Isospin mixing in the lowest $T=2$ state of ^{8}Be

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Partial decay widths of the lowest $T=2$ state of ^{8}Be are calculated using $(1p)^4$ shell-model wave functions. Agreement with experiment is improved if there is, in addition to the Coulomb force, a charge-dependent component of the nuclear force with magnitude close to that suggested by nucleon-nucleon scattering data. The calculations are able to account for the empirical values of the isobaric multiplet mass equation b and c coefficients for $A=8$, $T=2$, and provide an explanation of why the quadratic form of the isobaric multiplet mass equation is inadequate for this multiplet.

[NUCLEAR STRUCTURE Isospin mixing in mass 8, charge-dependent forces, IMME coefficients.]

I. INTRODUCTION

The lowest $T=2$ state at 27.5 MeV in ^{8}Be has several isospin-forbidden particle decay branches in addition to the weak isospin-allowed $M1$ γ decay branch to the $T=1$ 1^+ state at 17.6 MeV. An attempt to measure branching ratios for these decays was made recently by Freedman *et al.*[1] with mixed success; the data for some of the weaker decay channels have very large assigned errors, and the ratios sum to well over 100%. The branching ratio for the dominant ^{6}Li + d channel appears to be accurately known, however, with Freedman *et al.* finding

$$\frac{\Gamma_{d_0}}{\Gamma}=0.28\pm0.04 .$$

Different measurements[2–4] of the total width of the 27.5-MeV state are not in agreement, but Noé *et al.*[4] have given reasons why the most recent measurement of $\Gamma=5.5\pm2.0$ keV should be the most accurate, and we adopt their value in this paper.

Introducing the Coulomb force into a $(1p)^4$ shell-model calculation, Barker and Kumar[5] calculated partial widths for some of the decay channels at a time when virtually no experimental data were available. Their model has since been criticized[4] because some important features of the decay, such as the Γ_{d_0}/Γ ratio, are not well reproduced by their calculation. The present work was undertaken to test whether more care in the choice of model parameters could improve agreement with experiment, and to determine the effect of refinements to the model.

In Sec. II we present results of the new calculations of decay widths for the lowest $T=2$ state of ^{8}Be, and discuss the various charge-dependent terms which can contribute to isospin-mixing matrix elements. In Sec. III we consider the related problem of the isobaric multiplet mass equation for the $T=2$ states of mass 8. The findings are summarized in Sec. IV.

II. DECAY WIDTHS

The calculated spectrum of $(1p)^4$ 0^+ states of ^{8}Be is shown in Fig. 1. The interaction used was that of Kumar,[6] chosen to optimize energy levels in nuclei of mass 6–9. As noted by Barker and Kumar,[5] the third $T=0$ 0^+ state (hereafter referred to as the 0_3) lies very close to the $T=2$ state and therefore might be expected to be the most important contributor to isospin mixing. This conjecture is supported by a calculation of Coulomb matrix elements using harmonic oscillator radial functions. With an oscillator parameter of 1.65 fm (this value being proposed for $A=8$ by Barker[7]), matrix elements connecting $T=0$ and $T=1$ 0^+ states to the $T=2$ state are as listed in Table I. The largest matrix element is for the 0_3.

The admixture of 0_3 into the $T=2$ state will obviously depend sensitively on the precise value of the energy gap between these states, and this can be varied appreciably by quite minor changes to the two-body interaction matrix elements. It is therefore unrealistic to expect the unmodified Kumar interaction (or any other interaction chosen from other considerations) to give a 0_3 admixture which will exactly reproduce empirical decay widths. A more reasonable approach is to assume that the 0_3 wave function is as given by the calculation, but allow its component in the $T=2$ state to be a parameter

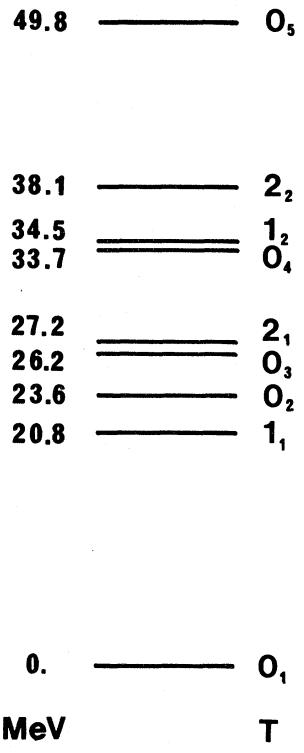

FIG. 1. Calculated spectrum of $(1p)^4$ 0^+ states. Levels are labeled by their isospin.

chosen to optimize agreement with experiment. Components of the other, less important, 0^+ states can be calculated with more confidence, since the energy gaps for these are much larger, and the fractional change caused by modification of the interaction will be much smaller than for the 0_3.

