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Isospin dependence of second-order pion-nucleus optical potential

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A universal form for the dependence of the second-order pion-nuclear optical potential on the nuclear and pionic isospin and on the densities ρ and $\Delta\rho$ is derived, where ρ is the total density and $\Delta\rho$ is the valence neutron density. The result is characterized by five complex parameters for each partial wave channel contributing to the optical potential. Our result applies to scattering in the vicinity of the (3,3) resonance, and the parameters may be most cleanly determined phenomenologically by applying the theory to elastic, and single- and double-charge-exchange data for medium to heavy weight $J=0$, spherical nuclei. The relationship between the parameters and a microscopic cluster decomposition of the optical potential is explored. The parameters are calculated theoretically for selected second-order processes to obtian a first orientation to the magnitude of the terms. The results show, in particular, that large contributions to double charge exchange arise from nonanalog intermediate nuclear states and that these contributions have a characteristic isospin dependence which is different from that found in the simple models previously studied.

[NUCLEAR REACTIONS Scattering theory for elastic, and single- and double-charge-exchange to IAS in the region of the p_{33} resonance. Second-order effects.]

I. INTRODUCTION

Pion-nucleus elastic, and single- and double-charge exchange scattering to the single- and double-isobaric analog states are related to the isospin symmetry of strong interactions. To the extent that isospin breaking effects can be ignored, these scattering processes can be treated theoretically on the basis of an optical potential of the following form:

$$\hat{U}=U_0+U_1(\vec{\phi}\cdot\vec{\mathrm{T}})+U_2(\vec{\phi}\cdot\vec{\mathrm{T}})^2\ , \tag{1.1}$$

where $\vec{\phi}$ is the pion and $\vec{\mathrm{T}}$ the nuclear isospin operator. This form of the pion-nucleus optical potential is analogous to the Lane[1] form of the nucleon-nucleus optical potential. However, for pions there is an additional term in the potential, the "isotensor" term U_2, which is allowed because in this case the isospin of the projectile is unity. The quantities U_0 and U_1 are the isoscalar and isovector terms. In principle, U_0, U_1, and U_2 may be calculated microscopically by an expansion in terms of the number of active nucleons, commonly referred to as a density expansion. We express this expansion as

$$U_i=U_i^{(1)}+U_i^{(2)}+\cdots\ , \tag{1.2}$$

where the superscript indicates the number of active nucleons, so that $U^{(1)}$ is the first-order optical potential, $U^{(2)}$ the second, etc.

In the recent past, numerous theories of pion-nucleus scattering have been proposed, some of which are given in Refs. 2–8. All modern approaches such as these include second-order terms of one form or another. The second-order terms are of fundamental interest because they measure the extent to which multiple scattering differs from a sequence of free elastic scatterings of the projectile with nucleons in their ground state. Most of the theoretical effort has been directed toward characterizing the second-order *isoscalar* terms. These are now recognized as being large, and necessary for a correct theoretical interpretation of the scattering data.

For nuclei with a neutron excess, the isovector and isotensor terms contribute, in addition to the isoscalar. Because most nuclei have a neutron excess, a restricted treatment of the isospin dependence of the optical potential limits the applicability of the theory to a small subset of possible nuclear targets. To treat $T \neq 0$ nuclei with as much care as $T=0$ nuclei, a substantially more complicated operator $U^{(2)}$ is required: Each of the terms $U_0^{(2)}$, $U_1^{(2)}$, and $U_2^{(2)}$ becomes a quadratic function of the nuclear densities ρ and $\Delta\rho$, as well as a function of the nuclear isospin T. However, now that single- and double-charge-exchange experiments[9] are being done with very high precision, the data warrant the introduction of these complexities. Indeed, because double-charge exchange is driven by terms quadratic in the density, a more sensitive analysis of $U^{(2)}$ is possible than would be with only elastic scattering. The prospect of a direct determination of the isovector and isotensor terms has a practical consequence for nuclear structure, because these terms also describe the details of pion elastic scattering, which must be understood in order to take advantage of the unique features of the pion as a probe of the neutron halo in nuclei.

In the description of higher-order scattering by Eq. (1.1), scattering through intermediate single- and double-isobaric analog states is singled out, and the repeated transitions of this nature are treated explicitly by the scattering equation. Thus, $U_i^{(2)}$ describes in a single action of the optical potential all processes which do not involve isobaric analog transitions. Miller and Spencer[10] and Ericson and Ericson,[11] among others, have stressed the role of certain terms in charge-exchange reactions. Examples of second-order effects include the Pauli principle, the Lorentz-Lorenz Ericson-Ericson effect, $\pi\pi$ interactions,[12] and various Δ-nucleus interactions.[13] Second- (and higher) order terms are also generated when U is calculated self-consistently[14]; such a treatment is important for elastic scattering[15] and also presumably for charge exchange in the resonance region. An often neglected term in $U^{(2)}$ which we find to be important is the effect of nonanalog intermediate nuclear states in the double scattering of the pion from two nucleons through the elementary pion-nucleon scattering amplitude.

Because so little attention has been paid to the isospin dependence of the second-order optical potential, we feel that it is appropriate in this paper to limit our ambitions. We will not attempt a complete calculation of second-order effects based on the most up-to-date theory; indeed, as there exists no complete microscopic model of pion-nucleus scattering theory, such a calculation is not possible at the present time. Rather, we set as our main purpose in this paper to seek a *form* for the dependence of the second-order optical potential on the total nuclear isospin and on the densities ρ and $\Delta\rho$ appropriate for $J=0$ spherical nuclei. Our main result is a universal form for this dependence characterized by five complex coefficients at a given energy. Such a result should prove useful as a phenomenological basis for data analysis. To obtain a first orientation for their size, we calculate values for these coefficients corresponding to a few important second-order processes. The sensitivity of single- and double-charge exchange cross sections to these terms will be demonstrated in a subsequent paper.

To accomplish our goal, we will assume that the underlying dynamics is described in terms of meson fields interacting with a collection of A fixed nucleons.[16] The important coupling of the pion directly to the isobar Δ_{33} is encompassed in this approach. Because we are working in the context of a field theory, our optical potential is crossing symmetric and is used in a Klein-Gordon equation in order to calculate phase shifts. For zero-range couplings to the nucleon, we are led to a second-order potential that can be incorporated easily into the computer code PIRK (Ref. 17); for finite-range couplings an adaptation of the code PIPIT (Ref. 18) can accomodate our results.

The outline of the paper is the following. In Sec. II we briefly discuss the theoretical framework which we adopt for our multiple scattering theory. Since this framework has been developed for elastic pion-nucleus scattering, we need a method of extending it to include the isobaric analog states. We do this in Sec. III for a general pion-two-nucleon interaction. In Sec. IV an approximation scheme is proposed in which a nuclear matter calculation is used to estimate the scattering from a finite nucleus. The resulting local density approximation is different from the one normally employed and contains an important correction arising from the rate of fall-off of the density in the nuclear surface. In Sec. V we discuss in detail specific examples of terms which contribute to the second-order optical potential and we obtain our main result, Eq. (5.35), which we argue applies to more general second-order diagrams that we have not explicitly evaluated. The specific cases are evaluated numerically and the results are given in Sec. VI; Sec. VII summarizes the main results of the paper.

II. SCATTERING THEORY

We consider the optical potential within the framework of the fixed scatterer field theory of Ref. 16. In this reference, the optical potential for elastic scattering was discussed in terms of the spectator expansion,[19] which lead directly to the classification scheme described below Eq. (1.1). The various terms in U are given by a diagrammatic expansion similar in appearance to Feynman diagrams and evaluated in terms of propagators and elementary pion-nucleon interactions, the rules for which being fully specified in Ref. 16. A few of the terms contributing to the second-order optical potential are shown in Figs. 1 and 2. In these diagrams, upper case letters A and B denote single particle states in the normally occupied Fermi sea, that is, we envision describing the nuclear ground state $|\Phi_0\rangle$ by

$$|\Phi_0\rangle = \mathscr{A}\prod f_{ij}\,|A,B,C,\ldots\rangle,\ \ i<j\ , \tag{2.1}$$

where $\mathscr{A}$ is the antisymmetrization operator, f_{ij} is a two-body correlation function acting between particles i and j, and $|A,B,\ldots\rangle$ is a product of single-particle orbitals representing the ground state configuration of the nucleus.

Figures 1(a) and (b) are the direct and exchange matrix elements of the double scattering terms in Fig. 4 of Ref. 16. Figure 1(c) is the once-iterated lowest-order optical potential which must be *subtracted* from the sum of Figs. 1(a) and (b) in order to obtain U, also shown explicitly in Fig. 4 of Ref. 16. According to the discussion of Ref. 16, a completely crossing symmetric optical potential is obtained by adding to the optical potential all *distinct* crossed terms. Figure 1(d) is the crossed process corresponding to Fig. 1(a). It is easy to verify, however,

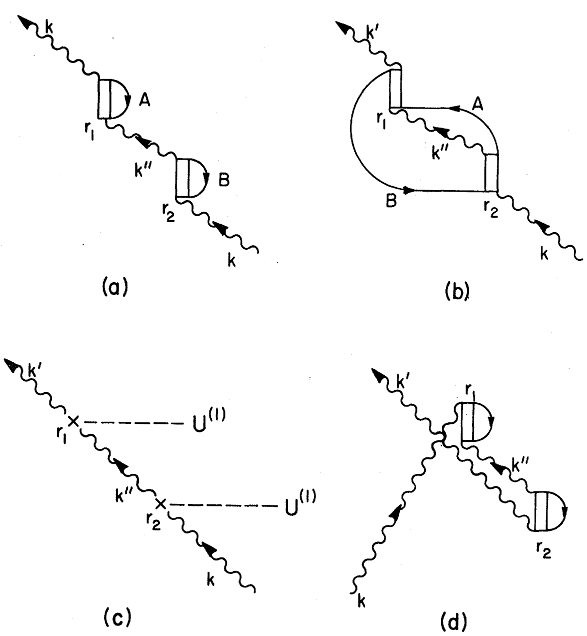

FIG. 1. Two-nucleon processes contributing to the pion-nucleus optical potential. These terms are second order in the pion-nucleon scattering amplitude and are referred to as sequential scattering processes.

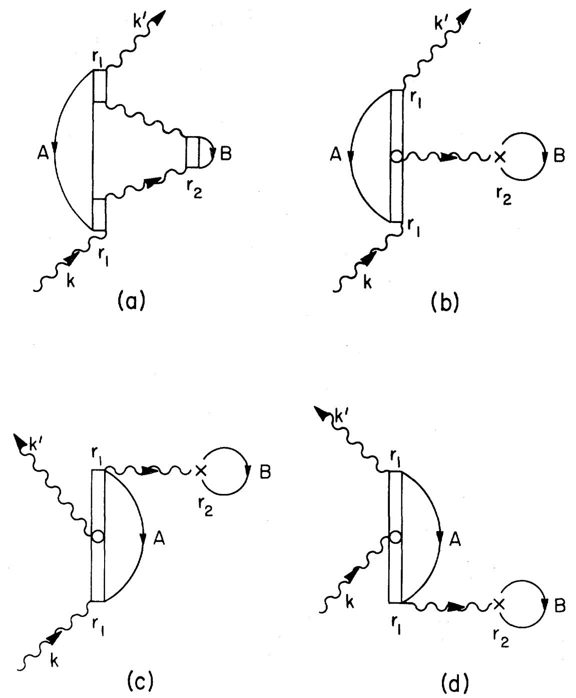

FIG. 2. Additional two-nucleon processes contributing to the pion-nucleus optical potential. (a) is third order in the pion-nucleon scattering amplitude and is referred to as a reflection process, whereas (b)–(d) involve various isobar-medium effects. Each process has a corresponding exchange and crossed piece.

that the terms 1(a), (b), and (c) are automatically crossing symmetric if the pion-nucleus amplitude (represented by the rectangle in Figs. 1 and 2) is crossing symmetric. We shall assume that this is the case, and hence there are no explicit crossed pieces needed corresponding to Fig. 1. We shall calculate and study in detail the contribution of Fig. 1 to the second-order optical potential in succeeding sections.

