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Symmetry relations between the polarization, analyzing power, and spin rotation functions in (p,p') reactions

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The symmetries of the collision matrix for the (p,p') reaction are studied in the framework of the adiabatic (frozen nucleus) approximation. Two examples of resulting relationships between the proton polarization observables are discussed in detail. The equality of the polarization and the analyzing power is found as well as the relations between the spin rotation functions.

[NUCLEAR REACTIONS Relations between scattering observables for (p,p') reactions in the adiabatic approximation.]

For the elastic scattering of spin $\frac{1}{2}$ particles by a target of arbitrary spin the equality between the polarization P and the analyzing power A_y was established by Wolfenstein and Ashkin.[1] This result follows from the invariance of the interaction under time reversal. For the case of inelastic scattering, arguments based on time-reversal symmetry lead to the polarization-analyzing power theorem, i.e., the equality between the analyzing power in the direct (excitation) reaction and the polarization in the inverse (deexcitation) reaction.[2] Also, more general relations between the polarization and analyzing power tensors were derived in Ref. 3. However, general symmetry arguments—made without recourse to a model for the reaction mechanism—do not guarantee that $P-A_y$ vanishes in a given channel (direct or inverse) for (p,p') transitions.

The purpose of this paper is to show that in the framework of any theory for the (p,p') reaction which is based on the adiabatic (frozen nucleus) approximation,[4–7] symmetry of the scattering operator under the inversion of the projectile motion leads to an equality between P and A_y for a given (p,p') transition as well as to a relation between two spin rotation observables D_{zx} and D_{xz} defined below. The second relation can also be expressed in terms of the experimentally determined spin rotation functions $D_{LS'}$ and $D_{SL'}$ (A and R' in Wolfenstein's notation[8]). (S,N,L) and (S',N',L') represent right handed coordinate frames where N and N' are in the direction $\vec{k}_L \times \vec{k}_{L'}$, and L and L' are in the respective directions of $\vec{k}_L$ and $\vec{k}_{L'}$, $\vec{k}_L$ and $\vec{k}_{L'}$ being the initial and final laboratory momenta of the proton, respectively. Preliminary results of this work were reported in Ref. 9.

Many characteristics of the adiabatic approximation have been known for a long time and discussed by several authors.[4–7,10–12] The present study, however, provides the first derivation of the relations between the proton spin observables with the full spin dependence of the projectile-target nucleus interaction taken into account. In the adiabatic approximation, it is assumed that

$$\frac{\text{(target excitation energy)}}{\text{(projectile kinetic laboratory energy)}} \to 0 \quad , \tag{1}$$

for all the excited states of the target nucleus which, thus, can be treated as being degenerate in energy. This degeneracy implies that the target degrees of freedom are frozen during the passage of the projectile through the target and that the elastic and inelastic transition amplitudes are given as the matrix elements of the same collision operators. In particular, the presence of the knock-on (exchange) terms is not allowed in the adiabatic limit. The adiabatic approximation is an underlying assumption of, for example, Glauber theory[7] as well as of its extended version[13] which corrects for finite energy effects. From (1) it follows that, for inelastic scattering in the adiabatic approximation,

$$|\vec{k}_i| = |\vec{k}_f| \quad , \tag{2}$$

where $\vec{k}_i$ and $\vec{k}_f$ are the initial and final projectile momenta in the center of mass frame. Because of condition (2) it is natural to expect that some symmetries of the elastic scattering reaction will also be valid for inelastic reactions. However, considerable confusion exists in the literature concerning the conditions under which relations of the type $P = A_y$ are valid for (p,p') reactions.

Let us consider the scattering process

$$p(S) + A(S_i) \to p(S) + A^*(S_f) \ , \tag{3}$$

where p is the projectile A with spin $S = 1/2$, and A and A^* are the respective initial and final target states with spins S_i and S_f, respectively. The general form of the scattering amplitude for this reaction can be written, using rotational invariance, as the scalar product of two tensor operators acting in the projectile and target spin spaces, respectively[14]:

$$F = \hat{F}_0\sigma_0 + \hat{F}_x\sigma_x + \hat{F}_y\sigma_y + \hat{F}_z\sigma_z \ , \tag{4}$$