Admixtures of $T=0$ and $T=1$ states other than the 0_3 into the $T=2$ state were evaluated using first-order perturbation theory with the matrix elements of Table I. Decay widths were calculated with standard methods, using conventional values for channel radii and dimensionless reduced widths, and the formulae of Smirnov and Chlebowska[8] for spectroscopic amplitudes. The ground states of ^{3}H, ^{3}He, and ^{4}He were assumed to have maximum spacial symmetry, while the first excited state of the α particle was taken to be the mixed-symmetry state proposed by Godsadze and Kopaleishvili.[9] As discussed above, Γ_{d_0} is the most precisely known partial width, and since the ^{6}Li + d channel has isospin $T=0$, this width is suitable for determining the 0_3 component. Use of Γ_{d_0}/Γ from Freedman *et al.*[1] and the total width Γ measured by Noé *et al.*[4] gives $\Gamma_{d_0}=1.55\pm0.8$ keV. If components of other 0^+ states are calculated using the Coulomb matrix elements listed in Table I, this value of Γ_{d_0} requires a 0_3 component of 0.4% in the $T=2$ state.

Calculated values for a number of partial widths are listed in Table II. The $M1$ γ width is in good

TABLE I. Matrix elements connecting the lowest $T=2$ state of ^{8}Be to other 0^+ states. (a) Coulomb matrix element; (b) surface contribution ($a=4$ fm); (c) charge-dependent nuclear contribution; (d) total matrix element.

	Matrix element (keV)			
State	(a)	(b)	(c)	(d)
0_1	29		22	51
0_2	21		31	52
0_3	39		63	102
0_4	24		50	74
0_5	5		14	19
1_1	3	6		9
1_2	17	52		71

TABLE II. Partial decay widths (in keV) of the lowest $T=2$ state of ^{8}Be. (a) With isospin mixing due to Coulomb force only; (b) mixing due to Coulomb plus surface contribution; (c) mixing due to Coulomb plus surface contribution plus charge-dependent nuclear interaction.

		Calc.		
Channel	Expt.	(a)	(b)	(c)
^{6}Li(0)+d	1.55 ±0.8	1.55	1.55	1.55
^{6}Li(1)+d	0.5 ±0.3	0.50	0.50	1.05
$\alpha+\alpha$	0.01 ±0.02	0.11	0.11	0.03
$\alpha+\alpha^*$	0.6 ±0.25	0.45	0.45	0.65
^{7}Li(0)+p	0.03 ±0.04	0.13	0.13	0.37
^{7}Li(1)+p	0.14 ±0.06	0.05	0.03	0.23
^{7}Be(0+1)+n	0.8 ±0.5	0.19	0.20	0.63
^{5}Li(0)+t	0.9 ±0.4	0.11	0.12	0.28
^{5}He(0)+^{3}He	0.5 ±0.2	0.10	0.08	0.36
^{8}Be(17.6 MeV)+γ	0.022±0.003	0.022	0.022	0.022
Total width	5.5 ±2.0	3.2	3.2	5.2

agreement with the empirical value of Noé *et al.*,[4] giving support to the belief that the $T=2$ component of the 27.5-MeV state (and the 17.6-MeV $T=1$ 1^+ state) is well described by the $(1p)^4$ wave function generated by Kumar's interaction. The $\Gamma_{d_1}/\Gamma_{d_0}$ ratio is also well reproduced; this ratio is sensitive to the form of the 0_3 wave function and is calculated to be much too small (<0.1) for either the Barker[10] or Cohen-Kurath[11] interactions. Calculated widths for particle channels other than deuteron channels are mostly too small, leading to a calculated Γ_{d_0}/Γ ratio which is a factor of 2 too large. If the 0_3 component is increased, these other widths tend to get larger, but Γ_{d_0} increases even faster, and Γ_{d_0}/Γ is in worse agreement with experiment.

