\vec{p})+m_N}{2m_N}\right]^{1/2}\begin{bmatrix}-\dfrac{\vec{\sigma}\cdot\vec{p}}{E_N(\vec{p})+m_N}\chi_s\\ \chi_s\end{bmatrix} , \tag{3.2}$$

which is equal to $v(-\vec{p},-s)$ of Bjorken and Drell.[16] The definition

$$\epsilon(\vec{p})=E_N(\vec{p})+m_N$$

will be useful in the following.

We now define

$$\langle\vec{k}',s'\,|\,\Sigma^{++}(W)\,|\,\vec{k},s\rangle=\bar{u}(\vec{k}',s')\langle\vec{k}'\,|\,\Sigma(W)\,|\,\vec{k}\rangle u(\vec{k},s) , \tag{3.3}$$

$$\langle\vec{k}',s'\,|\,\Sigma^{+-}(W)\,|\,\vec{k},s\rangle=\bar{u}(\vec{k}',s')\langle\vec{k}'\,|\,\Sigma(W)\,|\,\vec{k}\rangle w(\vec{k},s) , \tag{3.4}$$

$$\langle\vec{k}',s'\,|\,\Sigma^{-+}(W)\,|\,\vec{k},s\rangle=\bar{w}(\vec{k}',s')\langle\vec{k}'\,|\,\Sigma(W)\,|\,\vec{k}\rangle u(\vec{k},s) , \tag{3.5}$$

$$\langle\vec{k}',s'\,|\,\Sigma^{--}(W)\,|\,\vec{k},s\rangle=\bar{w}(\vec{k}',s')\langle\vec{k}'\,|\,\Sigma(W)\,|\,\vec{k}\rangle w(\vec{k},s) . \tag{3.6}$$

These are the natural extensions of the matrix elements defined in the study of nuclear matter.[5]

We may also write, with $\epsilon=\epsilon(\vec{k})$ and $\epsilon'=\epsilon(\vec{k}')$,

$$\langle\vec{k}',s'\,|\,\Sigma^{++}(W)\,|\,\vec{k},s\rangle=\left\langle s'\left|\left[S_1^{++}+\frac{i\vec{\sigma}\cdot(\vec{k}'\times\vec{k})}{\epsilon'\epsilon}S_2^{++}\right]\right|s\right\rangle , \tag{3.7}$$

$$\langle\vec{k}',s'\,|\,\Sigma^{+-}(W)\,|\,\vec{k},s\rangle=\left\langle s'\left|\left[\vec{\sigma}\cdot\left[\frac{\vec{k}}{\epsilon}+\frac{\vec{k}'}{\epsilon'}\right]S_1^{+-}+\vec{\sigma}\cdot\left[\frac{\vec{k}'}{\epsilon'}-\frac{\vec{k}}{\epsilon}\right]S_2^{+-}\right]\right|s\right\rangle , \tag{3.8}$$

$$\langle\vec{k}',s'\,|\,\Sigma^{-+}(W)\,|\,\vec{k},s\rangle=\left\langle s'\left|\left[\vec{\sigma}\cdot\left[\frac{\vec{k}}{\epsilon}+\frac{\vec{k}'}{\epsilon'}\right]S_1^{-+}+\vec{\sigma}\cdot\left[\frac{\vec{k}'}{\epsilon'}-\frac{\vec{k}}{\epsilon}\right]S_2^{-+}\right]\right|s\right\rangle , \tag{3.9}$$

and

$$\langle\vec{k}',s'\,|\,\Sigma^{--}(W)\,|\,\vec{k},s\rangle=\left\langle s'\left|\left[S_1^{--}+\frac{i\vec{\sigma}\cdot(\vec{k}'\times\vec{k})}{\epsilon\epsilon'}S_2^{--}\right]\right|s\right\rangle . \tag{3.10}$$