Figure 2 shows a few of the additional processes which are second order and encompassed within the framework of Ref. 16. Figure 2(a) shows a piece of the triple scattering term of Fig. 4 of Ref. 16. This is also called the "local field correction" in the notation of Ref. 20, and when evaluated self-consistently can account in large part for the observed spreading of the width of the Δ_{33} resonance.[15] Figure 2(b)–(d) are contributions to $U^{(2)}$ arising from isobar interactions with the nuclear medium, which may give important contributions to double charge exchange.[21] We will not calculate explicitly here any of the terms of Fig. 2 or their crossed counterparts, but we will give arguments that these and all other second-order terms may be cast into the same universal form as those of Fig. 1.

In addition to the processes shown in Figs. 1 and 2, it is also possible to exchange a ρ meson between the two nucleons. This process is believed to make a large contribution at low energy.[22] We have estimated the effect to be small for sequential scattering at resonance energy[23] and it has therefore not been included here.

Reference 16 deals with elastic scattering, and to derive U of the form of Eq. (1.1) requires an extension of the ideas expressed there. The main difference is that the U in Ref. 16 is projected onto the nuclear target ground state, whereas in Eq. (1.1) there are also matrix elements of U connecting to the isobaric analog states. Thus, we shall need to utilize isospin symmetry to express the three quantities U_0, U_1, and U_2 in terms of ground state matrix elements discussed in Ref. 16. Secondly, we shall have to be careful to restrict the intermediate states in the subtracted term of Fig. 1(c) to the ground state, and single analog. Since Figs. 1(a) and (b) include all nuclear intermediate states through the use of closure, the difference, Fig. 1(a) + Fig. 1(b) − Fig. 1(c), is the contribution of the nonanalog, double step route to elastic and single- and double-charge exchange.

III. GENERAL ISOSPIN DEPENDENCE

The assumption of total isospin invariance can be used to obtain the explicit isospin dependence that results from the pion-two-nucleon (π-2N) interaction. It can also be used to relate the π-2N isospin dependence to the pion-nucleus isospin dependence. Let $\hat{D}_{12}^{(\gamma)}$ be an operator on the π-2N isospin space whose matrix element results from a specific process contributing to the first two terms in Fig. 4 of Ref. 16, examples of which are shown in Figs. 1 and 2 of this paper. (Throughout this paper a caret over a symbol shall indicate an operator on the pion's isospin.) This operator will be a function of the isospin operators for the pion ($\vec{\phi}$) and the two nucleons ($\vec{\tau}_1$ and $\vec{\tau}_2$). Total isospin invariance implies that $\hat{D}_{12}^{(\gamma)}$ must transform like a scalar under rotations in total isospin space; thus its dependence on $\vec{\phi}$, $\vec{\tau}_1$, and $\vec{\tau}_2$ is constrained. Given the explicit $\vec{\tau}_1$ and $\vec{\tau}_2$ dependence of $\hat{D}_{12}^{(\gamma)}$, the next step is to make contact with the nuclear isospin operator $\vec{\mathrm{T}}$. Toward this end, we first need to explicitly display the isospin dependence of the nuclear isobaric analog states (IAS). For example, we denote the ground state as

$$|\Phi_0\rangle = |\alpha;T=T_0,M=-T_0\rangle , \tag{3.1}$$

where α contains all of the remaining quantum numbers necessary to describe the target, and $T_0=(N-Z)/2$ in terms of the number of neutrons (N) and protons (Z). Within our model of isospin invariance, we assume that the single- and double-analog states are also described in Eq. (3.1) except that $M=-T_0+1$ and $M=-T_0+2$, respectively. From our two-body operator $\hat{D}_{12}^{(\gamma)}$ we can construct an operator on the entire nuclear space,

$$\hat{D}^{(\gamma)} = \sum_{i\neq j} \hat{D}_{ij}^{(\gamma)} .$$

We may identify the optical potential U as $\sum_\gamma \hat{D}^{(\gamma)}$ on the space of nuclear ground and analog states by taking appropriate linear combinations of elastic matrix elements of $\hat{D}^{(\gamma)}$ between physical states. The relationships are[24]

$$\hat{U} = \sum_\gamma [U_0(\gamma)+U_1(\gamma)(\vec{\phi}\cdot\vec{\mathrm{T}})+U_2(\gamma)(\vec{\phi}\cdot\vec{\mathrm{T}})^2] , \tag{3.2}$$

where

$$U_2(\gamma)=[\langle D^{(\gamma)}\rangle^+ + \langle D^{(\gamma)}\rangle^- - 2\langle D^{(\gamma)}\rangle^0]/[T_0(2T_0-1)] , \tag{3.3}$$

$$U_1(\gamma)=[\langle D^{(\gamma)}\rangle^0 - \langle D^{(\gamma)}\rangle^+]/T_0 + T_0U_2(\gamma) , \tag{3.4}$$

$$U_0(\gamma)=\langle D^{(\gamma)}\rangle^0 - T_0U_2(\gamma) , \tag{3.5}$$

and the elastic matrix elements are defined as

$$\langle D^{(\gamma)}\rangle^m = \langle \alpha T_0,-T_0;\vec{\mathrm{k}}\,'1,m\,|\hat{D}^{(\gamma)}|\,\alpha T_0,-T_0;\vec{\mathrm{k}}1,m\rangle . \tag{3.6}$$

In Eq. (3.6) $\vec{\mathrm{k}}$ ($\vec{\mathrm{k}}\,'$) denotes the pion's initial (final) momentum and m denotes the pion's charge state. Also, when summing over diagrams in Eq. (3.2), we remind the reader that exchange graphs enter with a minus sign.

For the purpose of identifying the optical potential of Eq. (1.1), our above prescription is to set up the problem in the physical elastic channels. As we have stressed, the advantage of this approach is that we may use well known diagrammatic rules for calculating the optical potential.

Our goal is now to explicate the general structure of the second-order optical potential, which is, for example, simply related to the diagrams in Figs. 1 and 2. According to the results of Ref. 16 and Sec. II, we may write for each process depicted in these figures

$$\langle D^{(\gamma)}(\vec{k}\,',\vec{k})\rangle^m = \int d\vec{r}_1 d\vec{r}_2 e^{-i\vec{k}\,'\cdot\vec{r}_i}\Delta^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2)e^{i\vec{k}\cdot\vec{r}_j} , \tag{3.7}$$

where the indices i and j take the value 1 or 2 depending on the specific diagram. From this expression, we see that the specific dependence of $\hat{D}_{12}^{(\gamma)}$ on $\vec{\phi}$, $\vec{\tau}_1$, and $\vec{\tau}_2$ is contained within $\Delta^{(\gamma)}$. We find the most general form for $\Delta^{(\gamma)}$ by building overall scalars out of various tensor products of $\vec{\phi}$, $\vec{\tau}_1$, and $\vec{\tau}_2$. Since $\Delta^{(\gamma)}$ is defined by ground state matrix elements, the combinations of $\vec{\tau}_1$ and $\vec{\tau}_2$ are further constrained from changing the total isospin of the $2N$ subsystem. These scalars can be constructed by using standard recoupling techniques,[25] and then taking combinations of scalars that are symmetric under interchange of the nucleon labels ($\hat{D}_{12}=\hat{D}_{21}$). Our result is

$$\Delta^{(\gamma)} = \sum_{AB} \langle m,t_A(1)t_B(2)\,|\,\{a_{AB}^{(\gamma)}+b_{AB}^{(\gamma)}\vec{\tau}_1\cdot\vec{\tau}_2+c_{AB}^{(\gamma)}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2)+d_{AB}^{(\gamma)}\hat{\tau}_{12}\}\,|\,t_A(1)t_B(2),m\rangle , \tag{3.8}$$

where

$$\hat{\tau}_{12} \equiv \tfrac{1}{2}(\vec{\phi}\cdot\vec{\tau}_1\vec{\phi}\cdot\vec{\tau}_2+\vec{\phi}\cdot\vec{\tau}_2\vec{\phi}\cdot\vec{\tau}_1) . \tag{3.9}$$

Alternatively, we could have used the π-$2N$ isospin projection operators to decompose $\hat{D}_{12}$ onto channels of total isospin and $2N$ isospin. The conclusion is the same: There are four linearly independent terms that characterize the isospin dependence of the π-$2N$ operator appropriate for analog transitions. Of course the coefficients a, b, c, and d contain spin and spatial dependences, and the details that result from these additional degrees of freedom will be different for different processes. We have summed over all quantum numbers without restriction in Eq. (3.8), recalling that there is no overcounting in diagrams provided direct and exchange are both calculated.

By using Eqs. (3.7) and (3.8) together with the procedure given in Eqs. (3.2)—(3.6), we may identify the corresponding contribution of a specific diagram to the isoscalar, isovector, and isotensor optical potentials $[U_i(\gamma);1=0,1,2]$. That is, the $U_i(\gamma)$'s are given by Eq. (3.7) with $D^{(\gamma)}$ replaced by $U_i(\gamma)$ and $\Delta(\gamma)$ replaced by $\Delta_i(\gamma)$. The $\Delta_i^{(\gamma)}$ in turn are constructed by taking the appropriate linear combinations of elastic matrix elements of Eq. (3.8), and we obtain

$$\Delta_0^{(\gamma)} = a^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2)+b^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) - \frac{2d^{(\gamma)}(k,k;r_1,r_2)}{[2T_0-1]} , \tag{3.10a}$$

$$\Delta_1^{(\gamma)} = -\left[\frac{1}{T_0}\right]c^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) + \left[\frac{1}{T_0[2T_0-1]}\right]d^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) , \tag{3.10b}$$

$$\Delta_2^{(\gamma)} = \left[\frac{2}{T_0[2T_0-1]}\right]d^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) , \tag{3.10c}$$

where

$$a^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) \equiv \sum_{AB} a_{AB}^{(\gamma)} , \tag{3.11a}$$

$$b^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) \equiv \sum_{AB} b_{AB}^{(\gamma)}4t_A t_B , \tag{3.11b}$$

$$c^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) \equiv \sum_{AB} c_{AB}^{(\gamma)}2(t_A+t_B) , \tag{3.11c}$$

$$d^{(\gamma)}(\vec{k}\,',\vec{k};\vec{r}_1,\vec{r}_2) \equiv \sum_{AB} d_{AB}^{(\gamma)}4t_A t_B , \tag{3.11d}$$

and where $t=\pm\frac{1}{2}$ is the nuclear isospin.

