Brief Reports

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Fusion cross sections for the $^{165}Ho+^{56}Fe$ reaction

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Fusion cross sections are reported for the $^{165}Ho+^{56}Fe$ reaction at bombarding energies of 351, 421, 462, and 510 MeV. These results, along with those for the $^{165}Ho+^{40}Ar$ and $^{165}Ho+^{84}Kr$ reactions, are in agreement with a recent model of Swiatecki.

[NUCLEAR REACTIONS $^{165}Ho(^{56}Fe$, fusion-fission), $E_{lab}=351$, 421, 462, and 510 MeV. Deduced fusion cross sections; comparison to dynamical calculations.]

The purpose of this report is to present fusion cross sections for the $^{165}Ho+^{56}Fe$ reaction at the four bombarding energies 351, 421, 462, and 510 MeV. A study of the damped reactions between $^{165}Ho+^{56}Fe$ has been reported previously.[1]

Self-supporting targets of ^{165}Ho were bombarded with ^{56}Fe beams produced by the Lawrence Berkeley Laboratory SuperHILAC accelerator. Fusion-fission fragments were detected by a ΔE-E silicon surface-barrier-detector telescope.[1] Angular distributions of the products from fusion-fission reactions were measured at each energy. Over the angular range $18° \leq \theta_{c.m.} \leq 100°$, the value of $d\sigma_{fus}/d\theta_{c.m.}$ is constant within experimental error.[1] This result is equivalent to a $1/\sin\theta$ dependence of $d\sigma_{fus}/d\Omega_{c.m.}$.

The definition of fusion used here is analogous to that described earlier[2] where fusion includes the compound nucleus and fusionlike events. This is illustrated schematically in Fig. 1. If the experimental fusion cross sections are interpreted in terms of a sharp cutoff model such that

$$\sigma_{fus}=\pi\lambda\!\!\!^{-2} l_f(l_f+1) \quad , \tag{1}$$

where l_f is the maximum angular momentum that leads to fusion, one observes that l_f may exceed considerably the critical value, $l_{crit}=l_{RLDM}$, predicted by the rotating liquid drop model[3] as a limiting value for nuclear stability against the fission mode. For the present system $l_{RLDM}=66\hbar$, corresponding at $E_{lab}=462$ MeV to $\sigma_{fus}=\pi\lambda\!\!\!^{-2} l_{RLDM}(l_{RLDM}+1)=150$ mb, a value considerably less than the experimental fusion cross section of (720 ± 104) mb. Hence, at this energy only about $\frac{1}{5}$ of the fusion cross section can contribute to compound nucleus formation while the remaining $\frac{4}{5}$ of the cross section is in the fusionlike category (see Fig. 1).

For light and intermediate heavy-ion systems, fusion cross sections are remarkably well reproduced by a simple classical dynamical model[2] based on the one-dimensional proximity nuclear potential[4] and the one-body dissipation concept[5–7] using present knowledge about nuclear radii.[3,8] However, the simple one-dimensional model is known to fail for heavier systems. As shown by the various curves in

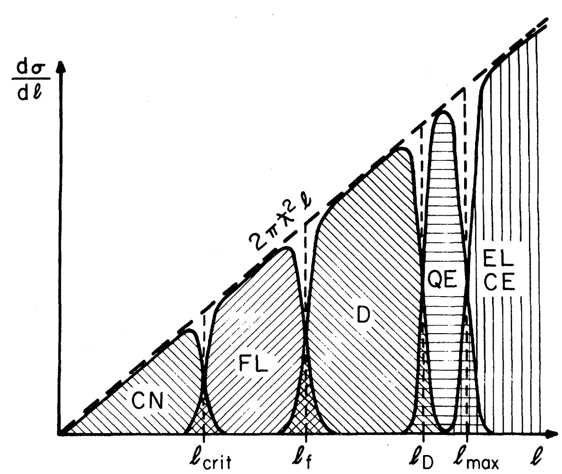

FIG. 1. Schematic illustration of the l dependence of the partial cross section for compound-nucleus (CN), fusionlike (FL), damped (D), quasielastic (QE), Coulomb-excitation (CE), and elastic (EL) processes.

