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Elastic and rearrangement scattering between two interacting deuterons as a four-body problem

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A four-body model is used in studying the four-nucleon system. This model is solvable and can be applied to nuclear reactions involving four nucleons. Intermediate quasiparticle states are used for the two- and three-body scattering leading to T matrices which are separable. The model leads to four-body equations which, by using partial wave decomposition, are reduced to single variable integral equations. Numerical calculations of the differential cross sections of the nuclear reactions ^{2}H$(d,p)^3$H, ^{2}H$(d,n)^3$He, and ^{2}H$(d,d)^2$H are carried out for different deuteron laboratory energies between 6.1 and 51.5 MeV. Inclusion of the p-wave three-body amplitudes is tested. Also, the simultaneous exchange of two nucleons between the incoming deuterons is investigated. The theoretically calculated angular distributions are in good agreement with the experimental measurements. The extracted values of the spectroscopic and normalization factors are reasonable.

[NUCLEAR REACTIONS ^{2}H$(d,p)^3$H, ^{2}H$(d,n)^3$He, ^{2}H$(d,d)^2$H; $E_d=6.1-51.5$ MeV; four-body model. Calculated $\sigma(\theta)$. Extracted spectroscopic factors.]

I. INTRODUCTION

Nuclear reactions have been found to be one of the most interesting tools for studying the static properties of nuclei. The success of the three-body formulations[1] in describing three-body systems, being exact solutions, led to the study of nuclear reactions as a three-body problem.[2] In parallel with the development of considering the Faddeev formalism[3] for the three-body systems, many authors[4] introduced integral equations for the N-body problem. Approximating the off-shell scattering amplitudes by using separable expressions in the separable potential model, the integral equations of the four-nucleon system can be expressed by multichannel two-particle Lippmann-Schwinger equations. For the four-nucleon system, the bound state[5] is represented in a practical way available for computations. The elastic scattering, rearrangement, and breakup processes between four nucleons had been calculated numerically.[6] With this formalism, the bound state of four-bound alpha particles had been calculated.[7] The elastic and rearrangement scattering of four alpha particles had also been considered.[8] The binding energies of light nuclei had been calculated[9] according to a cluster expansion of the nuclei as an N-cluster problem. The separable approximation is used in the kernel of the four-body equations, reducing them to single variable integral equations by partial wave decomposition. A small number of terms is sufficient to obtain good results for the four-body bound states. In the case of four-body scattering, rearrangement, and breakup processes, increased separable terms for the subamplitude are needed.

The off-shell one plus three and two plus two subamplitudes are needed to be used as input in the four-body equations. The spin and isospin effects are to be included in the four-nucleon system. In the two-nucleon sector of the model, we have two quasiparticles to be introduced in the s-wave coupling of the two nucleons. The two quasiparticle states of two interacting nucleons are either the spin triplet state, which is a physical particle (the deuteron), or the spin singlet state, which is an unphysical particle. The two-nucleon physical state (the deuteron) is denoted by d, while the two-nucleon unphysical state is denoted by ϕ. In the three-nucleon sector, both the total spin and the total isospin have two possible values, $\frac{1}{2}$ and $\frac{3}{2}$. Then, for zero total orbital angular momentum for the three-nucleon system, both the total angular momentum and the total spin are conserved. Then, we have three independent amplitudes according to the doublet D (for the value $\frac{1}{2}$) state, and to the quartet Q (for the value $\frac{3}{2}$) state. These states are (D,D), (D,Q), and

(Q,D) and are denoted by t, t', and t'', respectively. The t state is the physical state (the triton ^{3}H or the ^{3}He), while the t' and t'' are the unphysical states of the coupled $N+d$ and $N+\phi$, respectively. In the four-nucleon sector, a partial wave decomposition is introduced. Then, one-dimensional integral equations are obtained for all the elastic scattering and rearrangement processes, especially the processes $dd \to dd$, $dd \to n\,^3$He, and $dd \to p\,^3$H.