We see from these general results that our task reduces to determing the coefficients $a_{AB}^{(\gamma)}$–$d_{AB}^{(\gamma)}$ of Eq. (3.8). As mentioned above, these coefficients depend upon $\vec{k}\,'$, $\vec{k}$, $\vec{r}_1$, and $\vec{r}_2$; however, we may separate the nuclear density dependence of these coefficients. We shall proceed by ignoring the spin density matrices, and hence the initial and final states of the coefficients a through d below have the same spins. As we discuss more fully in Sec. VII, the contribution of the spin densities is small in medium to heavy weight $J=0$, spherical nuclei. For light nuclei, we have no strong argument that these terms are negligible; however, the methods we have outlined can also be applied to these cases, and further investigations of this point would be useful. Without making further assumptions, these coefficients can be shown to have the following form:

$$a_{AB}^{(\gamma)} = \Gamma_{AB}(\vec{r}_1,\vec{r}_2)n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)\langle s_A(1)s_B(2)\,|\,a_{\vec{k}\,',\vec{k}}^{(\gamma)}(\vec{r}_1-\vec{r}_2)\,|\,s_A(1)s_B(2)\rangle , \tag{3.12a}$$

$$b_{AB}^{(\gamma)} = \Gamma_{AB}(\vec{r}_1,\vec{r}_2)n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)\langle s_A(1)s_B(2)\,|\,b_{\vec{k}\,',\vec{k}}^{(\gamma)}(\vec{r}_1-\vec{r}_2)\,|\,s_A(1)s_B(2)\rangle , \tag{3.12b}$$

$$c_{AB}^{(\gamma)}=\Gamma_{AB}(\vec{r}_1,\vec{r}_2)n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)\langle s_A(1)s_B(2)\,|\,c_{\vec{k}',\vec{k}}^{(\gamma)}(\vec{r}_1-\vec{r}_2)\,|\,s_A(1)s_B(2)\rangle\ , \tag{3.12c}$$

$$d_{AB}^{(\gamma)}=\Gamma_{AB}(\vec{r}_1,\vec{r}_2)n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)\langle s_A(1)s_B(2)\,|\,d_{\vec{k}',\vec{k}}^{(\gamma)}(\vec{r}_1-\vec{r}_2)\,|\,s_A(1)s_B(2)\rangle\ , \tag{3.12d}$$

where Γ_{AB} is the radial distribution function, $n_{AB}^{(\gamma)}$ is a product of single-particle density matrices, and s refers to the z projection of the nucleon's spin. There are only two combinations of density matrices that arise, direct and exchange. For the direct combination [Fig. 1(a)].

$$n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)=\langle s_A\,|\,\rho^{(t_A)}(\vec{r}_1,\vec{r}_1)\,|\,s_A\rangle\langle s_B\,|\,\rho^{(t_B)}(\vec{r}_2,\vec{r}_2)\,|\,s_B\rangle\ , \tag{3.13a}$$

whereas for the exchange combination [Fig. 1(b)],

$$n_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)=\langle s_A\,|\,\rho^{(t_A)}(\vec{r}_2,\vec{r}_1)\,|\,s_A\rangle\langle s_B\,|\,\rho^{(t_B)}(\vec{r}_1,\vec{r}_2)s_B\rangle\ . \tag{3.13b}$$

In these expressions, $\rho^{(t)}(\vec{r}_1,\vec{r}_2)$ is the single-particle density matrix for nucleons of isospin z-projection t,

$$\langle s_1\,|\,\rho^{(t)}(\vec{r}_1,\vec{r}_2)\,|\,s_2\rangle=\sum_{A\in t}\langle\vec{r}_2,s_2\,|\,\psi_A\rangle\langle\psi_A\,|\,\vec{r}_1,s_1\rangle\ , \tag{3.14}$$

where $\langle\vec{r},s\,|\,\psi_A\rangle$ is the single-particle wave function.

The results obtained thus far are exact (within the fixed scatterer framework), and are applicable to finite nuclei. To proceed further we must adopt specific models for the nuclear density matrices and for the πN interaction. In the following sections we shall make a series of approximations which are physically motivated and which simplify our final results. We point out that our subsequent approximations are most appropriate for intermediate energy pions, where absorption is large.

IV. MODEL FOR THE NUCLEAR DENSITY MATRIX

In this section our goal is to adopt a simple treatment of the nuclear density matrices that is quantitatively accurate. Since we are mainly concerned with pion scattering in the resonance region, where interactions are surface dominated, we seek an approximation that can accurately describe the density matrix in the nuclear surface. We begin by considering the usual local density approximation. Then we refine this approximation to incorporate the exponential falloff of the nuclear wave functions.

By the local density approximation one would usually mean the following: The interaction of a pion with a cluster in a finite nucleus is the same as the interaction of the pion with the same cluster in an infinite nucleus of the same neutron and proton densities. This is accomplished in two steps. The first is to replace the finite nuclear wave functions by wave functions appropriate to an infinite nucleus

$$\langle\vec{r}\,|\,\psi_A\rangle\to\frac{e^{i\vec{k}_A\cdot\vec{r}}}{\sqrt{\Omega}}\,|\,t_A\rangle\,|\,s_A\rangle\ , \tag{4.1}$$

where Ω is the volume of the box and $|\,s_A\rangle$ and $|\,t_A\rangle$ are the spin and isospin wave functions. The second step is to replace the sum over states in a finite nucleus by that in an infinite nucleus according to the replacement

$$\Omega^{-1}\sum_A\to\sum_{t_As_A}\int\frac{d\vec{k}_A}{(2\pi)^3}\theta[k_F(t_a)-k_A]\ , \tag{4.2a}$$

where $k_F(t)$ is the Fermi momentum of nucleons of isospin projection t. We evaluate $k_F(t)$ as

$$k_F{}^3(t)=3\pi^2\rho_t(R)\ , \tag{4.2b}$$

where R is the location of the cluster in the nucleus. Also we denote density matrices in the local density approximation (LDA) with a tilde, i.e.,

$$n_{AB}(\vec{r}_1,\vec{r}_2)\overset{\text{LDA}}{\longrightarrow}\tilde{n}_{AB}(R,r)\ , \tag{4.3}$$

where we define

$$\vec{R}\equiv(\vec{r}_1+\vec{r}_2)/2 \tag{4.4}$$

and

$$\vec{r}\equiv\vec{r}_1-\vec{r}_2\ . \tag{4.5}$$

The substitution of Eqs. (4.1) and (4.2) into Eq. (3.14) is

$$\tilde{n}_{AB}^{(\gamma)}(R,r)=(\tfrac{1}{4})\rho_{t_A}(R)\rho_{t_B}(R) \tag{4.6a}$$

for the direct combination and

$$\tilde{n}_{AB}^{(\gamma)}(R,r)=(\tfrac{1}{4})\rho_{t_A}(R)S[k_F(t_A)r]\times\rho_{t_B}(R)S[k_F(t_B)r] \tag{4.6b}$$

for the exchange combination, where $S(x)$ is the Slater function

$$S(x)=\frac{3}{x^3}(\sin x - x\cos x)\simeq 1-\frac{x^2}{10}+\frac{x^4}{280}\ . \tag{4.6c}$$

It is shown in Ref. 26 that the Slater approximation to the density matrix is an excellent approximation in the interior of the nucleus. On the surface a new effect occurs, however, namely the exponential falloff of the nucleon wave functions. As a result, the Slater approximation *overestimates* the actual density matrix. (See Fig. 1 of Ref. 26.) In order to account for the exponential falloff of the nuclear wave functions we apply the correction described next.

Consider now the dependence on r_1 and r_2 of the functions $\Delta_i^{(\gamma)}$ in Eq. (3.10). Near resonance, the dominant contributions of the optical potential occur for r_1 and r_2 large and corresponding to the 10% density region of the nucleus. In the surface region (R large) an improvement over the Slater approximation would be to take

$$\tilde{n}_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)=\tilde{n}_{AB}^{(\gamma)}(R,r)e^{(R-r_1)/a}e^{(R-r_2)/a}\ , \tag{4.7}$$

where a is a characteristic diffuseness and the exponentials account for the falloff of the nuclear wave functions in the surface of the nucleus. The diffuseness $a(R)$ will presumably approach a constant value as $R\to\infty$ on the order of the diffuseness parameter of the charge density determined in electron scattering, and become very large as $R\to 0$. We now write for large R

$$r_1=\left|\vec{R}+\frac{\vec{r}}{2}\right|\simeq R+\frac{r^2}{8R}+\frac{\vec{R}\cdot\vec{r}}{2R} \tag{4.8a}$$

and

$$r_2=\left|\vec{R}-\frac{\vec{r}}{2}\right|\simeq R+\frac{r^2}{8R}-\frac{\vec{R}\cdot\vec{r}}{2R}\ , \tag{4.8b}$$

so Eq. (4.7) becomes

$$\tilde{n}_{AB}^{(\gamma)}(\vec{r}_1,\vec{r}_2)\simeq\tilde{n}_{AB}^{(\gamma)}(R,r)e^{-r^2/4Ra}\ . \tag{4.9}$$

These arguments apply to the density matrix, and the form of the result is different from that in Ref. 26. The reduction term found in Eq. (4.9) gives a correction of 0.75 at $r=2.5$ fm in Pb, for $R=7.5$ fm and

$$a(R)\equiv-\rho(R)/\rho'(R)=0.68\ .$$

This is very close to the size of the required correction, as deduced from Ref. 26, Fig. 1.

Now we may combine Eqs. (3.11), (3.12), (4.6), and (4.9) to obtain in our local density approximation

$$a^{(\gamma)}(\vec{k}',\vec{k};\vec{r}_1,\vec{r}_2)=\Gamma(r)\tilde{n}_1^{(\gamma)}(R,r)e^{-r^2/4Ra}\langle a^{(\gamma)}_{\vec{k}',\vec{k}}(\vec{r})\rangle_{av}\ , \tag{4.10a}$$

$$b^{(\gamma)}(\vec{k}',\vec{k};\vec{r}_1,\vec{r}_2)=\Gamma(r)\tilde{n}_2^{(\gamma)}(R,r)e^{-r^2/4Ra}\langle b^{(\gamma)}_{\vec{k}',\vec{k}}(\vec{r})\rangle_{av}\ , \tag{4.10b}$$

$$c^{(\gamma)}(\vec{k}',\vec{k};\vec{r}_1,\vec{r}_2)=\Gamma(r)\tilde{n}_3^{(\gamma)}(R,r)e^{-r^2/4Ra}\langle c^{(\gamma)}_{\vec{k}',\vec{k}}(\vec{r})\rangle_{av}\ , \tag{4.10c}$$

$$d^{(\gamma)}(\vec{k}',\vec{k};\vec{r}_1,\vec{r}_2)=\Gamma(r)\tilde{n}_2^{(\gamma)}(R,r)e^{-r^2/4Ra}\langle d^{(\gamma)}_{\vec{k}',\vec{k}}(\vec{r})\rangle_{av}\ , \tag{4.10d}$$

where we have assumed that the pair distribution function is state independent and a function of the relative coordinate only, i.e., $\Gamma_{AB}(r_1,r_2)=\Gamma(r)$. In these expressions, we have defined the spin average,

$$\langle 0\rangle_{av}\equiv(\tfrac{1}{4})\sum_{s_A s_B}\langle s_A(1)s_B(2)\,|\,0\,|\,s_A(1)s_B(2)\rangle\ , \tag{4.11}$$

and we have defined the six functions of neutron $[\rho_n(R)]$ and proton $[\rho_p(R)]$ densities:

$$\tilde{n}_1^{(\mathrm{dir})}(R,r)=[\rho_n+\rho_p]^2\ , \tag{4.12a}$$

$$\tilde{n}_2^{(\mathrm{dir})}(R,r)=[\rho_n+\rho_p]^2\ , \tag{4.12b}$$

$$\tilde{n}_3^{(\mathrm{dir})}(R,r)=-2[\rho_n+\rho_p][\rho_n-\rho_p]\ , \tag{4.12c}$$

$$\tilde{n}_1^{(\mathrm{ex})}(R,r)=[\rho_n S(k_{Fn}r)+\rho_p S(k_{Fp}r)]^2\ , \tag{4.12d}$$

$$\tilde{n}_2^{(\mathrm{ex})}(R,r)=[\rho_n S(k_{Fn}r)-\rho_p S(k_{Fp}r)]^2\ , \tag{4.12e}$$

$$\tilde{n}_3^{(\mathrm{ex})}(R,r)=-2[\rho_n S(k_{Fn}r)+\rho_p S(k_{Fp}r)][\rho_n S(k_{Fn}r)-\rho_p S(k_{Fp}r)]\ . \tag{4.12f}$$