A possible explanation of the discrepancies is suggested by the fact that empirical widths appear to be very different for neutron and proton channels, whereas they are calculated to be similar. Neutron and proton penetrabilities are quite similar, so large differences in widths can arise only from interference between $T=0$ and $T=1$ components, which would require much larger $T=1$ admixtures than are given by the calculation. A correction to the Coulomb matrix elements which in principle could lead to much larger $T=1$ admixtures is the "surface contribution" considered by Dalton and Robson[12] and by Barker.[7] This arises due to the difference in asymptotic behavior of neutron and proton radial wave functions, and can be large when a state lies close to threshold for nucleon decay as does the $T=2$ state in ^{8}Be; the thresholds of ^{7}Li$(T=\frac{3}{2})+p$ and ^{7}Be$(T=\frac{3}{2})+n$ lie just 1.0 and 2.4 MeV above it. The surface contribution to the matrix element connecting a $T=1$ state to the $T=2$ state arising from a $T=\frac{3}{2}$ channel i is

$$L=\frac{\hbar^2}{4ma}u_1u_2(S_n-S_p)\mathscr{S}_2{}^{1/2}\mathscr{S}_1{}^{1/2}\,.$$

Here, a is the channel radius of R-matrix theory, m is the reduced mass $7m_p/8$, and S_n and S_p are logarithmic derivatives at $r=a$ of a $1p$ neutron bound by

$$E^*(i,n)-E^*(T=2)$$

and a $1p$ proton bound by

$$E^*(i,p)-E^*(T=2)\,.$$

The factors $\mathscr{S}_2(i)^{1/2}$ and $\mathscr{S}_1(i)^{1/2}$ are spectroscopic amplitudes for the $T=2$ and $T=1$ states, and u_T is the mean value at $r=a$ of radial functions for a $1p$ neutron bound by $E^*(i,n)-E^*(T)$ and a $1p$ proton bound by $E^*(i,p)-E^*(T)$ when these functions are normalized to unity over $0<r<a$. The $E^*(i,p)$ and $E^*(i,n)$ referred to here are excitation energies in ^{8}Be of ^{7}Li$(i)+p$ and ^{7}Be$(i)+n$. Other than the lowest $J=\frac{3}{2}$ channel, $T=\frac{3}{2}$ states which can contribute are $J=\frac{3}{2}$ levels calculated to lie 5.5 and 12 MeV higher, and a $J=\frac{1}{2}$ level calculated to lie 3 MeV higher.

The $(1p)^4$ calculation gives two $T=1$ 0^+ states, lying at about 21 and 35 MeV in ^{8}Be. The S_n, S_p, u_1, and u_2 in the expression above were calculated using Woods-Saxon potentials having range and diffuseness parameters of 2.5 and 0.65 fm, and a Coulomb potential appropriate to a uniform charge of radius 2.5 fm. The second $T=1$ state is particle unstable with respect to some of the $T=\frac{3}{2}$ channels, so a modified prescription had to be used to calculate u_1. We followed Barker's[7] suggestion by choosing the radial function to be that of a $1p$ nucleon which has a logarithmic derivative at $r=a$ equal to the shift factor at $r=a$ of a p-wave nucleon unbound by the appropriate energy. (By "nucleon" we mean here an average of values obtained for a neutron and a proton.) This prescription gives a value for u_1 which matches smoothly onto the value obtained for bound channels as the energy approaches zero. The resulting surface contribution to the matrix elements connecting the two $T=1$ states to the $T=2$ state are in phase with the Coulomb matrix elements given in Table I, and are 6 and 52 keV if a is chosen to be 4 fm and 3 and 45 keV if a is 5 fm. Although these contributions increase the $T=1$ admixtures in the $T=2$ state by factors of 3 and 4, the admixtures are still far too small to account for the discrepancies in Table II; there are only very minor changes to the calculated widths.

It has been suggested[13] that a charge-dependent component of the nuclear interaction is required to account for the masses of isobaric multiplets in the $1p$ shell. What evidence there is for such a force from nucleon-nucleon scattering data suggests that its spin singlet componet is essentially charge symmetric, with both the neutron-neutron force and proton-proton nuclear force being about 2% weaker than the neutron-proton force. If it is assumed that all spin components behave similarly, this charge-dependent force is isotensor, and as such will give no contribution to $(T=1)/(T=2)$ mixing in ^{8}Be. The contributions to $(T=0)/(T=2)$ mixing have been estimated by taking 2% of the matrix elements given by the neutron-proton part of Kumar's interaction, and these are listed in Table I. All these terms are in phase with the Coulomb matrix elements, and lead to total matrix elements increased by an average factor of more than 2.5. If partial decay widths are recalculated using these new matrix elements (last column of Table I), again choosing the

0_3 component to give $\Gamma_{d_0}=1.55$ keV, results are as given in the last column of Table II.