Using the definitions given in Eqs. (3.3)—(3.6) and Eq. (2.13) we have, with

$$N'=\left[\frac{\epsilon'}{2m_N}\right]^{1/2}, \quad N=\left[\frac{\epsilon}{2m_N}\right]^{1/2}, \tag{3.11}$$

$$\begin{aligned}S_1^{++}(\vec{k}',\vec{k})=NN'\Bigg\{&A\left[1-\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right]+B\left[1+\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right]+\frac{C}{2m_N}\left[\frac{\vec{k}'^2}{\epsilon'}+\frac{\vec{k}^2}{\epsilon}+\frac{\vec{k}\cdot\vec{k}'(\epsilon+\epsilon')}{\epsilon\epsilon'}\right]\\&+iD\left[1+\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right]\left[\frac{E_N(\vec{k}')-E_N(\vec{k})}{2m_N}\right]\\&+\frac{E}{2m_N}\left[\frac{\vec{k}'^2}{\epsilon'}+\frac{\vec{k}^2}{\epsilon}-\frac{\vec{k}\cdot\vec{k}'(\epsilon+\epsilon')}{\epsilon\epsilon'}\right]+\frac{iF}{2m_N}\left[1-\frac{\vec{k}\cdot\vec{k}'}{\epsilon\epsilon'}\right][E_N(\vec{k})-E_N(\vec{k}')]\\&-\frac{G}{m_N^2}\left[\frac{\vec{k}'^2\vec{k}^2-(\vec{k}\cdot\vec{k}')^2}{\epsilon\epsilon'}\right]-\frac{H}{m_N^2}\left[\frac{\vec{k}'^2\vec{k}^2-(\vec{k}\cdot\vec{k}')^2}{\epsilon\epsilon'}\right]\Bigg\},\end{aligned} \tag{3.12}$$

$$\begin{aligned}S_2^{++}(\vec{k}',\vec{k})=NN'\Bigg\{&-A+B+\left[\frac{\epsilon+\epsilon'}{2m_N}\right]C+\frac{iD}{2m_N}[E_N(\vec{k}')-E_N(\vec{k})]-\frac{E}{2m_N}(\epsilon+\epsilon')\\&-iF\left[\frac{E_N(\vec{k}')-E_N(\vec{k})}{2m_N}\right]+\frac{G}{m_N^2}(\epsilon\epsilon'+\vec{k}\cdot\vec{k}')-\frac{H}{m_N^2}(\epsilon\epsilon'-\vec{k}'\cdot\vec{k})\Bigg\}.\end{aligned} \tag{3.13}$$

In order to obtain S_1^{--} from S_1^{++} one should change the sign of A, C, D, and G. In order to obtain S_2^{--} from S_2^{++} one should again change the sign of A, C, D, and G.

We also have

$$\begin{aligned}S_1^{+-}(\vec{k}',\vec{k})=NN'\Bigg\{&-A+C-\frac{E}{2m_N}[E_N(\vec{k}')-E_N(\vec{k})]\\&-\frac{iF}{2m_N}[E_N(\vec{k}')+E_N(\vec{k})]-\frac{G}{m_N^2}\frac{\epsilon\epsilon'}{2}\left|\frac{\vec{k}'}{\epsilon'}-\frac{\vec{k}}{\epsilon}\right|^2-\frac{H}{m_N}(\epsilon'-\epsilon)\Bigg\}\end{aligned} \tag{3.14}$$

and

$$\begin{aligned}S_2^{+-}(\vec{k}',\vec{k})=NN'\Bigg\{&B+\frac{C}{2m_N}[E_N(\vec{k}')-E_N(\vec{k})]\\&+\frac{iD}{2m_N}[E_N(\vec{k}')+E_N(\vec{k})]-E+\frac{G}{m_N^2}\frac{\epsilon\epsilon'}{2}\left[\left[\frac{\vec{k}'}{\epsilon'}\right]^2-\left[\frac{\vec{k}}{\epsilon}\right]^2\right]\\&+\frac{H}{m_N^2}[E_N(\vec{k}')E_N(\vec{k})-m_N^2+\vec{k}'\cdot\vec{k}]\Bigg\}.\end{aligned} \tag{3.15}$$

In order to obtain S_1^{-+} from S_1^{+-} one should change the signs of E, F, and H. In order to obtain S_2^{-+} from S_2^{+-} one should change the signs of B, E, and H.