V. DETAILED FORMULAS

To complete our analysis, we need to obtain the spin-averaged matrix elements appearing in Eq. (4.10). We shall identify these quantities by writing the specific operator expressions associated with the diagrams of Fig. 1. Briefly stated,[16] the operator expression corresponding to each direct diagram can be calculated by taking a πN scattering operator $[4\pi\hat{f}_i]$ for each vertex ($\vec{r}_i$), integrating over a free pion propagator

$$\left[\int d\vec{k}''(2\pi)^{-3}g_0(\vec{k}'',\vec{r}_i-\vec{r}_j)\right]$$

for each internal pion line ($\vec{k}''$), and then forming the average combination that is symmetric under interchange of vertex labels. For example, from Fig. 1(a) we have

$$\hat{D}_{12}^{1(a)}(\vec{k}',\vec{k};\vec{r}_1-\vec{r}_2)=\int\frac{d\vec{k}''}{(2\pi)^3}\{[-4\pi\hat{f}_1(\vec{k}',\vec{k}'')]g_0(\vec{k}'',\vec{r}_1-\vec{r}_2)[-4\pi\hat{f}_2(\vec{k}'',\vec{k})]$$
$$+[-4\pi\hat{f}_2(\vec{k}',\vec{k}'')]g_0(\vec{k}'',\vec{r}_2-\vec{r}_1)[-4\pi\hat{f}_1(\vec{k}'',\vec{k})]\}(\tfrac{1}{2})\ . \qquad (5.1)$$

To obtain the corresponding exchange operator expression, we only have to multiply the negative direct expression by the product of spin, isospin, and position exchange operators P_{12}^{σ}, P_{12}^{τ}, and P_{12}^{r}, respectively, where

$$P_{12}^{\sigma}=(1+\vec{\sigma}_1\cdot\vec{\sigma}_2)/2 \qquad (5.2a)$$

and

$$P_{12}^{\tau}=(1+\vec{\tau}_1\cdot\vec{\tau}_2)/2\ . \qquad (5.2b)$$

To take account of P_{12}^{r}, we substitute $\tilde{n}^{(\mathrm{ex})}$ for $\tilde{n}^{(\mathrm{dir})}$ in the final formulas. Continuing the example for Fig. 1(b), we then obtain

$$\hat{D}_{12}^{1(b)}(\vec{k}',\vec{k};\vec{r}_1-\vec{r}_2)=-\hat{D}_{12}^{1(a)}(\vec{k}',\vec{k};\vec{r}_1-\vec{r}_2)P_{12}^{\sigma}P_{12}^{\tau}P_{12}^{r}\ . \qquad (5.3)$$

In our subsequent illustrations, we shall assume that one partial wave (the P-wave) dominates the πN scattering amplitude; however, the final results may be easily generalized to several l values. For the fixed scatterer πN scattering operator, we then take

$$4\pi\hat{f}_j(\vec{k}',\vec{k})=\frac{v(k')v(k)}{v^2(k_0)}k_0^2[\hat{\lambda}_j^{\rho}\vec{e}_{k'}\cdot\vec{e}_k+i\hat{\lambda}_j^{\sigma}\vec{\sigma}_j\cdot(\vec{e}_{k'}\times\vec{e}_k)]\ , \qquad (5.4a)$$

or in terms of spherical harmonics

$$\hat{f}_j(\vec{k}',\vec{k})=\frac{v(k')v(k)}{v^2(k_0)}\left[\frac{k_0^2}{3}\right]\sum_{mn}[\hat{\lambda}_j^{\rho}\delta_{mn}+\hat{\lambda}_j^{\sigma}\sqrt{2}\sum_{\nu}(lm,1\nu\,|\,\mathrm{ln})\sigma_{j\nu}]Y_{lm}(\vec{e}_{k'})Y_{\mathrm{ln}}^{*}(\vec{e}_k)\ , \qquad (5.4b)$$

where $\vec{e}_k\equiv\vec{k}/|\vec{k}|$ is a unit vector and $(lm,\ l\nu\,|\,\mathrm{ln})$ is a Clebsch-Gordan coefficient. The isospin dependence is contained within $\hat{\lambda}^{\rho}$ and $\hat{\lambda}^{\sigma}$, i.e.,

$$\hat{\lambda}_j^{\rho}=\lambda_{00}+\tfrac{1}{2}\lambda_{01}\vec{\phi}\cdot\vec{\tau}_j \qquad (5.5)$$

and

$$\hat{\lambda}_j^{\sigma}=\lambda_{10}+\tfrac{1}{2}\lambda_{11}\vec{\phi}\cdot\vec{\tau}_j\ , \qquad (5.6)$$

where λ_{00}, λ_{01}, λ_{10}, and λ_{11} characterize the scalar-isoscalar, scalar-isovector, vector-isoscalar, and vector-isovector πN scattering amplitudes.

The basic manipulation in evaluating these diagrams is to expand the pion propagator in terms of spherical harmonics, i.e.,

$$g_0(\vec{k}'',\vec{r})=\frac{e^{-i\vec{k}''\cdot\vec{r}}}{k^2-(k'')^2+i\eta}=4\pi\sum_{LM}(i)^L\left[\frac{j_L(k''r)}{k^2-(k'')^2+i\eta}\right]Y_{LM}(\vec{e}_{k''})Y_{LM}^{*}(\vec{e}_r)\ , \qquad (5.7)$$

and use the identity

$$\int d\Omega_{\vec{k}''} Y_{LM}(\vec{e}_{k''}) Y^*_{1m}(\vec{e}_{k''}) Y_{1n}(\vec{e}_{k''}) = (-)^M \left[\frac{2L+1}{4\pi}\right]^{1/2} (L-M, 1m \mid 1n)(L0, 10 \mid 10) . \tag{5.8}$$

This eliminates the angular integration over the internal pions, and we are left with radial integrals that we define by the functions

$$H_L(k,r,\epsilon) \equiv \frac{i2}{k\pi} \int_0^\infty \frac{t^2 dt\, j_L(tr)}{k^2 - t^2 - \epsilon + i\eta} \left[\frac{v^2(t)}{v^2(k)}\right] . \tag{5.9}$$

These integrals are evaluated analytically for special cases in Ref. 14.

In the following three subsections (V A—V C), we write detailed expressions for the Δ's that correspond to diagrams 1(a)—(c). These subsections contain straightforward, but tedious, manipulations involving no further approximations. The casual reader may wish to jump ahead to subsection V E, where we accumulate our results of the previous sections and obtain our main result, Eq. (5.35).

A. The diagram shown in Figure 1(a)

By using the technique mentioned above, we may write Eq. (5.1) for the direct sequential-scattering diagram as

$$\hat{D}_{12}^{1(a)}(\vec{k}', \vec{k}; \vec{r}) = \omega(k', k, k_0)[H_0(k_0, r, 0)\hat{\Lambda}_0 + H_2(k_0, r, 0)\hat{\Lambda}_2] , \tag{5.10}$$

where all of the spin and isospin dependence is contained within the operators $\hat{\Lambda}_0$ and $\hat{\Lambda}_2$, and we have defined the recurring combination

$$\omega(k', k, k_0) = k_0{}^5 v(k') v(k) / [v^2(k_0) 12 i\pi] .$$

With the use of various tensor product identities, we evaluate these operators in the Appendix. Upon taking the spin average of $\hat{\Lambda}_0$ and $\hat{\Lambda}_2$, only the spin independent terms survive and we obtain

$$\langle \hat{\Lambda}_0 \rangle_{av} = \vec{e}_{k'} \cdot \vec{e}_k [\lambda_{00}{}^2 + \tfrac{1}{2}\lambda_{00}\lambda_{01} \vec{\phi} \cdot (\vec{\tau}_1 + \vec{\tau}_2) + \tfrac{1}{4}\lambda_{01}{}^2 \hat{\tau}_{12}] \tag{5.11}$$

and

$$\langle \hat{\lambda}_2 \rangle_{av} = K_{12} [\lambda_{00}{}^2 + \tfrac{1}{2}\lambda_{00}\lambda_{01} \vec{\phi} \cdot (\vec{\tau}_1 + \vec{\tau}_2) + \tfrac{1}{4}\lambda_{01}{}^2 \hat{\tau}_{12}] , \tag{5.12}$$

where we have defined the scalar combination of two tensors of rank two as

$$K_{12} \equiv \vec{e}_{k'} \cdot \vec{e}_k - 3\vec{e}_{k'} \cdot \vec{e}_r \vec{e}_k \cdot \vec{e}_r . \tag{5.13}$$

From Eqs. (5.10)—(5.12), we identify the spin-averaged matrix elements

$$\langle a_{\vec{k}', \vec{k}}^{1(a)}(\vec{r}) \rangle_{av} = (\lambda_{00}{}^2)\omega[\vec{e}_{k'} \cdot \vec{e}_k H_0 + K_{12} H_2] , \tag{5.14a}$$

$$\langle b_{\vec{k}', \vec{k}}^{1(a)}(\vec{r}) \rangle_{av} = 0 , \tag{5.14b}$$

$$\langle c_{\vec{k}', \vec{k}}^{1(a)}(\vec{r}) \rangle_{av} = (\tfrac{1}{2}\lambda_{00}\lambda_{01})\omega[\vec{e}_{k'} \cdot \vec{e}_k H_0 + K_{12} H_2] , \tag{5.14c}$$

$$\langle d_{\vec{k}', \vec{k}}^{1(a)}(\vec{r}) \rangle_{av} = (\tfrac{1}{4}\lambda_{01}{}^2)\omega[\vec{e}_{k'} \cdot \vec{e}_k H_0 + K_{12} H_2] . \tag{5.14d}$$

Now by using these expressions together with Eqs. (3.10) and (4.10), we may form the isoscalar, isovector, and isotensor contributions from this diagram.

$$\Delta_0^{1(a)} = \omega \Gamma(r) e^{-r^2/4Ra} \left\{ \vec{e}_{k'} \cdot \vec{e}_k H_0 \left[\tilde{n}_1^{(\mathrm{dir})} \lambda_{00}{}^2 - \tilde{n}_2^{(\mathrm{dir})} \frac{\lambda_{01}{}^2}{2(2T_0 - 1)}\right] + K_{12} H_2 \left[\tilde{n}_1^{(\mathrm{dir})} \lambda_{00}{}^2 - \tilde{n}_2^{(\mathrm{dir})} \frac{\lambda_{01}{}^2}{2(2T_0 - 1)}\right] \right\} , \tag{5.15a}$$

$$\Delta_1^{1(a)} = \frac{\omega}{2T_0}\Gamma(r)e^{-r^2/4Ra}\left\{ \vec{e}_{k'}\cdot\vec{e}_k H_0 \left[\tilde{n}_2^{(\mathrm{dir})}\frac{\lambda_{01}{}^2}{2(2T_0-1)} - \tilde{n}_3^{(\mathrm{dir})}\lambda_{00}\lambda_{01} \right] + K_{12}H_2 \left[\tilde{n}_2^{(\mathrm{dir})}\frac{\lambda_{01}{}^2}{2(2T_0-1)} - \tilde{n}_3^{(\mathrm{dir})}\lambda_{00}\lambda_{01} \right] \right\}, \tag{5.15b}$$

$$\Delta_2^{1(a)} = \omega\Gamma(r)e^{-r^2/4Ra}\{\vec{e}_{k'}\cdot\vec{e}_k H_0 + K_{12}H_2\}\tilde{n}_2^{(\mathrm{dir})}\frac{\lambda_{01}{}^2}{2T_0(2T_0-1)} . \tag{5.15c}$$