With this simple charge-dependent nuclear force, the widths of neutron, triton, ^{3}He, and α channels are all much closer to empirical values, and Γ_{d_0}/Γ at 0.30 is now in agreement with the observed ratio[1] of 0.28 ± 0.04. Agreement with experiment is worsened for Γ_{d_1}, but as pointed out above, this width can readily be decreased by minor changes to the interaction. The one remaining large discrepancy with experiment concerns Γ_{p_0}, which is now calculated to be in even poorer agreement with the result of Freedman *et al.*[1] The 0_3 component required to give $\Gamma_{d_0}=1.55$ keV is 0.42%, which together with the calculated matrix element of 102 keV connecting the 0_3 to the $T=2$ state implies that the 0_3 lies 1.6 MeV below the $T=2$ state (i.e., at 25.9 MeV). The large width of this state for decay into $\alpha+\alpha^*$ (α^* being the 20.8-MeV first excited state of the α particle) may be responsible for the large peak in the $\alpha+\alpha^*$ yield observed at about this energy in the ^{2}H(^{6}Li,α)α^* reaction by Warner *et al.*[14]

III. THE IMME FOR $A=8$, $T=2$

The lowest $T=2$ states of mass 8 are of interest because their masses, unlike those of virtually all multiplets which have been carefully measured, cannot be fitted by the quadratic isobaric multiplet mass equation (IMME)

$$M=a+bT_z+cT_z^2 .$$

To obtain agreement with experiment it is necessary to include either a cubic term dT_z^3, a quartic term eT_z^4, or both. To investigate the IMME coefficients we have calculated in the neutron-proton (np) scheme the energies of $(1p)^4$ 0^+ states of ^{8}C, ^{8}B, ^{8}Be, ^{8}Li, and ^{8}He, focusing attention on the states which are predominantly $T=2$. The interaction used was that of Kumar,[6] modified firstly in such a way that it becomes charge dependent while remaining charge symmetric; all neutron-neutron (nn) and proton-proton (pp) matrix elements are reduced by a factor $(1-x)$, while neutron-proton matrix elements are unchanged. This gives an isotensor contribution to the interaction. To mock up the surface contribution to matrix elements, which is an isovector effect when only nucleon decay channels are included, we have allowed the nn and pp matrix elements to be multiplied by additional factors of $(1+y)$ and $(1-y)$, respectively. This latter modification is what Lawson[15] finds to be the effect on two-body matrix elements of different neutron and proton wave functions, and it can be argued that the Lawson effect and surface contributions are essentially the same.

The c coefficient (the coefficient of the quadratic term in the IMME) arises almost entirely from the expectation value of the isotensor component of the charge dependent force. As pointed out by Lawson,[15] charge-dependent nuclear forces ($x>0$) give a positive contribution to c, and agreement with experiment for p-shell multiplets is improved if these forces are assumed to exist. However, with Coulomb matrix elements given by harmonic oscillator wave functions and an oscillator parameter of 1.65 fm, we find that the c coefficient for the $A=8$, $T=2$ states is 0.255 MeV with $x=0$, already larger than the empirical value of $c\simeq0.22$ MeV. The unreasonably large oscillator parameter of 1.9 fm is required to give $c=0.22$ MeV. Coulomb matrix elements are smaller if they are calculated using radial functions given by a Woods-Saxon potential with parameters as specified in Sec. II, with depth chosen to give a $1p$ proton binding energy of 1 MeV [equal to the energy by which the $T=2$ state in ^{8}Be is below the ^{7}Li($T=\frac{3}{2}$)$+p$ threshold]. The calculated c coefficient is then 0.189 MeV with $x=0$, and the most reasonable value of $x=0.02$ increases c to 0.217 MeV, in good agreement with experiment.

Values of the linear IMME coefficient b calculated from $(1p)^4$ interaction energies must be corrected for the neutron-proton mass difference and the difference $\Delta\epsilon$ between proton and neutron $1p$ single-particle energies before comparison can be made with the empirical value deduced from observed masses. The mass 5 nuclei ^{5}He and ^{5}Li are both unbound to particle emission, so the value of $\Delta\epsilon=1.08$ MeV deduced from their masses is probably not a reliable estimate. Use of the binding energies of ^{6}He and ^{6}Li($T=1$) gives $\Delta\epsilon=0.836$ MeV; if, however, the neutron-neutron $(1p)^2$ interaction is 2% weaker than the neutron-proton interaction (i.e., $x=0.02$), this value is corrected to $\Delta\epsilon=0.89$ MeV. We have used this latter value, which requires that 0.11 MeV be subtracted from values of b given by $(1p)^4$ calculations using $\Delta\epsilon=0$ and equal neutron and proton masses before they can be compared with empirical values.

