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# Influence of $\alpha$-cluster formation on $\alpha$ decay

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Dependence of $\alpha$-cluster and *pp*- and *nn*-cluster formation on high-lying configurations (continuum) in nuclei is studied. Its importance for $\alpha$-decay calculations is discussed.

[RADIOACTIVITY $\alpha$-decay, $\alpha$-transfer reactions, high-lying configurations (continuum).]

The understanding of nucleon clustering in nuclei is one of the standing problems in nuclear theory. Especially in $\alpha$ decay and $\alpha$-transfer reactions the formation of the $\alpha$ particle in the mother (or residual) nucleus is crucial for the correct description of these processes. Since these clusters are formed at the surface of the nucleus or beyond it, a microscopic description of them requires the inclusion of the continuum part of the single-particle spectrum. Therefore the solution even of the most simple problems (e.g., when the mother nucleus is frozen and only very few degrees of freedom are relevant) becomes a difficult undertaking.

In order to analyze this problem one has first to define the concept of clustering more precisely. The mere presence of the central field induced by the core tends to produce a clustering of the surface nucleons towards the center of the nucleus. Thus two particles with center of mass at a fixed distance from the nucleus center would have a relative wave function with a maximum at vanishing distance between them. This feature stems simply from the fact that the single-particle wave function of each particle diminishes exponentially with the distance from the center of the nucleus. In other words, these nucleons tend to approach the center of the nucleus and therefore also to approach each other. Such a clustering, which takes place even without any interaction among the clustering nucleons, is not the one in which we are interested in $\alpha$ decay or similar processes. In our understanding a "real" clustering occurs when the probability amplitude of the cluster becomes larger as the particles approach each other moving on the surface of a sphere centered at the nucleus center. Within this definition the central field does not produce any clustering, since the amplitude probability of a "cluster" of noninteracting particles would not depend upon the distances between these particles.

The most simple cluster we can consider is a pair of nucleons. Let us consider a wave function of two identical particles outside a magic core

$$\psi_{n_1}(\lambda\mu)=(\chi_1\chi_2)_0\sum_{p\leq q}X(pq;\mu\lambda)\,\hat{j}_p\hat{j}_q[C(pq;\vec{r}_1\vec{r}_2)-(-1)^{p+q-\lambda}C(qp;\vec{r}_1\vec{r}_2)]\ , \tag{1}$$

where

$$C(pq;\vec{r}_1\vec{r}_2)=R_p(r_1)\,R_q(r_2)(-1)^{l_q+1/2+j_p+\lambda}\begin{Bmatrix} l_p & j_p & \frac{1}{2}\\ j_q & l_q & \lambda\end{Bmatrix}\sum_{m_p,m_q}\langle l_pm_pl_qm_q|\lambda\mu\rangle\,Y_{l_pm_p}(\Omega_1)\,Y_{l_qm_q}(\Omega_2)\ ,$$

where we consider only the singlet ($S=0$) component of the wave function. The coordinates of the particles are $\vec{r}_1$ and $\vec{r}_2$; $R(r)$ is the radial wave function, $X$ is the two-particle wave function amplitude, and the rest of the notation is standard. The summation over $p$ and $q$ corresponds to the different two-particle configurations.

The calculation of $X$ may be done as usual by using the shell-model representation with a given nucleon-nucleon interaction.[1] In fact, as pointed out above, we should also include the continuum part of the representation. But in order to see the influence of high-lying single-particle states, we use the Bayman procedure[2] of binding the independent particle with $\frac{1}{2}$ of the pairing binding energy in a Wood-Saxon potential. This method was used in Ref. 3 to analyze absolute $\alpha$-decay rates in Po isotopes.

We applied Eq. (1) to study the angular distribution of the $^{210}$Po and $^{210}$Pb two-particle wave functions. The amplitudes $X$ were calculated within the single-particle states of Ref. 4. using a pairing residual interaction.

In Fig. 1 we show the calculated angular distributions as a function of the number of configurations for neutrons and protons, respectively. We notice that indeed the pairing interaction tends to produce a clustering of the two particles, as one would expect.[5] Yet the influence of high-lying configurations (i.e., the continuum) to produce the pairing cluster is very remarkable. These results may justify one to assume that a complete treatment of this problem would give a real $\delta$-like cluster. The question is now whether the more difficult case of $\alpha$ clustering exhibits the same features. In fact, most treatments of $\alpha$ decay up to now have treated the $\alpha$ particle as a point particle (the so-called $\delta$ approximation).[1] Moreover, recent theories of $\alpha$ decay treat the formation of the $\alpha$ particle inside the mother nucleus in terms of either very few single-particle configurations[6] or in terms of $2p$-$2n$ pairing vibrations.[3,7] In these last papers the $\alpha$ decay of $^{212}$Po was studied. The $^{212}$Po ground state was written as

$$^{212}\mathrm{Po(g.s.)}\rangle = |^{210}\mathrm{Pb(g.s.)} \otimes {}^{210}\mathrm{Po(g.s.)}\rangle \ , \qquad (2)$$