B. The diagram shown in Figure 1(b)

For the exchange sequential-scattering diagram we need to consider the spin average of $\hat{\Lambda}_i P_{12}^{\sigma} P_{12}^{\tau}$ for $i=0,2$. We consider first just the spin degrees of freedom. By using the Pauli identity, it is easy to show that there are only three types of spin dependence in $\hat{\Lambda}_i$ that survive the exchange spin averaging; constants, $\vec{\sigma}_1\cdot\vec{\sigma}_2$ terms, and $\vec{\sigma}_1\cdot\vec{p}\,\vec{\sigma}_2\cdot\vec{q}$ type terms. The spin averages of these type terms are

$$\langle P_{12}^{\sigma}\rangle_{\mathrm{av}} = \tfrac{1}{2} , \tag{5.16}$$

$$\langle \vec{\sigma}_1\cdot\vec{\sigma}_2 P_{12}^{\sigma}\rangle_{\mathrm{av}} = \tfrac{3}{2} , \tag{5.17}$$

and

$$\langle \vec{\sigma}_1\cdot\vec{p}\,\vec{\sigma}_2\cdot\vec{q}P_{12}^{\sigma}\rangle_{\mathrm{av}} = \tfrac{1}{2}\vec{p}\cdot\vec{q} . \tag{5.18}$$

From Eqs. (A15) and (A16) for Λ_0 and Λ_2 given in the Appendix, we then obtain

$$\langle \hat{\Lambda}_0 P_{12}^{\sigma}\rangle_{\mathrm{av}} = \tfrac{1}{2}\langle \hat{\Lambda}_0\rangle_{\mathrm{av}} + \frac{\vec{e}_{k'}\cdot\vec{e}_k}{4}[4\lambda_{10}{}^2 + 2\lambda_{10}\lambda_{11}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2) + \lambda_{11}{}^2\hat{\tau}_{12}] \tag{5.19}$$

and

$$\langle \hat{\Lambda}_2 P_{12}^{\sigma}\rangle_{\mathrm{av}} = \tfrac{1}{2}\langle \hat{\Lambda}_2\rangle_{\mathrm{av}} - \frac{K_{12}}{8}[4\lambda_{10}{}^2 + 2\lambda_{10}\lambda_{11}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2) + \lambda_{11}{}^2\hat{\tau}_{12}] . \tag{5.20}$$

These equations may now be multiplied by the isospin exchange operator, and through the use of the identities

$$\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2)P_{12}^{\tau} = \vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2) \tag{5.21}$$

and

$$\hat{\tau}_{12}P_{12}^{\tau} = 1 - \vec{\tau}_1\cdot\vec{\tau}_2 + \hat{\tau}_{12} , \tag{5.22}$$

we identify the spin-averaged matrix elements

$$\langle a_{\vec{k}',\vec{k}}^{1(b)}(\vec{r})\rangle_{\mathrm{av}} = -\tfrac{1}{4}[\langle a_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}} + 2\langle d_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}}] - \tfrac{1}{8}W(2\lambda_{10}{}^2 + \lambda_{11}{}^2) , \tag{5.23a}$$

$$\langle b_{\vec{k}',\vec{k}}^{1(b)}(\vec{r})\rangle_{\mathrm{av}} = -\tfrac{1}{4}[\langle a_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}} - 2\langle d_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}}] - \tfrac{1}{8}W(2\lambda_{10}{}^2 - \lambda_{11}{}^2) , \tag{5.23b}$$

$$\langle c_{\vec{k}',\vec{k}}^{1(b)}(\vec{r})\rangle_{\mathrm{av}} = -\tfrac{1}{2}\langle c_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}} - \tfrac{1}{4}W\lambda_{10}\lambda_{11} , \tag{5.23c}$$

$$\langle d_{\vec{k}',\vec{k}}^{1(b)}(\vec{r})\rangle_{\mathrm{av}} = -\tfrac{1}{2}\langle d_{\vec{k}',\vec{k}}^{1(a)}(\vec{r})\rangle_{\mathrm{av}} - \tfrac{1}{8}W\lambda_{11}{}^2 , \tag{5.23d}$$

where

$$W \equiv \omega[2\vec{e}_{k'}\cdot\vec{e}_k H_0 - K_{12}H_2] . \tag{5.23e}$$

These results together with Eqs. (3.10) and (4.10) give us

$$\Delta_0^{1(b)} = -\frac{\omega}{8}\Gamma(r)e^{-r^2/4Ra}\left\{\vec{e}_{k'}\cdot\vec{e}_k H_0\left[\tilde{n}_1^{(ex)}(2\lambda_{00}{}^2+4\lambda_{10}{}^2+\lambda_{01}{}^2+2\lambda_{11}{}^2)\right.\right.$$
$$\left.+\tilde{n}_2^{(ex)}\left[2\lambda_{00}{}^2+4\lambda_{10}{}^2-\lambda_{01}{}^2-2\lambda_{11}{}^2-\frac{2(\lambda_{01}{}^2+2\lambda_{11}{}^2)}{(2T_0-1)}\right]\right]$$
$$+K_{12}H_2\left[\tilde{n}_1^{(ex)}(2\lambda_{00}{}^2-2\lambda_{10}{}^2+\lambda_{01}{}^2-\lambda_{11}{}^2)\right.$$
$$\left.\left.+\tilde{n}_2^{(ex)}\left[2\lambda_{00}{}^2-2\lambda_{10}{}^2-\lambda_{01}{}^2+\lambda_{11}{}^2-\frac{2(\lambda_{01}{}^2-\lambda_{11}{}^2)}{(2T_0-1)}\right]\right]\right\}, \tag{5.24a}$$

$$\Delta_1^{1(b)} = \frac{-\omega}{8T_0}\Gamma(r)e^{-r^2/4Ra}\left\{\vec{e}_{k'}\cdot\vec{e}_k H_0\left[\tilde{n}_2^{(ex)}\frac{(\lambda_{01}{}^2+2\lambda_{11}{}^2)}{(2T_0-1)}-2\tilde{n}_3^{(ex)}(\lambda_{00}\lambda_{01}+2\lambda_{10}\lambda_{11})\right]\right.$$
$$\left.+K_{12}H_2\left[\tilde{n}_2^{(ex)}\frac{(\lambda_{01}{}^2-\lambda_{11}{}^2)}{(2T_0-1)}-2\tilde{n}_3^{(ex)}(\lambda_{00}\lambda_{01}-\lambda_{10}\lambda_{11})\right]\right\}, \tag{5.24b}$$

$$\Delta_2^{1(b)} = \frac{-\omega}{4T_0(2T_0-1)}\Gamma(r)e^{-r^2/4Ra}[\vec{e}_{k'}\cdot\vec{e}_k H_0(\lambda_{01}{}^2+2\lambda_{11}{}^2)+K_{12}H_2(\lambda_{01}{}^2-\lambda_{11}{}^2)]\tilde{n}_2^{(ex)}. \tag{5.24c}$$

C. The diagram shown in Figure 1(c)

This diagram is an iteration of the first-order optical potential

$$\hat{U}^{(1)}(\vec{k}',\vec{k}) = \int d\vec{r}\sum_{s_A t_A}\langle s_A t_A|[-4\pi\hat{f}(\vec{k}',\vec{k})]|s_A t_A\rangle\langle s_A|\rho^{(t_A)}(\vec{r},\vec{r})|s_A\rangle e^{-i\vec{r}\cdot(\vec{k}'-\vec{k})}, \tag{5.25}$$

which can be written in terms of the nuclear isospin

$$\hat{U}^{(1)}(\vec{k}',\vec{k}) = \frac{-v(k')v(k)}{v^2(k_0)}k_0{}^2\vec{e}_{k'}\cdot\vec{e}_k\int d\vec{r}e^{-i\vec{r}\cdot(\vec{k}'-\vec{k})}\left[\lambda_{00}\rho(r)+\frac{\lambda_{01}}{2T_0}\Delta\rho(r)\vec{\phi}\cdot\vec{T}\right], \tag{5.26}$$

where $\rho\equiv\rho_n+\rho_p$ and $\Delta\rho\equiv\rho_n-\rho_p$. In going from Eq. (5.25) to (5.26), we have omitted the spin dependence of $U^{(1)}$ because contributions that result from these terms vanish upon spin averaging. With $U^{(1)}$ in the form of Eq. (5.26), we may immediately write the isospin contributions from Fig. 1(c) in the form of Eq. (3.2),

$$\Delta_0^{1(c)} = \frac{v(k')v(k)}{v^2(k_0)}k_0{}^4\lambda_{00}{}^2\rho(r_1)\rho(r_2)\int\frac{d\vec{k}''}{(2\pi)^3}\vec{e}_{k'}\cdot\vec{e}_{k''}g_0(\vec{k}'',\vec{r})\vec{e}_{k''}\cdot\vec{e}_k\left[\frac{v^2(k'')}{v^2(k_0)}\right], \tag{5.27a}$$

$$\Delta_1^{1(c)} = \frac{v(k')v(k)}{v^2(k_0)2T_0}k_0{}^4\lambda_{00}\lambda_{01}[\rho(r_1)\Delta\rho(r_2)+\rho(r_2)\Delta\rho(r_1)]\int\frac{d\vec{k}''}{(2\pi)^3}\vec{e}_{k'}\cdot\vec{e}_{k''}g_0(\vec{k}'',\vec{r})\vec{e}_{k''}\cdot\vec{e}_k\left[\frac{v^2(k'')}{v^2(k_0)}\right], \tag{5.27b}$$

$$\Delta_2^{1(c)} = \frac{v(k')v(k)}{v^2(k_0)4T_0{}^2}k_0{}^4\lambda_{01}{}^1\Delta\rho(r_1)\Delta\rho(r_2)\int\frac{d\vec{k}''}{(2\pi)^3}\vec{e}_{k'}\cdot\vec{e}_{k''}g_0(\vec{k}'',\vec{r})\vec{e}_{k''}\cdot\vec{e}_k\left[\frac{v^2(k'')}{v^2(k_0)}\right]. \tag{5.27c}$$

If we now adopt the approximations for the densities discussed in Sec. IV, we obtain from these expressions

$$\Delta_0^{1(c)} = \omega e^{-r^2/4Ra}[\vec{e}_{k'}\cdot\vec{e}_k H_0+K_{12}H_2]\tilde{n}_1^{(dir)}\lambda_{00}{}^2, \tag{5.28a}$$

$$\Delta_1^{1(c)} = \frac{-\omega}{2T_0}e^{-r^2/4Ra}[\vec{e}_{k'}\cdot\vec{e}_k H_0+K_{12}H_2]\tilde{n}_3^{(dir)}\lambda_{00}\lambda_{01}, \tag{5.28b}$$

$$\Delta_2^{1(c)} = \frac{\omega}{4T_0{}^2}e^{-r^2/4Ra}[\vec{e}_{k'}\cdot\vec{e}_k H_0+K_{12}H_2]\tilde{n}_2^{(dir)}\lambda_{01}{}^2. \tag{5.28c}$$

One obvious difference between these results and those of Figs. 1(a) and (b) is the omission of the factor $\Gamma(r)$, which occurs because no correlations can arise between iterations of the optical potential in solving the Klein-Gordon equation.