The b coefficient arises mainly from the expectation value of the isovector component of the charge-dependent force. It is therefore almost independent of x, being -0.68 MeV with the Woods-Saxon Coulomb matrix elements referred to above. This is quite far from the value of about -0.89 MeV found by Tribble *et al.*,[16] but a y of just under 0.04 brings the calculation into agreement with experiment. Based on the work of Lawson,[15] this is very close to the value of y expected to result from differences in the neutron and proton wave functions.

Concerning the higher-order IMME coefficients,

second-order perturbation theory shows that $T=1$ admixtures in the $T=2$ states can result in both d and e being nonzero. There is also a contribution to e from $T=0$ admixtures in the $T=2$ state of ^{8}Be, equal to $0.25\Delta M$ if this mixing shifts the energy of the $T=2$ state by ΔM. Our $A=8$ calculations suggest that $T=1$ mixing is very small, so the fact that the empirical e coefficient is positive shows that the $T=2$ state in ^{8}Be is pushed *up* by mixing with $T=0$ states, in agreement with our conclusion that mixing is dominated by the 0_3 state which is calculated to be *below* the $T=2$ state. The final ^{8}Be calculation described in Sec. II, using the matrix elements listed in the last column of Table I, gives $e=1.7$ keV when this is estimated from the energy shift of the $T=2$ state. The $A=8$ calculations using Woods-Saxon Coulomb matrix elements, together with Kumar's interaction made charge dependent by setting $x=0.02$, gives $e=2.2$ keV and d close to zero for any reasonable value of y. The difference in e is due mainly to the fact that the 0_3 state was shifted down by about 600 keV in the ^{8}Be calculation so as to give $\Gamma_{d_0}=1.55$ keV. If d *is* close to zero, the appropriate least-squares fit[16] to empirical masses suggests that $e=4.3\pm1.7$ keV, somewhat larger than calculated.

IV. SUMMARY AND DISCUSSION

The calculations reported in this paper appear to provide additional evidence for the existence of a charge-dependent component of the strong nuclear interaction. The isospin-forbidden decay of the lowest $T=2$ state in ^{8}Be is poorly reproduced by calculations using just the Coulomb force, even when off-diagonal matrix elements are corrected for effects of the different asymptotic form of neutron and proton radial functions. Almost all calculated partial widths are improved by the addition of a simple isotensor charge-dependent force with strength as suggested by low-energy nucleon-nucleon scattering data. A similar composite force is also able to account for the magnitudes of the linear and quadratic IMME coefficients for the $A=8$, $T=2$ multiplet, and provide an explanation of why the quadratic form of the IMME is inadequate.

The calculations for $A=8$ nuclei described in Sec. III used Woods-Saxon Coulomb matrix elements which were smaller (by about 30%) than the matrix elements from harmonic oscillator wave functions used for the ^{8}Be calculations of Sec. II. However, using the Woods-Saxon matrix elements would not cause any large changes in the ^{8}Be results, because the total off-diagonal matrix elements listed in Table I would be reduced on the average by only 10%.

There remains a discrepancy between the calculations for ^{8}Be and experiment in that the calculated proton width is too large when compared with the measurements of Freedman *et al.*[1] As pointed out in Sec. II, this appears to require a much larger $T=1$ component than given by the calculation. The two $(1p)^4$ $T=1$ 0^+ states lie several MeV from the $T=2$ state, so even quite large matrix elements connecting them give only small admixtures, but it is feasible that states arising from $2\hbar\omega$ excitations lie sufficiently close to the $T=2$ state to become important. The only $2\hbar\omega$ states which can contribute to Γ_{p_0} are those in which a $1p$ nucleon is excited into the $1f\,2p$ shell. This configuration could also be responsible for the decays observed to $\frac{5}{2}^-$ and $\frac{7}{2}^-$ states of mass 7 in ^{7}Li $+p$ and ^{7}Be $+n$ which the $(1p)^4$ calculation cannot reproduce. The penetrabilities of $l=3$ nucleons are so low, however, that it is difficult to see how the decay widths of these channels can be as large as measured by Freedman *et al.*[1]

Enhanced $T=1$ admixtures could also improve agreement with experiment for the higher-order IMME coefficients, by leading to a nonzero d. Fits to empirical masses[16] require smaller values of e, in better agreement with the calculations, when d is greater than zero.

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