where $\sigma_0 = 1$, and the Pauli spin operators σ_η, $\eta = x$, y, and z, act on the spin $\frac{1}{2}$ projectile in the Cartesian frame where the x, y, and z axes are parallel to $\vec{k}_f - \vec{k}_i$, $\vec{k}_i \times \vec{k}_f$, and $\vec{k}_i + \vec{k}_f$, respectively. The tensor components $\hat{F}_\eta = \hat{F}_\eta(\vec{k}_i, \vec{k}_f, S_i, S_f)$, $\eta = 0, x, y,$ and z, are operators which can be represented as $(2S_f + 1)(2S_i + 1)$ dimensional matrices connecting the spin spaces of the initial and final nuclear states. In the frame of reference we have chosen the invariance of the collision operator F, under the space reflection imposes the following transformation properties on the operators $\hat{F}_\eta$:

$$\hat{F}_0 \to +\hat{F}_0, \quad \hat{F}_x \to -\hat{F}_x, \quad \hat{F}_y \to +\hat{F}_y, \quad \hat{F}_z \to -\hat{F}_z \ , \tag{5}$$

We shall now discuss the consequences of the above behavior of the $\hat{F}_\eta$ on the projectile spin observables in the scattering process where the initial target state (S_i) is unpolarized. All these observables can be expressed in terms of the functions D_{nm}

$$D_{nm}(\vec{k}_i, \vec{k}_f) = \frac{\mathrm{Tr}(F\sigma_m F^\dagger \sigma_n)}{\mathrm{Tr}(FF^\dagger)} \ , \tag{6}$$

where $m,n = 0, x, y,$ and z and the traces are taken with respect to the projectile and nuclear spin projections. The subscripts m and n refer to the spin projections in the entrance and exit channels, respectively. Using Eqs. (4) and (6) we can express the D_{nm} in terms of 16 traces with respect to the products of the tensor components $\hat{F}_\eta$

$$D_{nm} = \frac{1}{2} \sum_{\substack{\eta,\eta' = \\ 0,x,y,\text{ and } z}} \mathrm{Tr}''(\sigma_\eta \sigma_m \sigma_{\eta'} \sigma_n) h_{\eta\eta'}(\vec{k}_i, \vec{k}_f) \ , \tag{7}$$

where

$$h_{\eta\eta'}(\vec{k}_i, \vec{k}_f) = \frac{\mathrm{Tr}'(\hat{F}_\eta \hat{F}^\dagger_{\eta'})}{\mathrm{Tr}'\left[\sum_\xi \hat{F}_\xi \hat{F}^\dagger_\xi\right]} \ . \tag{8}$$

Here Tr′ and Tr″ indicate traces taken with respect to the target and the projectile spin projections, respectively. The functions $h_{\eta\eta'}$ have to transform under space reflection according to Eqs. (5). The resulting symmetry properties of the $h_{\eta\eta'}$ are listed in Table I. On the other hand, the $h_{\eta\eta'}$ are functions only of $\vec{k}_i$ and $\vec{k}_f$. Since it is impossible to construct a pseudoscalar function from $\vec{k}_i$ and $\vec{k}_f$, we conclude that, as a consequence of invariance under the parity transformation, the functions $h_{\eta\eta'}$ from the last two lines of Table I have to vanish (compare the discussion in Ref. 1). Consequently, it follows from Eq. (7) that D_{0x}, D_{x0}, D_{z0}, D_{0z}, D_{yx}, and D_{xy} are equal to zero.

Let us now consider four proton spin observables:

$$P = D_{y0}; \quad A_y = D_{0y}; \quad D_{zx}; \quad D_{xz} \ . \tag{9}$$

From (7) we can derive the following expressions for P, A_y, D_{zx}, and D_{xz} in terms of two functions h_{0y} and h_{xz}:

$$\begin{aligned} P &= 2\,\mathrm{Re}(h_{0y}) + 2\,\mathrm{Im}(h_{xz}) \ , \\ A_y &= 2\,\mathrm{Re}(h_{0y}) - 2\,\mathrm{Im}(h_{xz}) \ , \\ D_{zx} &= -2\,\mathrm{Im}(h_{0y}) + 2\,\mathrm{Re}(h_{xz}) \ , \\ D_{xz} &= 2\,\mathrm{Im}(h_{0y}) + 2\,\mathrm{Re}(h_{xz}) \ , \end{aligned} \tag{10}$$