We suggest the following procedure for the calculation of the functions $A, B, \ldots,$ etc. One may obtain the matrix elements of the self-energy defined in Eqs. (3.3) and (3.6). From these one may obtain the eight functions, $S_1^{++}, S_2^{++}, \ldots,$ etc., defined in Eqs. (3.9) and (3.10). In turn, the knowledge of these eight functions allow us to construct the eight functions, $A, B, \ldots,$ etc., *via* linear relations such as those given in Eqs. (3.12)–(3.15). Once we have obtained the latter set of functions we can study the role of functions other than A and B in the analysis of nucleon-nucleus scattering. [Thus far all fits to the data have been made with potentials of the form

$$\Sigma(r)=A(r)+\gamma^0 B(r),$$

where $A(r)$ and $B(r)$ are both complex and energy dependent.]

IV. THE NUCLEON SELF-ENERGY IN NUCLEAR MATTER AND FINITE NUCLEI: SOME NUMERICAL RESULTS

In nuclear matter, we need only retain the terms involving A, B, and C. In this case we have[17]

$$\langle s' | \Sigma^{++}(\vec{p}) | s\rangle = \delta_{ss'}\left[A(\vec{p}) + \frac{E_N(\vec{p})}{m_N}B(\vec{p}) + \frac{\vec{p}^{\,2}}{2m_N}C(\vec{p})\right], \quad (4.1)$$

$$\langle s' | \Sigma^{+-}(\vec{p}) | s\rangle = \langle s' | \vec{\sigma}\cdot\hat{p} | s\rangle \frac{|\vec{p}|}{m_N}[C(\vec{p}) - A(\vec{p})], \quad (4.2)$$

$$\langle s' | \Sigma^{-+}(\vec{p}) | s\rangle = \langle s' | \vec{\sigma}\cdot\hat{p} | s\rangle \frac{|\vec{p}|}{m_N}[C(\vec{p}) - A(\vec{p})], \quad (4.3)$$

and

$$\langle s' | \Sigma^{--}(\vec{p}) | s\rangle = \delta_{ss'}\left[-A(\vec{p}) + \frac{E_N(\vec{p})}{m_N}B(\vec{p}) - \frac{\vec{p}^{\,2}}{m_N^2}C(\vec{p})\right]. \quad (4.4)$$

Relativistic models of nuclear structure are characterized by having large negative values of A and large positive values for B. The quantities tend to cancel when constructing the potential energy of a nucleon and therefore this potential is rather small (~ 50 MeV) for a low-energy nucleon. On the other hand, the leading contribution to the spin-orbit splitting in finite nuclei is determined by S_2^{++}. From Eq. (3.13) we see that S_2^{++} involves $(-A+B)$, which is a large number (~ 700 MeV).

Values for A, B, and C have been calculated for nuclear matter for nucleons of kinetic energy of 0 to 200 MeV. (Some of the details of these calculations may be found in Refs. 4 and 5.) In Fig. 1 we present the values calculated using a relativistic Brueckner-Hartree-Fock approach.[4,5] Values for A and B in the energy region of 200 MeV to 1 GeV may be found in Ref. 7. These latter values were calculated using a relativistic impulse approximation. Good agreement is obtained between the theoretical potential and the parameters obtained in phenomenological studies.[1,2] Below 300 MeV, medium corrections to the nucleon-nucleon T matrix become important and the impulse approximation is inadequate.

As may be seen from Fig. 1 or from Ref. 7, the imaginary part of A is greater than zero, while the imaginary part of B is less than zero. This feature is found in phenomenological studies and is well reproduced in theoretical models.[5,7] Analytic studies of the Lorentz character of the imaginary part of the optical potential for nuclear matter have been performed by Horowitz.[18] He shows that the observed features may be obtained using