In the present work, we are interested in the four-nucleon systems. We study the elastic and rearrangement scattering processes which initially are between two interacting deuterons (a four-nucleon system), leading to a final state of also a four-nucleon system. The four-nucleon integral equations are approximated by using the separable expansion in the separable potential model approximation. Then, we are led to transition amplitudes which are the off-shell scattering and rearrangement amplitudes. Also, a partial wave decomposition is used to reduce these equations to one-dimensional equations. Then, only a few terms of the separable approximations are needed. The nuclear reactions considered in the present work are the elastic and rearrangement scattering between two incoming interacting deuterons. Thus, in the present work we investigate the inclusion of the p-wave three-body amplitudes. Also, the effect of simultaneous exchange of two nucleons between the two incoming deuterons is taken into account. Numerical calculations are carried out for the differential cross sections of the nuclear reactions ^{2}H$(d,p)^3$H, ^{2}H$(d,n)^3$He, and ^{2}H$(d,d)^2$H at different deuteron laboratory energies between 6.1 and 51.5 MeV. The theoretically calculated angular distributions of these nuclear reactions are compared with the experimental measurements. From the fitting between the calculated cross sections and the experimental data, the spectroscopic and normalization factors are extracted.

In Sec. II, the four-nucleon amplitudes and integral equations are introduced. Numerical calculations and results are given in Sec. III. Section IV is left for discussion and conclusions.

II. FOUR-NUCLEON AMPLITUDES AND INTEGRAL EQUATIONS

In the present work, we are interested in the four-nucleon system. To represent these systems, we introduce for them the four-nucleon amplitudes and their integral equations. As input for the four-nucleon amplitudes, we need the two-nucleon and three-nucleon amplitudes. The Aaron, Amado, and Yam[10] approach is used in defining the two- and three-nucleon amplitudes.

The two-nucleon scattering amplitudes which have a separable form in momentum space can be given by two independent s-wave amplitudes, one for the spin triplet pair

$$\langle \vec{k} \,|\, S^{1,0}(E) \,|\, \vec{k}' \rangle = \lambda_d f_d(k) t_d(E+\epsilon_d) f_d(k') \;, \quad (1)$$

and the other for the spin singlet pair

$$\langle \vec{k} \,|\, S^{0,1}(E) \,|\, \vec{k}' \rangle = \lambda_\phi f_\phi(k) t_\phi(E) f_\phi(k') \;, \quad (2)$$

where t_d and t_ϕ are the sums of self-energy bubbles. λ is the coupling constant for each interaction and $f(k)$ is the vertex function. In the Aaron, Amado[10,11] approach, the renormalization constant of the wave function for the case of the triplet interaction takes on the range of values between zero and one. The values of the parameters of the interaction are chosen in such a way so as to fit the low-energy triplet and singlet nucleon-nucleon data.

For the three-nucleon sector, a set of equations describing the particle quasiparticle scattering is given by Aaron, Amado, and Yam.[10] If the nucleon is denoted by N, then we are dealing with the scattering process $Nx \to Nx'$, where x and x' are either d or ϕ. The scattering amplitudes are given by integral equations which after partial wave decomposition have a form for each partial wave given as

$$T^{XYl}_{xx'}(k,k';E) = \Lambda^{XY}_{xx'} b^{l}_{xx'}(k,k';E) + \sum_{x''=d,\phi} \int_0^\infty dk'' \frac{k''^2}{2\pi^2} \Lambda^{XY}_{xx''} b^{l}_{xx''}(k,k'';E) t_{x''}(E+\epsilon_{x''} - \tfrac{3}{2}k''^2) T^{XYl}_{x''x'}(k'',k';E) \;. \quad (3)$$

In Eq. (3), $\Lambda^{XY}_{xx'}$ are the three-body spin-isospin recoupling coefficients introduced in Ref. 10 with their values. $b^{l}_{xx'}$ is the single nucleon exchange Born term. There are three independent amplitudes for each value of l according to the values of spin X and isospin Y. This combination of values is $(\frac{1}{2},\frac{1}{2})$, $(\frac{3}{2},\frac{1}{2})$, and $(\frac{1}{2},\frac{3}{2})$. For the first values, $X=Y=\frac{1}{2}$, both the Nd and $N\phi$ channels are included. For the second values, $X=\frac{3}{2}$ and $Y=\frac{1}{2}$, the contribution comes only from the Nd channel, while for the third values, $X=\frac{1}{2}$ and $Y=\frac{3}{2}$, and $N\phi$ channel gives the amplitude.