D. Other terms

After having seen numerous specific examples, it should be clear that the direct, exchange, and crossed terms associated with Fig. 2, as well as all other second-order processes driven by the Δ_{33} resonance, can be straightforwardly expressed in terms of $\Delta_i^{(\gamma)}$'s of similar structure to those corresponding to Figs. 1(a) and (b).

E. Accumulated results

From the results of the above subsections [Eqs. (5.15), (5.24), and (5.28)], we note that all of the Δ's have the same general structure,

$$\Delta_i^{(\gamma)}(\vec{k}',\vec{k};\vec{R},\vec{r})=\vec{e}_{k'}\cdot\vec{e}_k M_i^{(\gamma)}(R,r)+K_{12}N_i^{(\gamma)}(R,r)\ , \tag{5.29}$$

where the functions M and N for any particular diagram can be identified from the appropriate equations. The final step in obtaining the second-order optical potentials from these results involves an integration over the two nucleons's positions. If we use the center-of-mass and relative position variables given in Eqs. (4.4) and (4.5), this integration [Eq. (3.7)] for Fig. 1 becomes

$$U_i^{(\gamma)}(\vec{k}',\vec{k})=\int d\vec{R}\,e^{-i\vec{q}\cdot\vec{R}}\int d\vec{r}\,e^{-i\vec{p}\cdot\vec{r}}\Delta_i^{(\gamma)}(\vec{k}',\vec{k};\vec{R},\vec{r})\ , \tag{5.30}$$

where $\vec{q}=\vec{k}'-\vec{k}$ and $\vec{p}=(\vec{k}'+\vec{k})/2$. Now considering just the $d\vec{r}$ integration in Eq. (5.30), we observe from Eq. (5.29) that the angular part ($d\Omega_r$) may be performed analytically. To obtain our result, we expand $e^{-i\vec{p}\cdot\vec{r}}$ in terms of spherical harmonics and use the relation

$$\sum_m \int d\Omega_r Y_{lm}(\vec{e}_r)K_{12}Y_{lm}^*(\vec{e}_p)=\delta_{l2}(\vec{e}_{k'}\cdot\vec{e}_k-3\vec{e}_{k'}\cdot\vec{e}_p\vec{e}_k\cdot\vec{e}_p)\ , \tag{5.31}$$

which may be easily proved using Eqs. (5.8), (A4), (A7), and (A8). The general result for the $d\vec{r}$ integration is

$$\int d\vec{r}\,e^{-i\vec{p}\cdot\vec{r}}\Delta_i^{(\gamma)}(\vec{k}',\vec{k};\vec{R},\vec{r})=\frac{4\pi v(k')v(k)}{v^2(k_0)}k_0{}^2\int_0^\infty r^2dr[\,\vec{e}_{k'}\cdot\vec{e}_k j_0(pr)M_i^{(\gamma)}(R,r)+(3\vec{e}_{k'}\cdot\vec{e}_p\vec{e}_k\cdot\vec{e}_p-\vec{e}_{k'}\cdot\vec{e}_k)j_2(pr)N_i^{(\gamma)}(R,r)]\ . \tag{5.32}$$

This expression makes it clear that the contribution of any term to $U^{(2)}$ is, in general, nonlocal and contributes to all partial waves even if $f_{\pi N}$ consists of only one partial wave. We may simplify Eq. (5.32), however, by using the fact that at medium energies the scattering is diffractive, and therefore predominantly in the forward direction. Then by approximating $\vec{p}\simeq\vec{k}$ in Eq. (5.32), we use

$$3\vec{e}_{k'}\cdot\vec{e}_p\vec{e}_k\cdot\vec{e}_p-\vec{e}_{k'}\cdot\vec{e}_k\simeq 2\vec{e}_{k'}\cdot\vec{e}_k$$

to obtain

$$\int d\vec{r}\,e^{-i\vec{p}\cdot\vec{r}}\Delta_i^{(\gamma)}(\vec{k}',\vec{k};\vec{R},\vec{r})\simeq\frac{4\pi v(k')v(k)}{v^2(k_0)}k_0{}^2\vec{e}_{k'}\cdot\vec{e}_k\int_0^\infty r^2dr[j_0(kr)M_i^{(\gamma)}(R,r)+2j_2(kr)N_i^{(\gamma)}(R,r)]\ . \tag{5.33}$$

With the approximation of Eq. (5.33), we may easily combine $\hat{U}^{(1)}$ and $\hat{U}^{(2)}$ to obtain our *total* optical potential

$$\hat{U}(\vec{k}',\vec{k})=\frac{-v(k')v(k)}{v^2(k_0)}k_0{}^2\vec{e}_{k'}\cdot\vec{e}_k\int d\vec{R}\,e^{-i\vec{q}\cdot\vec{R}}\left\{\left[\lambda_{00}\rho(R)+\frac{\lambda_{01}}{2T_0}\Delta\rho(R)\vec{\phi}\cdot\vec{T}\right]+[\xi_0(k_0,R)+\xi_1(k_0,R)\vec{\phi}\cdot\vec{T}+\xi_2(k_0,R)(\vec{\phi}\cdot\vec{T})^2]\right\}\ . \tag{5.34}$$

We close this section by displaying the general density and isospin dependence of the functions ξ_i. These dependences result from the functions M and N and do *not* depend upon our particular approximation, Eq. (5.33). Subtracting the contribution of Fig. 1(c) from the sum of contributions of Figs. 1(a) and (b), we obtain

$$\xi_0(k,R)=\frac{\rho^2(R)}{\rho_0}\lambda_0(A,k,R)+\frac{\Delta\rho^2(R)}{\rho_0}\lambda_3(A,k,R)-\frac{\Delta\rho^2(R)}{(2T_0-1)\rho_0}\lambda_2(A,k,R)\ , \tag{5.35a}$$

$$\xi_1(k,R)=\frac{\rho(R)\Delta\rho(R)}{2T_0\rho_0}\lambda_1(A,k,R)+\frac{\Delta\rho^2(R)}{2T_0(2T_0-1)\rho_0}\lambda_2(A,k,R)\ , \tag{5.35b}$$

$$\xi_2(k,R)=\frac{\Delta\rho^2(R)}{T_0(2T_0-1)\rho_0}\lambda_2(A,k,R)+\frac{\Delta\rho^2}{T_0{}^2\rho_0}\lambda_4(A,k,R)\ , \tag{5.35c}$$

where we have introduced the constant density $\rho_0\equiv 0.16\ \text{fm}^{-3}$ so that the first- and second-order parameters will have the same units. We have not explicitly indicated a dependence of λ_i on $N-Z$ in Eq. (5.35), anticipating a main result of the next section that this dependence is very weak. Again, we emphasize that *all* second-order contributions driven by p-wave pion-nucleon scattering can be cast into this same universal form. In writing our results for the λ's, we wish to separate the short-range from the long-range correlation dependences of these functions, i.e., $\lambda=\lambda(\text{SR})+\lambda(\text{LR})$. This separation may be accomplished by introducing

$$G_{lj}(\text{LR};p,R)\equiv ik_0{}^3\int_0^\infty r^2dr\,e^{-r^2/4Ra}j_l(pr)H_l(k_0,r,0)\left[\frac{\tilde{n}_j^{(\text{ex})}(R,r)}{\tilde{n}_j^{(\text{dir})}(R,r)}\right]\ , \tag{5.36a}$$

and

$$G_{lj}(\text{SR};p,R)\equiv ik_0{}^3\int_0^\infty r^2dr\,e^{-r^2/4Ra}j_l(pr)H_l(k_0,r,0)\left[\frac{\tilde{n}_j^{(\text{ex})}(R,r)}{\tilde{n}_j^{(\text{dir})}(R,r)}\right][\Gamma(r)-1]\ , \tag{5.36b}$$

where we have extended the definitions of $\tilde{n}_j$ in Eq. (4.12) to include the label $j=0$, $\tilde{n}_0^{(\text{ex})}=\tilde{n}_0^{(\text{dir})}\equiv 1$. We then obtain the following expressions for the λ's:

$$\lambda_0(\text{SR};k,R)=\rho_0\lambda_{00}{}^2G_0^+(\text{SR})-\frac{\rho_0}{8}(2\lambda_{00}{}^2+\lambda_{01}{}^2)G_1^+(\text{SR})-\frac{\rho_0}{4}(2\lambda_{10}{}^2+\lambda_{11}{}^2)G_1^-(\text{SR})\ , \tag{5.37a}$$

$$\lambda_0(\text{LR};k,R)=-\frac{\rho_0}{8}(2\lambda_{00}{}^2+\lambda_{01}{}^2)G_1^+(\text{LR})-\frac{\rho_0}{4}(2\lambda_{10}{}^2+\lambda_{11}{}^2)G_1^-(\text{LR})\ , \tag{5.37b}$$

$$\lambda_1(\text{SR};k,R)=\rho_0\lambda_{00}\lambda_{01}[2G_0^+(\text{SR})-G_3^+(\text{SR})]-\rho_0 2\lambda_{10}\lambda_{11}G_3^-(\text{SR})\ , \tag{5.37c}$$

$$\lambda_1(\text{LR};k,R)=-\rho_0\lambda_{00}\lambda_{01}G_3^+(\text{LR})-\rho_0 2\lambda_{10}\lambda_{11}G_3^-(\text{LR})\ , \tag{5.37d}$$

$$\lambda_2(k,R)=\rho_0\frac{\lambda_{01}{}^2}{4}[2G_0^+-G_2^+]-\rho_0\frac{\lambda_{11}{}^2}{2}G_2^-\ , \tag{5.37e}$$

$$\lambda_3(k,R)=\frac{-\rho_0}{8}(2\lambda_{00}{}^2-\lambda_{01}{}^2)G_2^+-\frac{\rho_0}{4}(2\lambda_{10}{}^2-\lambda_{11}{}^2)G_2^-\ , \tag{5.37f}$$

$$\lambda_4(\text{SR};k,R)=0\ , \tag{5.37g}$$

$$\lambda_4(\text{LR};k,R)=-\rho_0\frac{\lambda_{01}{}^2}{4}G_0^+(\text{LR})\ , \tag{5.37h}$$

where the non-spin-flip (G^+) and spin-flip (G^-) combinations of G's are defined as

$$G_i^+(\text{SR})=\tfrac{1}{3}[G_{0i}(\text{SR};k,R)+2G_{2i}(\text{SR};k,R)]\ , \tag{5.38a}$$

$$G_i^+(\text{LR})=\tfrac{1}{3}[G_{0i}(\text{LR};k,R)+2G_{2i}(\text{LR};k,R)]\ , \tag{5.38b}$$

$$G_i^-(\text{SR})=\tfrac{1}{3}[G_{0i}(\text{SR};k,R)-G_{2i}(\text{SR};k,R)]\ , \tag{5.38c}$$

$$G_i^-(\text{LR})=\tfrac{1}{3}[G_{0i}(\text{LR};k,R)-G_{2i}(\text{LR};k,R)]\ . \tag{5.38d}$$

The omission of the (SR) or (LR) label from any of these equations just indicates the equation has the same form for either (SR) or (LR).