Figs. 66 and 67 of Ref. 2, there is no reasonable way to adjust the parameters of the one-dimensional calculation, including the radii and forces, to remedy the large discrepancy between data and model predictions. Recently, Swiatecki[9,10] has introduced a dynamical theory that explains a wide variety of fusion data,[11–16] including those for the heaviest systems. The "Swiatecki model" identifies three important configurations in the dynamical evolution of a nucleus-nucleus collision.[17] These are (a) the contact configuration (where the growth of a neck between two spherical nuclei at contact becomes energetically favorable), (b) the configuration of conditional equilibrium (a saddle-point in a multidimensional plot of the potential energy for a frozen mass asymmetry), and (c) the configuration of unconditional equilibrium (the fission saddle-point shape). These configurations allow one to divide (at least conceptually) nucleus-nucleus reactions into four more or less distinct categories as shown in Fig. 1.

For the heavier systems, or for lighter systems with sufficient angular momentum, an extra amount of radial energy above the threshold for fusion in the contact configuration is required to induce fusionlike reactions. This "extra push" energy E_X is parametrized for noncentral collisions as follows[10]:

$$E_X(l)=\begin{cases}0, & \text{for } (Z^2/A)_{\rm eff}+(fl/l_{\rm ch})^2\le(Z^2/A)_{\rm eff\,thr}\\ K[(Z^2/A)_{\rm eff}+(fl/l_{\rm ch})^2-(Z^2/A)_{\rm eff\,thr}]^2+\cdots & \text{for } (Z^2/A)_{\rm eff}+(fl/l_{\rm ch})^2>(Z^2/A)_{\rm eff\,thr}\ ,\end{cases} \quad (2)$$

where

$$(Z^2/A)_{\rm eff}=4Z_1Z_2/A_1^{1/3}A_2^{1/3}(A_1^{1/3}+A_2^{1/3})\ , \quad (3)$$

$$(Z^2/A)_{\rm eff\,thr}=b\{1-1.7826[(N_1-Z_1+N_2-Z_2)/(A_1+A_2)]^2\}\ , \quad (4)$$

$$l_{\rm ch}^2=0.010\,55A_1^{4/3}A_2^{4/3}(A_1^{1/3}+A_2^{1/3})^2/(A_1+A_2)\ , \quad (5)$$

and

$$K=7.601\times10^{-4}[A_1^{1/3}A_2^{1/3}(A_1^{1/3}+A_2^{1/3})^2/(A_1+A_2)]a^2\ \text{MeV}\ . \quad (6)$$

The magnitude of the extra push energy E_X given by Eq. (2) is l dependent, as this expression assumes an equivalent effect of the Coulomb and centrifugal forces of fusion of the two nuclei at contact in the entrance channel. An important feature of Eq. (2) is that all parameters are fixed, except a, b, and f. The slope and the threshold coefficients have been determined[17] empirically to be $a=12\pm2$ and $b=35.6\pm1.0$, respectively. The quantity f represents the fraction of the total angular momentum that remains orbital after contact and allows for the fact that some of the initial orbital angular momentum is converted into intrinsic fragment spin. In the present calculations, three types of collisions between spherical nuclei are assumed, sliding, rolling, and sticking. These, respectively, correspond to f values of 1, $\frac{5}{7}$, and

$$(A_1^{1/3}+A_2^{1/3})^2/[(A_1^{1/3}+A_2^{1/3})^2+(\tfrac{2}{5})(A_1+A_2)(A_1^{1/3}A_2^{-1}+A_2^{2/3}A_1^{-1})]\ .$$

The fusion cross section is then given by[12]

$$\sigma_{\rm fus}[l_f,E+E_X(l_f)]=\{E/[E+E_X(l_f)]\}\sigma_{\rm fus}^1(l_f,E)\ , \quad (7)$$

where $\sigma_{\rm fus}^1(l_f,E)$ is the fusion cross section calculated with the one-dimensional dynamical model[2] at a center-of-mass energy E, and l_f is the maximum model angular momentum that leads to fusion at energy E. An alternate and simpler procedure, not requiring a classical trajectory calculation, has been suggested by Swiatecki.[10] In this procedure the fusion cross section is given by