To avoid the numerical difficulties of the four-body problem, Fonseca and Shanley[6] inserts some approximations into the three-body amplitudes. Assuming that the three-body problem is characterized by the three $l=0$ three-body amplitudes (D,D), (Q,D), and (D,Q), they then approximated the three-body amplitudes to have a separable form in momentum space by specifying the total spin and the total isospin as

$$\langle \vec{k} \mid T^{XY}_{xx'}(E) \mid \vec{k}' \rangle = \lambda_{xy} g_{xy}(k) t_y(E+\epsilon_y) \times \lambda_{x'y} g_{x'y}(k') . \tag{4}$$

y is t, t', or t'', according to the total spin and total isospin of the channel interaction. λ is the coupling constant of each interaction and $g(k)$ is the vertex function, taken to be energy independent. ϵ_t is the triton binding energy observable as 8.48 MeV. With these approximations, the three-body amplitude is represented by a one term representation which includes the feature of the exact problem and with parameters which could be adjusted to the on-shell three-body data.

For the four-body sector, the total spin is S and the total isospin is I. We are studying the nuclear reactions which in the initial channel are initiated by two incoming interacting deuterons. For these reactions the total isospin I is purely specified. The four-body equations have to be expressed in such a way so that no quasiparticle-quasiparticle state appears as an off-shell external line. Thus, the four-body equations for the elastic and rearrangement scattering processes of two incoming deuterons could be written in the Lippmann-Schwinger form, identifying the Born terms and the intermediate propagators.

For these kind of processes we have $d+d \rightarrow N+y$ and $d+d \rightarrow d+d$. The elastic amplitude of $d+d \rightarrow d+d$ is the half-on-shell amplitude of the rearrangemnt amplitude of $d+d \rightarrow N+y$. The integral equation of the rearrangement processes is given by

$$\langle \vec{k} \mid \tau^{SI}_{ddy}(E) \mid \vec{k}' \rangle = \langle \vec{k} \mid B^{SI}_{ddy}(E) \mid \vec{k}' \rangle + \sum_{y''} \int d^3k'' \frac{1}{(2\pi)^3} \langle \vec{k} \mid A^{SI}_{yy''}(E) \mid \vec{k}'' \rangle \times t_{y''}(E+\epsilon_{y''} - \tfrac{4}{3} k''^2) \langle \vec{k}'' \mid \tau^{SI}_{ddy''}(E) \mid \vec{k}' \rangle . \tag{5}$$

$B_{ddy}(E)$ refer to the single-particle exchange Born terms. The summation over y'' is taken t, t', and t''. $A_{yy''}(E)$ is the driving term given by

$$A_{yy''}(E) = B_{yy''}(E) + C_{yy''}(E) + F_{yy''}(E) . \tag{6}$$

In Eq. (6), the $B_{yy''}(E)$ term is the two-nucleon, deuteron or/and ϕ particle exchange Born term. $C_{yy''}(E)$ and $F_{yy''}(E)$ corresponds to the sum over some amplitudes and to the sum over some two amplitudes which may have as intermediate states the two plus two channels of dd, $d\phi$, ϕd, and $\phi\phi$. The exchange of two correlated and fully interacting particles is described by the Born term $B_{yy''}(E)$, while $C_{yy''}(E)$ and $F_{yy''}(E)$ describe the exchange of two uncorrelated particles in a two-step process. The amplitudes of the elastic scattering of two incoming deuterons are given by integral equations related to the rearrangement process amplitudes as

$$\langle \vec{k} \mid \tau^{SI}_{dd,dd}(E) \mid \vec{k}' \rangle = \sum_{y} \int d^3k'' \frac{1}{(2\pi)^3} \langle \vec{k} \mid B^{SI}_{ddy}(E) \mid \vec{k}'' \rangle \tau_y(E+\epsilon_y - \tfrac{4}{3} k''^2) \langle \vec{k}'' \mid \tau^{SI}_{ddy}(E) \mid \vec{k}' \rangle . \tag{7}$$