Note that the (long range) contributions of the isoscalar and isovector terms for Fig. 1(c) cancel identically against corresponding terms in Fig. 1(a). This arises because the optical potential is defined in such a way that its iteration by the Klein-Gordon equation accounts for higher-order terms in the multiple scattering expansion. However, this cancellation does *not* occur in the isotensor potential. The reason is that U has matrix elements only between the ground state and single- and double-isobaric analog states. On the other hand, the second-order processes in Figs. 1(a) and (b) consist of *all possible* intermediate states. Thus, the contribution to the *isotensor potential that we have evaluated*

$$\text{Fig. 1(a)} + \text{Fig. 1(b)} - \text{Fig. 1(c)}$$

is precisely that arising from the sum to all nonanalog intermediate states. It is interesting to note that this contribution to double charge exchange has a characteristic T dependence which will clearly show up in cross sections. We shall show the effect on cross sections in a companion paper.

VI. NUMERICAL RESULTS

The coefficients of the second-order optical potential have been expressed in Eq. (5.37) in terms of 16 integrals defined by the functions G_i^+ and G_i^-. These integrals are of the form

$$G_i^+ = \frac{ik_0{}^3}{3}\int_0^\infty r^2 dr\, F_i(r,R)[\,j_0(kr)H_0(k,r,0) + 2j_2(kr)H_2(k,r,0)] \tag{6.1a}$$

and

$$G_i^- = \frac{ik_0{}^3}{3}\int_0^\infty r^2 dr\, F_i(r,R)[\,j_0(kr)H_0(k,r,0) - j_2(kr)H_2(k,r,0)]\ . \tag{6.1b}$$

We now want to study these integrals. First, however, we want to take into account the fact that in propagating the distance r in the nucleus, the pion undergoes interactions with the medium, which can be described by including self-energy insertions on the pion propagators in Fig. 1. We include this effect only in lowest order, by replacing the pion propagator in Eq. (6.1) by its renormalized, local-density equivalent. This is accomplished through the functions H_L [see Eq. (5.9)] by the substitution in Eq. (6.1)

$$H_L(k,r,0) \rightarrow H_L[k,r,U^{(1)}(R)]\ , \tag{6.2a}$$

where

$$U^{(1)}(R) = -k_0{}^2\rho(R)\lambda_{00}\left[1+\frac{\nabla^2\rho(R)}{2k_0{}^2\rho(R)}\right], \tag{6.2b}$$

which is the equivalent local form of the p-wave pion-nucleus optical potential.[27] The Laplacian term accounts for the effects of the nuclear surface.

There are now several issues of interest. One is to evaluate the sizes of the various parameters λ_i, and another is to see the extent to which they depend on A, ρ, $\Delta\rho$, and R.

The study of the $G_i^\pm$ functions is made easier by noting that there exist rather simple approximations to the Slater functions in the region of the resonance. The point is that $x = k_F r$ is small in the region of space where the dominant contributions to the $G_i^\pm$ integrals arise. For the $\lambda_i(\text{SR})$, this is clear because the integrals have a range of about 0.5 fm, and therefore $k_F r < 0.68$ taking $k_F = 1.36\ \text{fm}^{-1}$. We may therefore set $S(x) \simeq 1$ [see Eq. (4.6c)] and

$$\tilde{n}_i^{(\text{ex})}(R,r)/\tilde{n}_i^{(\text{dir})}(R,r) = 1 \tag{6.3a}$$

or

$$G_i^\pm(\text{SR}) \simeq G_0^\pm(\text{SR}) \equiv G^\pm(\text{SR})\ , \tag{6.3b}$$

which in turn gives us from Eq. (5.38)

$$\lambda_0(\text{SR};k,R) = \frac{\rho_0}{8}(6\lambda_{00}{}^2 - \lambda_{01}{}^2)G^+(\text{SR}) - \frac{\rho_0}{4}(2\lambda_{10}{}^2 + \lambda_{11}{}^2)G^-(\text{SR})\ , \tag{6.4a}$$

$$\lambda_1(\text{SR};k,R) = \rho_0\lambda_{00}\lambda_{01}G^+(\text{SR}) - \rho_0 2\lambda_{10}\lambda_{11}G^-(\text{SR})\ , \tag{6.4b}$$

$$\lambda_2(\text{SR};k,R) = \frac{\rho_0}{4}\lambda_{01}{}^2G^+(\text{SR}) - \frac{\rho_0}{2}\lambda_{11}{}^2G^-(\text{SR})\ , \tag{6.4c}$$

$$\lambda_3(\text{SR};k,R) = \frac{-\rho_0}{8}(2\lambda_{00}{}^2 - \lambda_{01}{}^2)G^+(\text{SR}) - \frac{\rho_0}{4}(2\lambda_{10}{}^2 - \lambda_{11}{}^2)G^-(\text{SR})\ . \tag{6.4d}$$

For the $\lambda_i(\mathrm{LR})$ it is necessary to take into account the first correction in Eq. (4.6c). We find

$$\tilde{n}_i^{(\mathrm{ex})}(R,r)/\tilde{n}_i^{(\mathrm{dir})}(R,r)=1-\gamma_i(k_F r)^2+\delta_i(k_F r)^4 , \tag{6.5}$$

where $\gamma_i=\frac{2}{10}, \frac{1}{3}, \frac{4}{15}$, and $\delta_i=\frac{1}{100}, \frac{1}{36}, \frac{1}{60}$ for $i=1$, 2, and 3, respectively. The corrections to this are of order $(\Delta\rho/\rho)^2$ and would contribute to the optical potential in higher orders than $U^{(2)}$. We therefore neglect these corrections, which we shall demonstrate below to be numerically small. This establishes that the explicit dependence on $\Delta\rho$ and T in Eq. (5.37) accounts for the dominant dependence on the neutron excess in $U^{(2)}$. Now combining Eqs. (5.36), (5.37), (6.1), and (6.5), we find

$$\lambda_0(\mathrm{LR};k,R)=\frac{-\rho_0}{8}(2\lambda_{00}{}^2+\lambda_{01}{}^2)(1-\mu_1^+)G^+(\mathrm{LR}) -\frac{\rho_0}{4}(2\lambda_{10}{}^2+\lambda_{11}{}^2)(1-\mu_1^-)G^-(\mathrm{LR}) , \tag{6.6a}$$

$$\lambda_1(\mathrm{LR};k,R)=-\rho_0\lambda_{00}\lambda_{01}(1-\mu_3^+)G^+(\mathrm{LR}) -2\rho_0\lambda_{10}\lambda_{11}(1-\mu_3^-)G^-(\mathrm{LR}) , \tag{6.6b}$$

$$\lambda_2(\mathrm{LR};k,R)=\frac{\rho_0}{4}\lambda_{01}{}^2(1+\mu_2^+)G^+(\mathrm{LR}) -\frac{\rho_0}{2}\lambda_{11}{}^2(1-\mu_2^-)G^-(\mathrm{LR}) , \tag{6.6c}$$

$$\lambda_3(\mathrm{LR};k,R)=\frac{-\rho_0}{8}(2\lambda_{00}{}^2-\lambda_{01}{}^2)(1-\mu_2^+)G^+(\mathrm{LR}) -\frac{\rho_0}{4}(2\lambda_{10}{}^2-\lambda_{11}{}^2)(1-\mu_2^-)G^-(\mathrm{LR}) , \tag{6.6d}$$

$$\lambda_4(\mathrm{LR};k,R)=\frac{\rho_0}{4}\lambda_{01}{}^2G^+(\mathrm{LR}) , \tag{6.6e}$$

where

$$\mu_i^+\equiv\gamma_i k_F{}^2\langle r^2\rangle^+-\delta_i k_F{}^4\langle r^4\rangle^+ , \tag{6.7a}$$

$$\mu_i^-\equiv\gamma_i k_F{}^2\langle r^2\rangle^--\delta_i k_F{}^4\langle r^4\rangle^- , \tag{6.7b}$$

$$\langle r^n\rangle^+\equiv\frac{ik_0{}^3}{3G^+(\mathrm{LR})}\int_0^\infty r^{2+n}dr\, e^{-r^2/4Ra}[j_0(kr)H_0(k,r,U^{(1)})+2j_2(kr)H_2(k,r,U^{(1)})] , \tag{6.7c}$$

$$\langle r^n\rangle^-\equiv\frac{ik_0{}^3}{3G^-(\mathrm{LR})}\int_0^\infty r^{2+n}dr\, e^{-r^2/4Ra}[j_0(kr)H_0(k,r,U^{(1)})-j_2(kr)H_2(k,r,U^{(1)})] , \tag{6.7d}$$

$$G^+(\mathrm{LR})\equiv\frac{ik_0{}^3}{3}\int_0^\infty r^2dr\, e^{-r^2/4Ra}[j_0(kr)H_0(k,r,U^{(1)})+2j_2(kr)H_2(k,r,U^{(1)})] , \tag{6.8a}$$

$$G^-(\mathrm{LR})\equiv\frac{ik_0{}^3}{3}\int_0^\infty r^2dr\, e^{-r^2/4Ra}[j_0(kr)H_0(k,r,U^{(1)})-j_2(kr)H_2(k,r,U^{(1)})] , \tag{6.8b}$$

$$G^+(\mathrm{SR})\equiv\frac{ik_0{}^3}{3}\int_0^\infty r^2dr\, e^{-r^2/4Ra}[\Gamma(r)-1][j_0(kr)H_0(k,r,U^{(1)})+2j_2(kr)H_2(k,r,U^{(1)})] , \tag{6.8c}$$

$$G^-(\mathrm{SR})\equiv\frac{ik_0{}^3}{3}\int_0^\infty r^2dr\, e^{-r^2/4Ra}[\Gamma(r)-1][j_0(kr)H_0(k,r,U^{(1)})-j_2(kr)H_2(k,r,U^{(1)})] . \tag{6.8d}$$

In all of the calculations that we are about to discuss, we have taken $v(k)$ to have the form

$$v(k)=k(1+k^2/\beta^2)^{-1} , \tag{6.9}$$

with $\beta=4.82\ \mathrm{fm}^{-1}$. We have also used the phase shift analysis of Ref. 28 to obtain the πN parameters, and at 180 MeV we find (in units of fm^3) $\lambda_{00}=0.52+9.15i$, $\lambda_{01}=0.82+9.13i$, $\lambda_{10}=0.73+4.53i$, and $\lambda_{11}=1.11+4.53i$. For the nuclear densities, we have taken a two-parameter Fermi shape with the same half-density radius ($R_c=1.1A^{1/3}$ fm) and diffuseness ($a_0=0.56$ fm) for both neutrons and protons. The radial distribution function we have used is a step function $\Gamma(R)=\theta(0.5-R)$.

To study the extent to which the parameters λ_i depend upon A and R, we have calculated them from Eqs. (5.36), (5.37), and (6.2) for various values of A and R. These results are displayed in Fig. 3, where

$$\lambda_i=\lambda_i(\mathrm{SR})+\lambda_i(\mathrm{LR}) .$$

The circles are our results for the targets ^{18}O, ^{58}Ni, ^{90}Zr, ^{120}Sn, and ^{208}Pb at the corresponding values

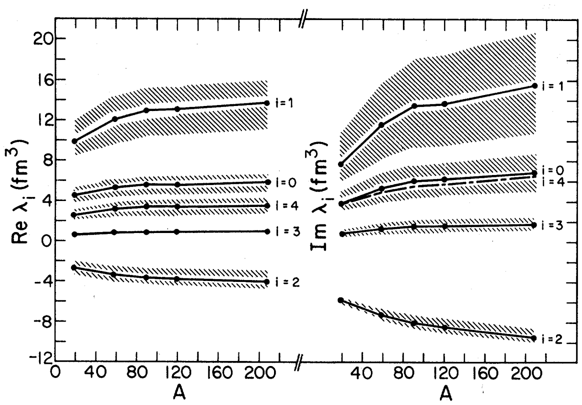

FIG. 3. Calculated parameters characterizing the second-order pion-nucleus optical potential at $T_\pi = 180$ MeV as a function of nuclear mass number A. The solid dots result from setting $\bar{R}$ to the values listed in Table II. The hatch marks show the variation as $\bar{R}$ is varied by ± 0.5 fm.