and many single-particle states were included to describe the two-particle system. Actually, this kind of wave function was already used in early sixtieth to analyze $\alpha$ decay.[8,9] But in these calculations only few configurations were used and no clustering would be evident within such a small shell-model subspace. An important result of the calculations in Refs. 3 and 7 was that great enhancements of $\alpha$-decay width were obtained. Yet this enhancement was not large enough to obtain the experimental value. As we have shown above, the inclusion of many configurations to calculate $^{210}$Pb(g.s.) and $^{210}$Po(g.s.) produces a pointlike cluster for each pair of particles. Therefore the $^{212}$At(g.s.) wave function (2) would not produce an $\alpha$-particle cluster, since the proton and the neutron pair are each one in an $s$ state. This implies that the wave function (2) would not depend upon the angle between neutrons and protons. Since such a description does not produce an $\alpha$-like structure in the mother nucleus, the overlap between the entrance and exit channel wave functions would be small.

As a conclusion, one can say that in order to get a good description of $\alpha$-decay processes within bases which include correlated states [as Eq. (2)] many two-particle states must be included. It is not enough to consider only one correlated configuration even if many single-particle states are used to evaluate the correlated states.

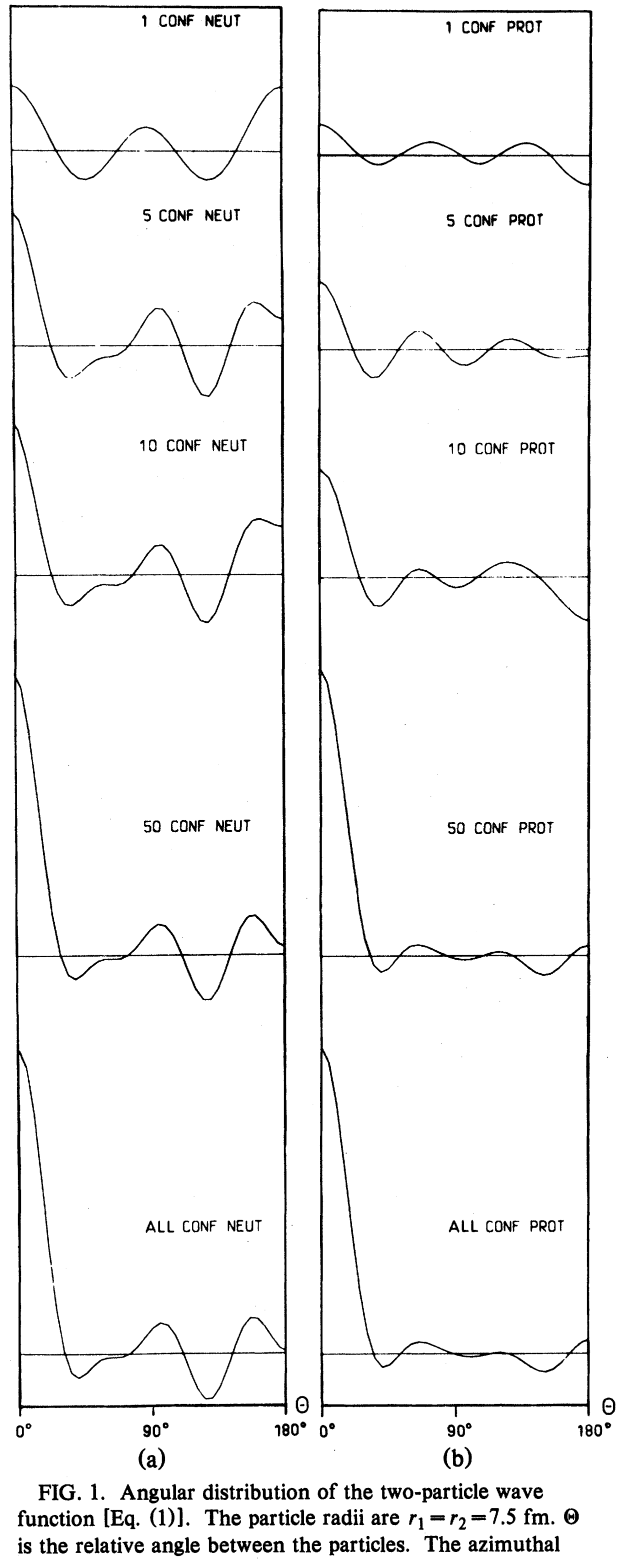


FIG. 1. Angular distribution of the two-particle wave function [Eq. (1)]. The particle radii are $r_1 = r_2 = 7.5$ fm. $\Theta$ is the relative angle between the particles. The azimuthal angles are $\phi_1 = \phi_2 = 0°$. The number of configurations included in the calculation is given in the figure. The single-particle states were taken from Ref. 4. (a) Neutrons ($^{210}$Pb), (b) protons ($^{210}$Po).

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