Equations (5) and (7) are represented graphically in Figs. 1(a) and (b), respectively. In Fig. 2(a), the graphical representation of the amplitude of the process $Nx \rightarrow Nx'$ is given, with y t, t', or t''. In Fig. 2(a) a graphical representation for the integral equation of the amplitudes is given by Eq. (6). In Fig. 2(b), the first term on the right-hand side is the two-particle exchange Born term $B_{ty'}(E)$. The second term on the right-hand side of Fig. 2(b) stands for the box amplitude $C_{ty'}(E)$. The sum of the two box amplitudes $F_{ty'}(E)$ is illustrated graphically by the third and fourth sums on the right-hand side of Fig. 2(b). In Fig. 2(b), we consider the approximation that y is the particle t.

With these equations, the three-body vertex functions and propagators have been well defined for the one plus three and two plus two subamplitudes. As long as the two-body and three-body inputs are well defined, then, the four-body amplitudes are given by four-body equations which are reduced to one variable integral equations.

The p-wave three-body amplitudes can be considered in the same way. It is easy and straightforward to include these p waves in the three-body amplitudes. The result gives many terms, which also can be managed on the computer for numerical cal-

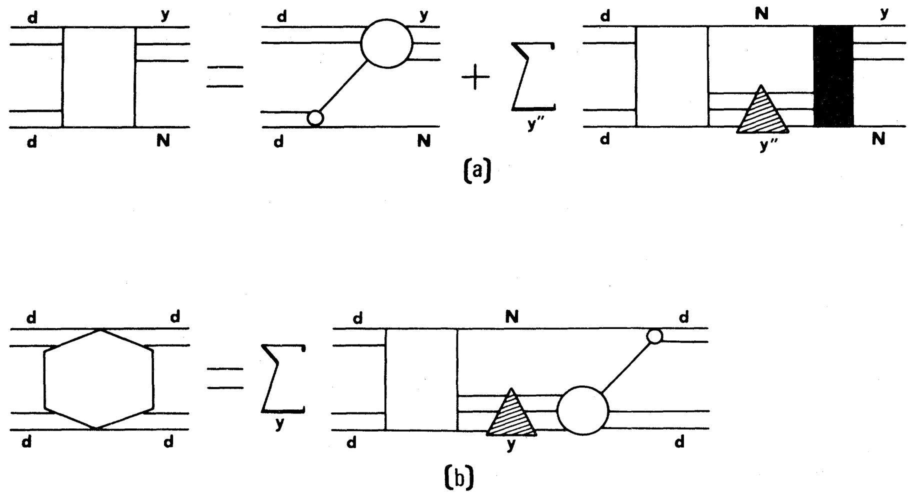

FIG. 1. (a) Graphical representation of the integral equation for the $d+d \rightarrow y+N$ amplitude. (b) Graphical representation of the integral equation relating the $d+d \rightarrow d+d$ amplitude and the half-on-shell $d+d \rightarrow y+N$ amplitude.

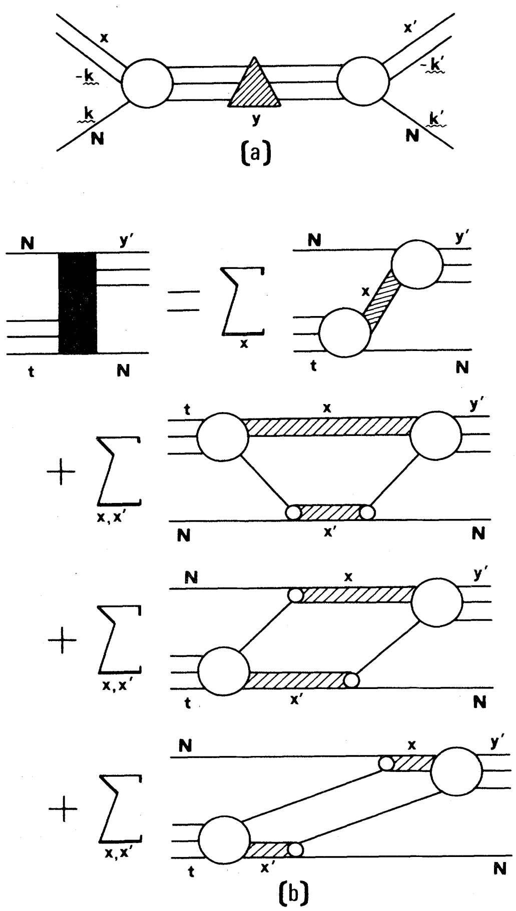

FIG. 2. (a) Graphical representation of the $N+x \rightarrow x'+N$ amplitude. (b) Graphical representation of the driving term in the integral equation for the $N+t \rightarrow y'+N$ amplitude.

culations. Also, the simultaneous exchange of two nucleons between the two incoming deuterons is treated. This is easily seen in Figs. 1 and 2.