where we have used the relations $h^*_{\xi\xi'} = h_{\xi'\xi}$. The observables D_{zx} and D_{xz} are generalizations of the spin rotation function Q introduced in the context of elastic scattering of protons from a spin zero nucleus by Glauber and Osland.[15] It is interesting to note that, for inelastic scattering, two such observables exist. For elastic scattering it follows from time-reversal invariance that

$$h_{xz} = h_{zx} = 0 \ , \tag{11}$$

because it is impossible to construct a scalar function $h_{\eta\eta'}(\vec{k}_i, \vec{k}_f)$ which is odd under time reversal $(\vec{k}_i \to -\vec{k}_f;\ \vec{k}_f \to -\vec{k}_i)$ from vectors $\vec{k}_i$ and $\vec{k}_f$ of equal length (under the time reversal transformation $\hat{F}_x \to +\hat{F}_x$ and $\hat{F}_z \to -\hat{F}_z$). Therefore a familiar result[1]

TABLE I. Behavior of the functions $h_{\eta\eta'}(\vec{k}_i, \vec{k}_f)$ under the space reflection transformation. S and PS denote scalar and pseudoscalar, respectively.

	$h_{\eta\eta'}$					
S	h_{00},	h_{xx},	h_{yy},	h_{zz},	h_{0y},	h_{y0}
S	h_{zx},	h_{xz}				
PS	h_{0x},	h_{x0},	h_{xy},	h_{yx}		
PS	h_{0z},	h_{z0},	h_{yz},	h_{zy}		

is obtained

$$P = A_y \ , \tag{12}$$

as well as

$$D_{zx} = -D_{xz} \ . \tag{13}$$

Now, if the (p,p') process is treated in the adiabatic approximation it turns out, by inspecting the behavior of the scattering operator F under the transformation of the inversion of the projectile motion $(\vec{k}_i \to -\vec{k}_f; \vec{k}_f \to -\vec{k}_i; \vec{\sigma} \to -\vec{\sigma})$ that the corresponding function h_{xz} also vanishes. Following Refs. 10 and 11 let us use the general decomposition of the operator F in the adiabatic approximation:

$$F = F^{(+)} + F^{(-)}$$

with $F^{(+)}$ and $F^{(-)}$ changing under the above mentioned transformation as follows:

$$F^{(\pm)}(\vec{k}_i, \vec{k}_f, \vec{\sigma}, S_i, S_f) \to \pm F^{(\pm)}(-\vec{k}_f, -\vec{k}_i, -\vec{\sigma}, S_i, S_f) \ .$$

By using the analogous decompositions for the components $\hat{F}_\eta$, $\eta = 0,x,y,z$, we find that only the products of both even and both odd components of $\hat{F}_x$ and $\hat{F}_z$ contribute to h_{xz}, i.e.,

$$h_{xz}(\vec{k}_i, \vec{k}_f) = \mathrm{Tr}'(\hat{F}_x^{(+)}\hat{F}_z^{(+)\dagger} + \hat{F}_x^{(-)}\hat{F}_z^{(-)\dagger}) \Big/ \mathrm{Tr}\left[\sum_\xi \hat{F}_\xi \hat{F}_\xi^\dagger\right] \ ,$$

because h_{xz} is a scalar function of two vectors of equal length. On the other hand by using the transformation properties of $\hat{F}^{(\pm)}$ and $\hat{F}^{(\pm)}$ under the inversion of the projectile motion and the rotation by 180° about the x axis[10,11] we find that the matrix elements of the components $\hat{F}_x^{(\pm)}$ and $\hat{F}_z^{(\pm)}$ between the nuclear states with spin projections M_i and M_f satisfy the following relations:

$$\langle M_f | \hat{F}_x^{(\pm)} | M_i \rangle = \pm (-1)^{S_f - S_i} \langle -M_f | \hat{F}_x^{(\pm)} | -M_i \rangle$$

and

$$\langle M_f | \hat{F}_z^{(\pm)} | M_i \rangle = \mp (-1)^{S_f - S_i} \langle -M_f | \hat{F}_z^{(\pm)} | -M_i \rangle \ .$$