$$\sigma_{\rm fus}=\frac{\pi R_B^2}{E}\left\{-\left(\frac{\alpha_X\beta_X+\frac{1}{2}}{\beta_X^2}\right)+\left[\left(\frac{\alpha_X\beta_X+\frac{1}{2}}{\beta_X^2}\right)^2-\left(\frac{\alpha_X^2+V_B-E}{\beta_X^2}\right)\right]^{1/2}\right\}\ , \quad (8)$$

where

$$\alpha_X=K^{1/2}\{(Z^2/A)_{\rm eff}-(Z^2/A)_{\rm eff\,thr}\}\ \text{MeV}^{1/2}\ , \quad (9)$$

$$\beta_X=(8/1.1755)K^{1/2}f^2/(A_1^{1/3}A_2^{1/3})\ \text{MeV}^{-1/2}\ . \quad (10)$$

Equation (8) gives explicitly the energy dependence of the fusion cross section for systems that require an additional amount of radial energy above the one-dimensional interaction barrier V_B to induce fusion. When no extra push energy is required, the fusion cross section is given by

$$\sigma = \frac{\pi R_B^2}{E}(E - V_B) \ . \quad (11)$$

The values of the radial position and height of the barrier, R_B and V_B, respectively, are determined with the proximity nuclear potential (see, for example, Table 1 in Ref. 2).

The fusion cross sections measured for the ^{165}Ho + ^{56}Fe reaction at bombarding energies of 351, 421, 462, and 510 MeV are shown in Fig. 2 along with the theoretical values based on Eq. (8). The dot-dashed, dashed, and dotted curves correspond to sticking, rolling, and sliding collisions, respectively. The solid line is calculated with Eq. (11).

In order to observe the trend in the fusion excitation function with mass of the projectile, excitation functions for a ^{165}Ho target with lighter[18] (^{40}Ar) and heavier[19,20] (^{84}Kr) projectiles are shown for comparison in Figs. 3 and 4, respectively. For the ^{165}Ho + ^{40}Ar reaction, the experimental fusion cross sec-

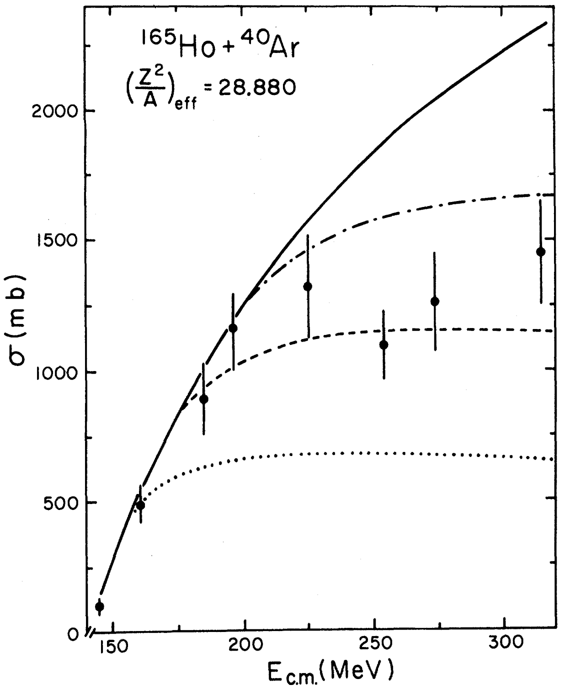

FIG. 3. Same as Fig. 2 except for the ^{165}Ho + ^{40}Ar reaction. Data are from Ref. 18.