The transition amplitudes of the four-nucleon elastic and rearrangement scattering have been given by these equations. Then, the nuclear reactions initiated with two incoming interacting deuterons have been studied. If i denotes the initial channel, while f stands for the final channel, then $T_{i,f}$ denotes the transition amplitudes for such reactions. Thus, the differential cross sections for the scattering and for the rearrangement nuclear reactions are given as

$$\frac{d\sigma}{d\Omega} \sim |T_{i,f}(\vec{k},\vec{k}')|^2 . \tag{8}$$

III. NUMERICAL CALCULATIONS AND RESULTS

In the present work, the four-nucleon problem is considered. In Sec. II, the four-nucleon amplitudes and integral equations were given for the case of elastic and rearrangement scattering between (incoming) two interacting deuterons. The four-nucleon amplitudes are given with inputs of the two- and three-body amplitudes. To calculate the four-body amplitudes, the coupling constants and vertex functions of the two- and three-body amplitudes have to be defined. These have been given by Eqs. (1), (2), and (4). These renormalized parameters of the interactions are calculated and determined in such a way so as to fit the most important observables of the two- and three-nucleon systems. Different observables are considered, especially the binding energies of these systems. The binding energy of the deuteron ϵ_d is taken as 2.225 MeV, while

TABLE I. The three-body T matrix parameters.

Three-nucleon system y / Parameters	t $X=\frac{1}{2}$, $Y=\frac{1}{2}$ Nd	t $N\phi$	t' $X=\frac{3}{2}$, $Y=\frac{1}{2}$ Nd	t'' $X=\frac{1}{2}$, $Y=\frac{3}{2}$ $N\phi$
β_1 (fm^{-1})	1.953	0.795	0.269	0.448
β_2 (fm^{-1})	1.891	2.219	1.837	2.318
β_3 (fm^{-1})	3.612	2.643	2.716	2.384
λ (λ^2=fm^{-13})	1156.478	-228.657	49.164	18.26
ϵ_t (MeV)	8.48			
$Z_t=0.0$	0.0			

we consider for the triton binding energy ϵ_t a value of 8.48 MeV. In Eq. (4), the vertex functions $g_{xy}(k)$ are chosen by Alt, Grassberger, and Sandhas[5] to have the same momentum dependence as the exact three-body functions. In their work, Alt *et al.*[5] introduced, with reasonable accuracy, a representation for these vertex functions in the case of $E < -\epsilon_d$ as

$$g(k;E)\approx\{[k^2+\beta_1^2(E)][k^2+\beta_2^2(E)]\times[k^2+\beta_3^2(E)]\}^{-1}, \quad (9)$$

where the parameters β_1, β_2, and β_3 depend upon the three-body center-of-mass energy E. Fonseca[6] used an expression similar to Eq. (9) but with β_1, β_2, and β_3 being energy independent. For simplicity we also use in the present calculations an expression similar to Eq. (9) for the vertex functions $g(k;E)$, with the parameters β_1, β_2, and β_3 being energy independent. Also, the coupling constants of the three-body systems are determined and calculated in such a way that the $N+d$ and $N+\phi$ states contribute with equal mixtures to the three-nucleon wave function. This means that the approximations of the triplet and singlet nucleon-nucleon interactions are considered[12] identical. Also, the rescattering continuum of the triplet nucleon-nucleon interactions is contained as well as the nucleon plus deuteron contributions. For the (D,D) case, the parameters are determined to fit the three-body bound-state energy.[13] For cases (Q,D) and (D,Q), where there are no three-body bound states, the parameters are determined to fit the vertex function at the two-body scattering threshold $E=-\epsilon_d$.