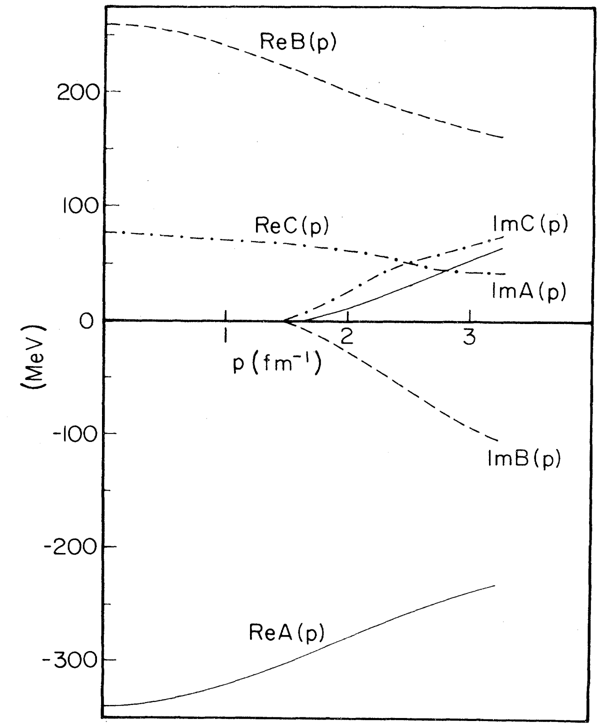

FIG. 1. Values of the real and imaginary parts of $A(\vec{p})$, $B(\vec{p})$, and $C(\vec{p})$ as a function of the quasiparticle momentum. The nucleon-nucleon interaction is that of Holinde, Erkelenz, and Alzetta (HEA) (Ref. 21). (See Ref. 5 for further details concerning these results.)

sigma and omega exchange in a second-order perturbation analysis. (Inclusion of pion exchange yields relatively small corrections if pseudovector coupling is used for the πNN vertex.[18])

V. SUMMARY

We have presented a general formalism for the study of the scattering of a relativistic nucleon from a spin-zero target. We find that there are eight scalar invariants that should be determined. In the case of nuclear matter the number of functions required to specify the self-energy is three if the Hamiltonian is invariant under the time reversal operation.

As we have remarked previously, excellent fits to the data have been obtained using only two terms in the self-energy, $\Sigma = A + \gamma^0 B$. In addition, the self-energy has been taken to be a local operator, $\Sigma(r) = A(r) + \gamma^0 B(r)$. As our nuclear matter calculations have shown, the quantity C is nonzero. (See Fig. 1.)

It is of interest to calculate the self-energy in its most general form and determine $A, B, C, \ldots,$ etc., for finite nuclei. Such calculations will aid in the understanding of the success of the current phenomenological models. These calculations may be performed for projectile energies less than about 300 MeV. Above this energy one requires a model of the nucleon-nucleon interaction, based upon field theory, that incorporates a description of the inelastic channels—that is, meson production processes.

APPENDIX A

In this appendix we discuss the reduction of the Bethe-Salpeter equation for the case of nucleon-nucleus scattering. (The reduction appropriate to the study of nucleon-nucleon scattering is reviewed in Refs. 19 and 20.) One may write the nucleon momentum as

$$p=\left\{\frac{W}{2}+k^0,\vec{k}\right\}$$

and the momentum of the nucleus as

$$P=\left\{\frac{W}{2}-k^0,-\vec{k}\right\}$$

in the center-of-mass frame. We consider the covariant equation

$$\langle k'|M(W)|k\rangle=\langle k'|K(W)|k\rangle+\int d^4k''\langle k'|K(W)|k''\rangle G_F(k''|W)\langle k''|M(W)|k\rangle \ , \tag{A1}$$

with

$$G_F(k''|W)=G_F(p,P)\equiv\frac{i}{(2\pi)}\frac{1}{\gamma\cdot p-m_{\rm N}+i\epsilon}\frac{1}{P^2-M_A^2+i\epsilon} \ . \tag{A2}$$

We now wish to obtain equations of the form of Eqs. (2.2) and (2.3). We choose to keep the massive nucleus on its mass shell. Therefore, to obtain $g_0^{(+)}$ we effectively replace $(P^2-M_A^2+i\epsilon)^{-1}$ by $-2\pi i\delta(P^2-M_A^2)$ and put

$$g_0^{(+)}=\frac{1}{\gamma\cdot p-m_{\rm N}+i\epsilon}\frac{1}{2E_A(\vec{P})}\delta(P^0-E_A(\vec{P})) \ . \tag{A3}$$

We specialize to the center-of-mass frame so that Eq. (2.2) becomes

$$\langle\vec{k}'|M(W)|\vec{k}\rangle=\langle\vec{k}'|\Sigma(W)|\vec{k}\rangle+\int d\vec{k}''\langle\vec{k}'|\Sigma(W)|\vec{k}''\rangle\frac{1}{\gamma\cdot k''-m_{\rm N}+i\epsilon}\frac{1}{2E_A(\vec{k}'')}\langle\vec{k}''|M(W)|\vec{k}\rangle \ . \tag{A4}$$

Here $k''=[W-E_A(\vec{k}''),\vec{k}'']$.