From the last two relations and Eq. (8) it follows trivially that $h_{xz} = h_{zx} = 0$. Consequently, Eqs. (12) and (13) hold for the (p,p') reaction in the adiabatic approximation. The spin rotation parameters $D_{LS'}$ and $D_{SL'}$ described earlier are given by

$$D_{LS'} = \tfrac{1}{2}[(D_{zx} + D_{xz}) - (D_{zx} - D_{xz})\cos\theta_L - (D_{xx} + D_{zz})\sin\theta_L] \ ,$$

$$D_{SL'} = \tfrac{1}{2}[(D_{zx} + D_{xz}) + (D_{zx} - D_{xz})\cos\theta_L + (D_{xx} + D_{zz})\sin\theta_L] \ ,$$

where θ_L is the laboratory angle of the scattered proton [we are assuming (projectile mass)/(target mass) $\to 0$]. From Eq. (13) it follows that $D_{LS'}$ and $D_{SL'}$ satisfy analogous relations as D_{zx} and D_{xz}:

$$D_{LS'} = -D_{SL'} \ . \tag{14}$$

Equation (14) is true for the p-nucleus elastic scattering, in general, and for the (p,p') reactions in the adiabatic approximation.

As a simple example of the relationships derived above let us consider the polarization, analyzing power, and spin rotation functions for the reaction $p(\frac{1}{2}) + A(0^+) \to p(\frac{1}{2}) + A^*(0^-)$. From the parity invariance alone it follows that the terms $\hat{F}_0$ and $\hat{F}_y$ in the general collision matrix [Eq. (4)] are both equal to zero; thus

$$F_{0^+ \to 0^-} = F_x \sigma_x + F_z \sigma_z \ .$$

Equations (7), (8), and (10) then lead to

$$\begin{aligned} &P = -A_y = 2\,\mathrm{Im}(h_{xz}) \ , \\ &D_{zx} = D_{xz} = D_{LS'} = D_{SL'} = 2\,\mathrm{Re}(h_{xz}) \ , \\ &D_{xx} = D_{SS'} = 1 - 2h_{zz} \ ; \ D_{yy} = D_{NN'} = 1 \ ; \\ &D_{zz} = D_{LL'} = -1 + 2h_{zz} \ . \end{aligned} \tag{15}$$

In the adiabatic approximation for the inelastic scattering process, time-reversal symmetry implies that F_z is equal to zero. Therefore $h_{xx}(\vec{k}_i, \vec{k}_f)$ is the only nonzero function $h_{\eta\eta'}$; and from Eq. (8), $h_{xx} = 1$. Then the only nonzero D_{nm} functions are

$$D_{00} \equiv 1 \ ; \ D_{xx} = 1 \ ; \ D_{yy} = 1 \ ; \ D_{zz} = -1 \ .$$

Thus, in the adiabatic approximation, the spin observables in a $0^+ \to 0^-$ (p,p') transition would be predicted to be

$$\begin{aligned} &P = A_y = D_{zx} = D_{xz} = D_{LS'} = D_{SL'} = 0 \ , \\ &D_{SS'} = 1 \ ; \ D_{NN'} = 1 \ ; \ D_{LL'} = -1 \ . \end{aligned} \tag{16}$$

Comparison of Eqs. (15) and (16) indicates that the measurement of the spin observables for a $0^+ \to 0^-$ (p,p') transition would provide an excellent test of the validity of the adiabatic approximation in the reaction mechanism.

Our results can thus be useful in understanding the observed relations between the projectile polarization observables, i.e., in distinguishing what follows from general symmetry principles and what is related to the specific excitation mechanism. Recent experimental developments in high resolution polarization experiments[16,17] in nuclear scattering of protons at intermediate energies allow very accurate tests of such relations. (Measurements of $P - A_y$ and $D_{LS'} + D_{SL'}$ for (p,p') reactions were recently proposed in order to obtain the information on the exchange processes.[18]) We also mention that, by proceeding in an analogous way in the context of a general reaction for a projectile of arbitrary spin Eq. (3), relations between the projectile spin tensor observables can be found.[19] Such relations can be tested, e.g., in the context of (d,d') reactions at the Saturne accelerator. Also, the validity of the relations discussed here can be extended in a straightforward way for (p,n) reactions.

Finally, a remark is in order here on other applications of our results. Although the above discussion was in terms of nuclear (p,p') reactions the symmetry relations derived above are also valid in other processes, provided that the underlying interactions are time-reversal invariant and the adiabacity condition is satisfied. Thus they hold in the context of inelastic electron scattering from atoms as well as in the scattering of elementary particles (for example, in the diffractive excitation of nucleon isobars at high energies).

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