Numerical calculations have been carried out for Eqs. (1), (2), and (4) to obtain the values of the coupling constants λ and the vertex functions $g(k)$. The phase shifts are also numerically calculated. From the comparison of the calculated phase shifts, with recent phase shift analyses, the values of the different parameters of the two-nucleon and three-nucleon interactions are obtained. The different values obtained for the different parameters are given in Table I. These values are different from but near the values obtained previously. The differences exist because we include in the present calculations the p-wave amplitudes as well as the simultaneous exchange of two nucleons between the two incoming interacting deuterons.

The differential cross sections of the rearrangement scattering between two incoming deuterons are calculated. Numerical calculations of the cross sections of the rearrangement processes $d+d\rightarrow{}^3\text{H}+p$ and $d+d\rightarrow{}^3\text{He}+n$ are carried out using the two- and three-body interaction parameters in Table I, for different deuteron laboratory energies between 6.1 and 51.5 MeV. The present theoretically calculated values of the angular distributions are shown in Figs. 3—5. In the same figures the experimental measurements[14–16] are introduced. Also, the differential cross sections of the elastic scattering between two incoming deuterons have been calculated theoretically using the equations given in Sec. II. The calculated angular distributions as well as the experimental measurements[17] are given in Fig. 6. For the purpose of comparison with other previous calculations, we introduced in Figs. 3—6 the differential cross sections without the p-wave three-body amplitudes. Neglect of the p-wave three-body amplitudes and the simultaneous exchange of two nucleons between the two incoming interacting deuterons had been suggested previously by Fonseca.[6] Theoretical calculations not including these two effects are performed for the present reactions and are shown by the dashed curves in Figs. 3—6. The present theoretical calculations of the angular distributions for the present reactions, including both of the p-wave three-body amplitudes as well as the simultaneous exchange of two nucleons between the two incoming interacting deuterons, are shown in Figs. 3—6 by the solid curves. Theoretical calculations of the differential cross sections including these two effects (solid curves), are closer to the experimental data than that calculation neglecting these two effects (dashed curves). Thus from Figs. 3—6, we see that the present theoretically calculated differential cross sections including the p-wave

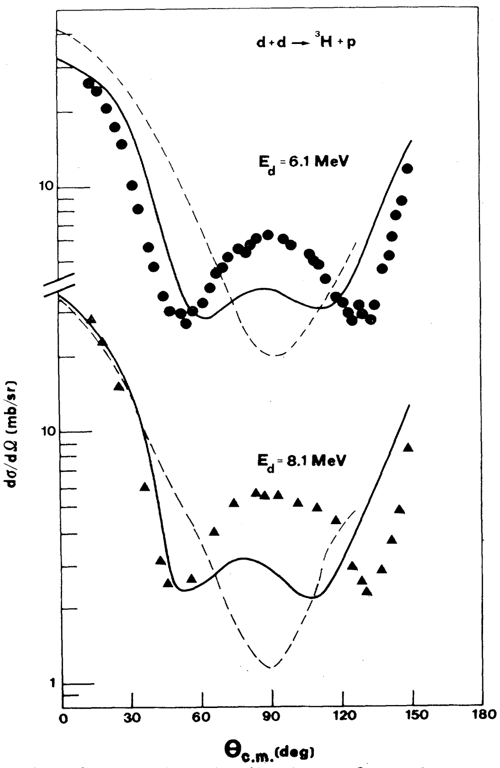

FIG. 3. The angular distributions of the $d+d \rightarrow {}^3H+p$ reactions at different deuteron laboratory energies. The solid curves are our present calculations including the p-wave three-body amplitudes and the simultaneous two nucleon exchange between the two deuterons. The dashed curves are the calculations without these effects. The three-body T matrix parameters are given in Table I. The points are the experimental data and are taken from Ref. 14.