It is then useful to define

$$\langle\vec{k}'|\overline{M}(W)|\vec{k}\rangle=\left[\frac{1}{2E_A(\vec{k}')}\right]^{1/2}\langle\vec{k}'|M(W)|\vec{k}\rangle\left[\frac{1}{2E_A(\vec{k})}\right]^{1/2} \tag{A5}$$

and

$$\langle\vec{k}'|\overline{\Sigma}(W)|\vec{k}\rangle=\left[\frac{1}{2E_A(\vec{k}')}\right]^{1/2}\langle\vec{k}'|\Sigma(W)|\vec{k}\rangle\left[\frac{1}{2E_A(\vec{k})}\right]^{1/2} \tag{A6}$$

to obtain

$$\langle\vec{k}'|\overline{M}(W)|\vec{k}\rangle=\langle\vec{k}'|\overline{\Sigma}(W)|\vec{k}\rangle+\int d\vec{k}''\langle\vec{k}'|\overline{\Sigma}(W)|\vec{k}''\rangle\frac{1}{\gamma\cdot k''-m_{\rm N}+i\epsilon}\langle\vec{k}''|\overline{M}(W)|\vec{k}\rangle \ . \tag{A7}$$

If we put

$$\overline{M}(W)|\vec{k}\rangle u(\vec{k},s)=\overline{\Sigma}(W)|\psi^{(+)}_{\vec{k},s}\rangle \ , \tag{A8}$$

we see that

$$\langle\vec{k}'|\psi^{(+)}_{\vec{k},s}\rangle=\delta(\vec{k}'-\vec{k})u(\vec{k},s)+\int d\vec{k}''\frac{1}{\gamma\cdot k'-m_{\rm N}+i\epsilon}\langle\vec{k}'|\overline{\Sigma}(W)|\vec{k}''\rangle\langle\vec{k}''|\psi^{(+)}_{\vec{k},s}\rangle \ , \tag{A9}$$

where $u(\vec{k},s)$ is a positive-energy spinor solution of the Dirac equation. Equation (A9) may also be written as

$$[\gamma^0(W-E_A(\vec{k}'))-\vec{\gamma}\cdot\vec{k}'-m_{\rm N}]\langle\vec{k}'|\psi^{(+)}_{\vec{k},s}\rangle=\int d\vec{k}''\langle\vec{k}'|\overline{\Sigma}(W)|\vec{k}''\rangle\langle\vec{k}''|\psi^{(+)}_{\vec{k},s}\rangle \ . \tag{A10}$$

APPENDIX B

For bound-state problems and elastic scattering at very low energies one would find that $V=\gamma^0\Sigma$ is Hermitian. Under transposition of the initial and final momenta the various bracketed quantities in Eq. (2.12) satisfy

$$\gamma^0(\quad)^\dagger\gamma^0=(\quad)\ , \tag{B1}$$

so that Hermitian character of V would imply, with $\nu\equiv\hat{W}\cdot\hat{q}$,

$$a^*(s,q^2,\hat{W}\cdot\hat{\pi},\nu)=a(s,q^2,\hat{W}\cdot\hat{\pi},-\nu)\ , \tag{B2}$$

$$b^*(s,q^2,\hat{W}\cdot\hat{\pi},\nu)=b(s,q^2,\hat{W}\cdot\hat{\pi},-\nu)\ , \tag{B3}$$

etc.

A requirement of time reversal invariance of the quantity $\langle p;P'\,|\,\Sigma\,|\,p,P\rangle$ yields the following relations:

$$a(s,q^2,\hat{\pi}\cdot\hat{W},\nu)=a(s,q^2,\hat{\pi}\cdot\hat{W},-\nu)\ , \tag{B4}$$

$$b(\nu)=b(-\nu)\ , \tag{B5}$$

$$c(\nu)=c(-\nu)\ , \tag{B6}$$

$$d(\nu)=-d(-\nu)\ , \tag{B7}$$

$$e(\nu)=e(-\nu)\ , \tag{B8}$$

$$f(\nu)=f(-\nu)\ , \tag{B9}$$

$$g(\nu)=-g(-\nu)\ , \tag{B10}$$

$$h(\nu)=h(-\nu)\ . \tag{B11}$$

In Eqs. (B5)—(B11) we have suppressed the variables which do not change sign under time reversal. [See Eq. (B4).]