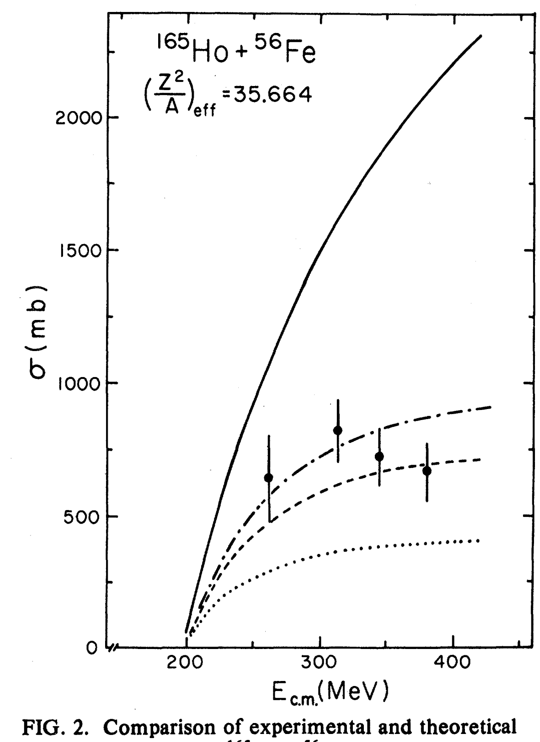

FIG. 2. Comparison of experimental and theoretical fusion cross sections of ^{165}Ho + ^{56}Fe reaction. The dotted, dashed, and dot-dashed curves are calculated with Eq. (8) and correspond to sliding, rolling, and sticking collisions, respectively. The solid curve is calculated with Eq. (11).

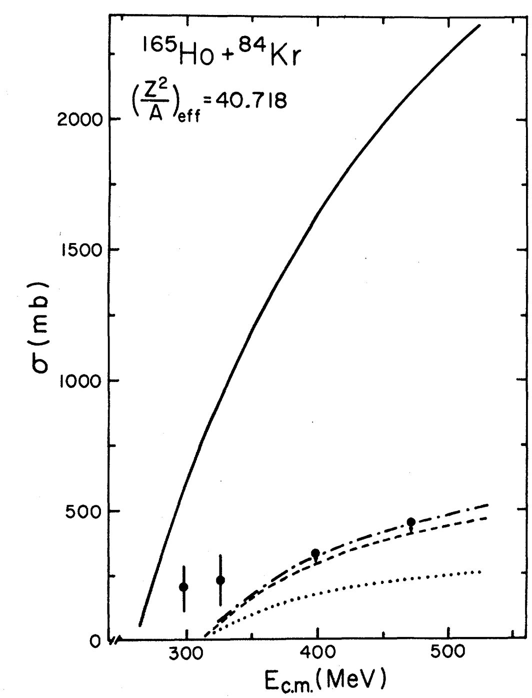

FIG. 4. Same as Fig. 2 except for the ^{165}Ho + ^{84}Kr reaction. Data are from Refs. 19 and 20.

tions follow the predictions of Eq. (11) at energies below 200 MeV, but deviate considerably from this simple formula with increasing energy, i.e., with increasing mean angular momentum. In examining a number of fusion excitation functions with $(Z^2/A)_{\mathrm{eff}} < 30$, including some for very light systems, the fusion cross sections appear to start deviating from the standard formula [Eq. (11)] at values of $(E - V_B)/\mu \geq (2.0 \pm 0.3)$ MeV per nucleon. In the one-dimensional model this limitation in the fusion cross section is associated with the disappearance of attractive pockets in the effective interaction potential. However, in the Swiatecki model at this energy per nucleon an extra push becomes necessary when $(Z^2/A)_{\mathrm{eff}} + (fl/l_{\mathrm{ch}})^2$ begins to exceed $(Z^2/A)_{\mathrm{eff\,thr}}$. The above observation may be useful in renormalizing the l_{ch} parameters for reactions between light heavy ions, where the procedure as outlined above does not give a good fit to the data.

For the heaviest systems, such as the ^{165}Ho + ^{84}Kr reaction, Swiatecki's model predicts a sizable value of E_X already for $l = 0$, as can be seen in Fig. 4. At the two highest bombarding energies of 602 and 714 MeV, only upper limits are reported for the experimental fusion cross sections, because there may be some contribution from the strongly damped reaction process.[20] At the two lowest bombarding energies of 492 and 525 MeV, finite cross sections for fusion were reported[19]; however, the results of these early and difficult measurements should probably also be considered as upper limits. Hence, not too much significance can at present be attributed to the apparent discrepancies between data and calculations near the fusion threshold.

The fusion data shown in Figs. 2–4 are consistent with the extra push model of Swiatecki. With the empirical values of the constants a and b listed previously and used in the present calculations, the data indicate that a sizable fraction of the orbital angular momentum is converted to intrinsic spin at contact. The data are in reasonable agreement with the theory based on rolling collisions ($f = \frac{5}{7}$), although additional transfer of angular momentum up to the sticking limit cannot be ruled out with the present data. On the other hand, the data are inconsistent with the limit of sliding collisions. This is in accord with earlier analyses of fusion excitation functions of lighter systems with one-dimensional models.[2,21]

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