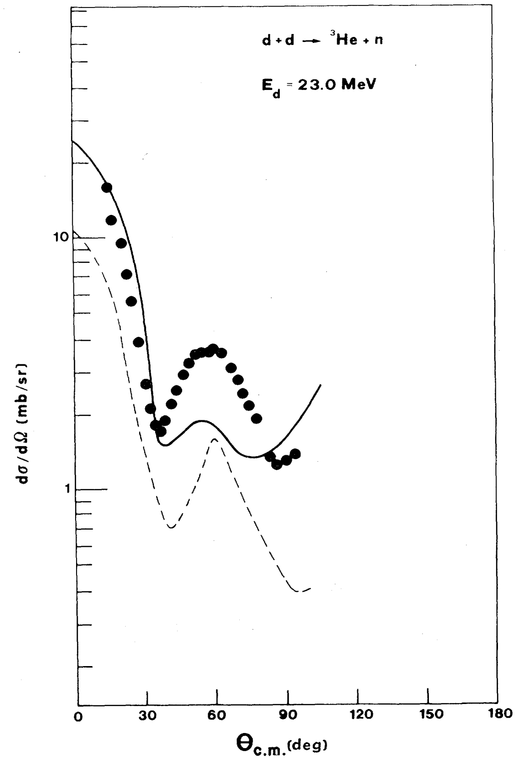

FIG. 4. The angular distributions of the $d+d \rightarrow {}^3He+n$ reactions at deuteron laboratory energy 23.0 MeV. The solid curve is our present calculations including the p-wave three-body amplitudes and the simultaneous two nucleon exchange between the two deuterons. The dashed curves are the calculations without these effects. The three-body T matrix parameters are given in Table I. The points are the experimental data and are taken from Ref. 15.

three-body amplitudes and also the simultaneous exchange of two nucleons between the two incoming interacting deuterons reasonably reproduce the shape and the magnitude of the experimentally measured values. Fitting the calculated angular distributions to the experimental data, the extracted values of the spectroscopic and normalization factors of these reactions are obtained. The extracted values of the spectroscopic and normalization factors for all the considered reactions are given in Table II.

IV. DISCUSSION AND CONCLUSIONS

In the present work, the four-body matrix elements and integral equations are introduced. These equations are particularly useful in studying the four-nucleon system of elastic and rearrangement scattering for two incoming interacting deuterons. We are interested here in such reactions. The present integral equations have been applied and numerical calculations of the differential cross sections have been computed. The four-body matrix elements are given in such a way so as to include the two- and three-body scattering processes. The two- and three-body scattering go through intermediate quasiparticles. The parameters of such two- and three-body interactions are obtained by fitting the two-and three-nucleon experimentally observed properties such as the binding energies and phase shifts. In the present calculations, the one plus three and two plus two subamplitudes are treated and taken into account. These subamplitudes are treated here and represented by a one separable term, which differs from previous considerations[13,18] of the subamplitudes which introduced a complete separable expansion of these subamplitudes. The p-wave three-body amplitudes are included in the present calculations. Moreover, the simultaneous exchange of two nucleons between the two incoming interacting deuterons has been taken into account. Calcula-

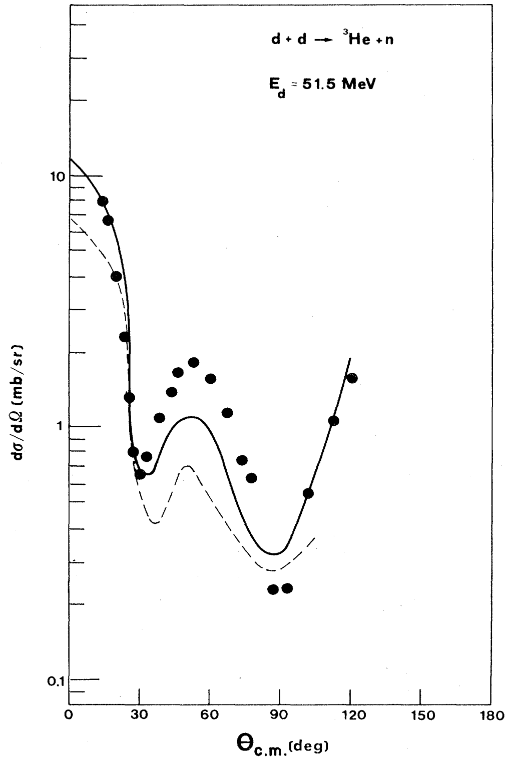

FIG. 5. The angular distributions of the $d+d\rightarrow{}^3\mathrm{He}+n$ reactions at deuteron laboratory energy 51.5 MeV. The solid curve is our present calculations including the p-wave three-body amplitudes and the simultaneous two nucleon exchange between the two deuterons. The dashed curves are the calculations without these effects. The three-body T matrix parameters are given in Table I. The points are the experimental data and are taken from Ref. 16.