APPENDIX C

The values of various Dirac matrix forms of Eq. (2.12) are given in the nucleon-nucleus center-of-mass as follows:

$$\gamma\cdot\hat{W}\to\gamma^0\ , \tag{C1}$$

$$\gamma\cdot\hat{\pi}\to\{\gamma^0[E_A(\vec{k}\,')+E_A(\vec{k})]+\vec{\gamma}\cdot(\vec{k}+\vec{k}\,')\}/(2M_A)\ , \tag{C2}$$

$$i\gamma\cdot\hat{q}\to i\{\gamma^0[E_A(\vec{k}\,')-E_A(\vec{k})]-\vec{\gamma}\cdot(\vec{k}-\vec{k}\,')\}/m_{\rm N}\ , \tag{C3}$$

$$i\sigma_{\mu\nu}\hat{q}_\mu\hat{\pi}_\nu\to-\frac{1}{M_A m_{\rm N}}[\gamma^0\vec{\gamma}\cdot\vec{k}E_A(\vec{k}\,')+\vec{\gamma}\cdot\vec{k}\,'\gamma^0E_A(\vec{k}) +(\vec{k}\cdot\vec{k}\,')+(\vec{\gamma}\cdot\vec{k}\,')(\vec{\gamma}\cdot\vec{k})]\ , \tag{C4}$$

$$i\sigma_{\mu\nu}\hat{q}_\mu\hat{W}_\nu\to-(\vec{\gamma}\cdot\vec{k}\,'\gamma^0+\gamma^0\vec{\gamma}\cdot\vec{k})/m_{\rm N}\ , \tag{C5}$$

$$\sigma_{\mu\nu}\hat{\pi}_\mu\hat{W}_\nu\to i(\vec{\gamma}\cdot\vec{k}\,'\gamma^0-\gamma^0\vec{\gamma}\cdot\vec{k})/(2M_A)\ , \tag{C6}$$

$$i\gamma_5\hat{W}_\mu\gamma_\nu\hat{q}_\rho\hat{\pi}_\sigma\epsilon_{\mu\nu\rho\sigma}\to-i\gamma^0[\vec{\Sigma}\cdot(\vec{k}\,'\times\vec{k})]/(m_{\rm N}M_A)\ . \tag{C7}$$

These relations may be written somewhat differently if we note the identity

$$\vec{k}\cdot\vec{k}\,'+(\vec{\gamma}\cdot\vec{k})(\vec{\gamma}\cdot\vec{k}\,')=-i\vec{\Sigma}\cdot(\vec{k}\,'\times\vec{k})\ . \tag{C8}$$

We are here using the notation of Bjorken and Drell so that

$$\vec{\Sigma}=\begin{bmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{bmatrix}\ . \tag{C9}$$

APPENDIX D

The passage from Eq. (2.12) to Eq. (2.13) using the results presented in Appendix C yields the following linear relations:

$$A=a\ , \tag{D1}$$

$$B=b+c\left[\frac{E_A(\vec{k})+E_A(\vec{k}\,')}{2M_A}\right]+id\left[\frac{E_A(\vec{k}\,')-E_A(\vec{k})}{m_{\rm N}}\right]\ , \tag{D2}$$

$$C=\frac{m_{\rm N}c}{M_A}\ , \tag{D3}$$

$$D=d\ , \tag{D4}$$

$$E=-2f-e\left[\frac{E_A(\vec{k}\,')+E_A(\vec{k})}{M_A}\right]\ , \tag{D5}$$

$$F=\frac{m_{\rm N}}{M_A}g-ie\left[\frac{E_A(\vec{k}\,')-E_A(\vec{k})}{M_A}\right]\ , \tag{D6}$$

$$G=\frac{m_{\rm N}}{M_A}e\ , \tag{D7}$$

$$H=\frac{m_{\rm N}}{M_A}h\ . \tag{D8}$$

Here $E_A(\vec{k})=(\vec{k}^{\,2}+M_A^2)^{1/2}$.

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