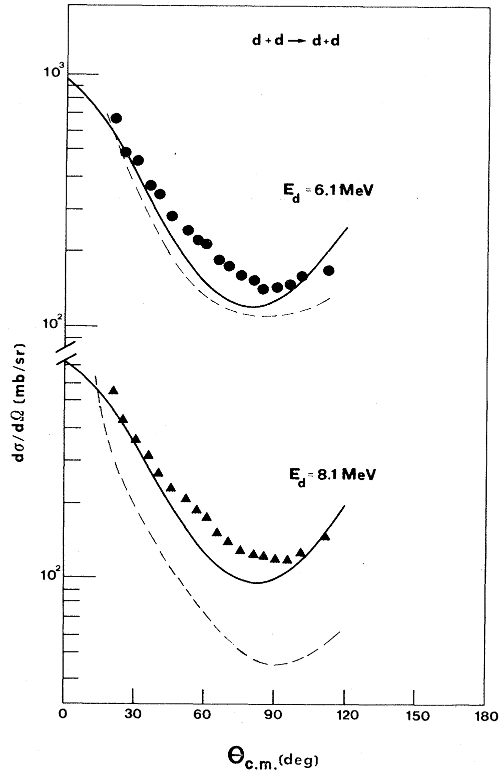

FIG. 6. The differential cross sections of the $d+d\rightarrow d+d$ elastic scattering reactions at different deuteron laboratory energies. The solid curves are our present calculations including the p-wave three-body amplitudes and the simultaneous two nucleon exchange between the two deuterons. The dashed curves are the calculations without these effects. The three-body T matrix parameters are given in Table I. The points are the experimental data and are taken from Ref. 17.

tions without these effects are shown by the solid curve. Thus, we see that by including the p-wave three-body amplitudes and taking into account the simultaneous exchange of two nucleons between the two incoming interacting deuterons have two clear features. These features in the differential cross sections are seen by comparing the solid and dashed curves with the experimental measurements. From Figs. 3—5, it is seen that including these effects (solid curves) produces the peaks, the minima, and the maxima of the experimental angular distributions which had not appeared in other theoretical calculations which neglected these effects (dashed curves). Also, and as another feature, the magnitudes of the differential cross sections calculated by including these effects (solid curves) are raised and are closer to that of the experimental data than those obtained for calculations done without these effects (dashed curves).

From Figs. 3—6, we see that the present calculations including the p-wave three-body amplitudes and also the simultaneous exchange of two nucleons between the two incoming interacting deuterons display the main features of the experimental angular distributions. Our present calculations also reproduce the values of the measured cross sections. Also, a glance at Table II shows that the extracted

TABLE II. Extracted spectroscopic and normalization factors.

Nuclear reaction	Incident energy (MeV)	Spectroscopic and normalization factors
$^2\mathrm{H}(d,p)^3\mathrm{H}$	6.1	1.0128
	8.1	1.2694
$^2\mathrm{H}(d,n)^3\mathrm{He}$	23.0	1.1246
	51.5	1.0839
$^2\mathrm{H}(d,d)^2\mathrm{H}$	6.1	1.2165
	8.1	1.2843

spectroscopic and normalization factors have reasonable values. Thus, we can conclude that the present calculations reproduce the experimental differential cross sections in both shape and magnitude.

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[1]For a review, see *Proceedings of the Seventh International Conference of Few-Body Problems in Nuclear and Particle Physics, Delhi, 1976,* edited by A. N. Mitra *et al.* (North-Holland, Amsterdam, 1976).

[2]For another review, see *Proceedings of the Eighth International Conference on Few Body Systems and Nuclear Forces II, Graz, 1978,* edited by H. Zingl, M. Haftel, and H. Zangel (Springer, Berlin, 1978), Vols. 82 and 87.

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