$\bar{R} = 3.50$, 5.50, 6.37, and 8.12 fm. The hatch marks indicate the variation in the λ's that correspond to a variaton of these average radii by ± 0.5 fm. These values of $\bar{R}$ were taken from Ref. 24, where a diffractive model of π-nucleus scattering was used to obtain them. In the diffractive theory, the parameter $\bar{R}$ corresponds to the impact parameter at which the derivative of the profile function for elastic scattering peaks. The variation of ± 0.5 fm corresponds approximately to the fullwidth at half maximum of the same quantity and therefore it is λ within this range of R which determines the scattering in semiclassical theories and presumably more exact theories as well. Note that for all λ_i the variation is small in this range, compared to the variation of the density over the same interval. Therefore, we shall approximate the R dependence by choosing λ_i to be evaluated by its value at $\bar{R}$; this is the value which should be compared to the phenomenologically determined one.

From Fig. 3 we observe that the mean values, $\lambda(\bar{R})$, show a very smooth dependence on A and are independent of $N-Z$. For the most part, the λ_i's are essentially independent of A. The exceptions are λ_1 and $\mathrm{Im}\lambda_2$, where the A dependence of these can be phenomenologically described as

$$\mathrm{Re}\lambda_1(A) \simeq 14(1+\exp[(-40-A)/60])^{-1} ,$$

$$\mathrm{Im}\lambda_1(A) \simeq 16(1+\exp[(10-A)/50])^{-1} ,$$

and

$$\mathrm{Im}\lambda_2(A) \simeq -3.2A^{2/10} .$$

In Table I we have listed the individual short-range (SR) and long-range (LR) components of the λ's for a typical case, ^{90}Zr with $R = \bar{R} = 6.37$ fm. From this table, we see that LR components dominate over the SR components. This LR dominance holds for all the values of A we have considered. Furthermore, the SR values listed in Table I are within 10% of *all* the SR values we have obtained by varying A and R.

There are two additional points we can make from the results listed in Table I that universally apply to *all* of our results. Firstly, by comparing the columns labeled total and approximate, we see that the approximate equations for λ_i, Eqs. (6.4) and (6.6), are accurate. This establishes the validity of the approximation in Eqs. (6.4) and (6.6) of dropping the $(\Delta\rho/\rho)^2$ correction terms from the λ_i's. Secondly, in considering the LR components, we observe the exchange-spin-flip contributions to be negligible with respect to the direct and exchange-non-spin-flip terms. Therefore, to qualitatively understand the variations of the λ's plotted in Fig. 3, we shall focus on the LR components of λ_i given in Eq. (6.6) and characterized by the $G^+(\mathrm{LR})$ function.

TABLE I. Individual components of calculated second-order optical potential parameters for ^{90}Zr at $T_\pi = 180$ MeV with $R = 6.37$ fm.

	Direct	Exchange (non-spin-flip)	Exchange (spin-flip)	Total	Approximate
λ_0(SR)	$1.89+0.12i$	$-0.70-0.03i$	$-0.34+0.07i$	$0.84+0.16i$	$0.84+0.16i$
λ_1(SR)	$3.78+0.12i$	$-1.88-0.06i$	$-0.91+0.20i$	$0.99+0.26i$	$0.97+0.26i$
λ_2(SR)	$0.95+0.00i$	$-0.47+0.00i$	$-0.23+0.07i$	$0.25+0.07i$	$0.25+0.07i$
λ_3(SR)		$-0.23-0.03i$	$-0.11+0.00i$	$-0.35-0.03i$	$-0.35-0.03i$
λ_0(LR)		$4.47+5.57i$	$0.28+0.33i$	$4.74+5.90i$	$4.75+5.73i$
λ_1(LR)		$11.75+12.99i$	$0.26+0.22i$	$12.01+13.21i$	$12.59+13.50i$
λ_2(LR)	$-6.92-11.05i$	$2.95+2.83i$	$0.08+0.14i$	$-3.89-8.08i$	$-3.72-7.85i$
λ_3(LR)		$1.28+1.60i$		$1.28+1.60i$	$1.35+1.82i$
λ_4	$3.46+5.52i$				

The quantities needed to calculate the $\lambda_i(\mathrm{LR})$ terms according to Eq. (6.6) are given in Table II. In the upper portion of Table II, we have listed, for different targets, the quantities $\langle r^2\rangle^+$, $\langle r^4\rangle^+$, and $G^+(\mathrm{LR})$ evaluated at the centroid value of the radius, $R=\bar{R}$. The combinations of these quantities that are appropriate for calculating $\lambda_0(\mathrm{LR})$, $\lambda_1(\mathrm{LR})$, $\lambda_2(\mathrm{LR})$, and $\lambda_3(\mathrm{LR})$ are listed in columns one through four, respectively, of the lower portion of Table II. We note that the A dependence of each column in the lower portion of Table II is essentially the same and, except for the imaginary terms of column three, the values in these columns are essentially the same for each nucleus. The factors that determine whether the A dependence of these columns is enhanced or suppressed are the different combinations of the πN parameters λ_{00} and λ_{01}. The combination appropriate for $\lambda_1(\mathrm{LR})[-\lambda_{00}\lambda_{01}]$ is much larger than any of the other combinations; thus the greater A dependence of λ_1.

We should point out that the isoscalar and isovector potential corresponding to Figs. 1(a)—(c) have been calculated in numerous contexts. The parameter $\lambda(SR)$ contains the physics of the Lorentz-Lorenz effect.[11] Eisenberg *et al.*[29] carefully studied the effect of the exchange terms on charge exchange scattering from ^{13}C. They found large enhancements, which is consistent with the large coefficient $\lambda_1(\mathrm{LR})$ we find. The dependence on the parameters characterizing the free pion-nucleon amplitude in these terms is the same as that found by Delorme and Ericson.[30]

No one, to our knowledge, has carefully worked out the isotensor correlations arising from the contributions in Fig. 1. It is clear from Fig. 3 that large isotensor contributions to the potential arise from the difference between the iterated pion-nucleon scattering amplitude and the iterated optical potential. These terms have a different dependence on T_N and will significantly affect the nuclear dependence of the double-charge-exchange cross section. Although we have found the λ_i coefficients that result from the processes in Fig. 1 to be large with respect to the πN coefficients, until the processes in Fig. 2 are evaluated we may not place any particular significance on the sign or magnitude of the overall terms in the second-order potential.

VII. SUMMARY AND DISCUSSION

The main goal of this paper was to obtain a theoretically motivated form for the isospin dependence of the second-order pion-nucleus optical potential, $\hat{U}^{(2)}$, for pion scattering near the (3,3) resonance. Assuming isospin invariance, we have taken $\hat{U}^{(2)}$ to have the most general dependence on $\vec{\phi}\cdot\vec{\mathrm{T}}$, namely

$$\hat{U}^{(2)}=U_0^{(2)}+U_1^{(2)}\vec{\phi}\cdot\vec{\mathrm{T}}+U_2^{(2)}(\vec{\phi}\cdot\vec{\mathrm{T}})^2 , \tag{7.1}$$

where $\vec{\phi}$ is the pion and $\vec{\mathrm{T}}$ the nuclear isotopic spin operators. Our main result is that *all* second-order processes contributing to $U_0^{(2)}$, $U_1^{(2)}$, and $U_2^{(2)}$ which are driven by the (3,3) resonance have the same dependence on T, $\rho=\rho_n+\rho_p$ and $\Delta\rho=\rho_n(r)-\rho_p(r)$, which is given by

$$U_i^{(2)}(\vec{\mathrm{k}}\,',\vec{\mathrm{k}})=\frac{-v(k')v(k)}{v^2(k_0)}k_0{}^2\vec{\mathrm{e}}_{k'}\cdot\vec{\mathrm{e}}_k \times\int dR\, e^{-i\vec{\mathrm{R}}\cdot(\vec{\mathrm{k}}\,'-\vec{\mathrm{k}})}\xi_i(R) , \tag{7.2}$$

where

TABLE II. Approximate quantities arising from long-range correlations that qualitatively describe our results of Fig. 3.

Target	$\bar{R}$ (fm)	ρ/ρ_0	$\langle r^2\rangle^+$	$\langle r^4\rangle^+$	G^+ (LR)
^{18}O	3.50	0.204	$4.02+2.47i$	$39.0+36.8i$	$0.55+1.22i$
^{58}NI	5.50	0.094	$5.68+2.91i$	$83.0+56.9i$	$0.67+1.61i$
^{90}Zr	6.37	0.071	$6.58+3.11i$	$114.2+68.5i$	$0.72+1.80i$
^{120}Sn	6.87	0.072	$6.97+3.26i$	$130.0+77.0i$	$0.71+1.86i$
^{208}Pb	8.12	0.057	$8.21+3.56i$	$182.7+98.1i$	$0.74+2.09i$

Target	$(1-\mu_1^+)G^+$(LR)	$(1-\mu_3^+)G^+$(LR)	$(1+\mu_2^+)G^+$(LR)	$(1-\mu_2^+)G^+$(LR)
^{18}O	$0.55+0.71i$	$0.53+0.63i$	$0.63+1.79i$	$0.47+0.65i$
^{58}Ni	$0.69+1.02i$	$0.68+0.91i$	$0.70+2.32i$	$0.64+0.90i$
^{90}Zr	$0.75+1.18i$	$0.73+1.04i$	$0.73+2.57i$	$0.71+1.03i$
^{120}Sn	$0.74+1.21i$	$0.73+1.07i$	$0.73+2.64i$	$0.69+1.08i$
^{208}Pb	$0.77+1.37i$	$0.77+1.23i$	$0.76+2.95i$	$0.72+1.23i$

$$\xi_0(R)=\frac{\rho^2(R)}{\rho_0}\lambda_0+\frac{\Delta\rho^2(R)}{\rho_0}\lambda_3 -\frac{1}{2T-1}\frac{\Delta\rho^2(R)}{\rho_0}\lambda_2 , \quad (7.3a)$$

$$\xi_1(R)=\frac{1}{2T}\frac{\rho(R)\Delta\rho(R)}{\rho_0}\lambda_1 +\frac{1}{2T(2T-1)}\frac{\Delta\rho^2(R)}{\rho_0}\lambda_2 , \quad (7.3b)$$

$$\xi_2(R)=\frac{1}{T(2T-1)}\frac{\Delta\rho^2(R)}{\rho_0}\lambda_2+\frac{\lambda_4}{T^2}\frac{\Delta\rho^2}{\rho_0}(R) . \quad (7.3c)$$

These results can be easily extended to include nonresonant pion-nucleon dynamics, resulting in a form similar to Eq. (7.3) but including additional π-2N partial wave contributions.

The five quantities $\lambda_0^{(2)}$–$\lambda_4^{(2)}$ depend upon the details of reaction dynamics and nuclear structure physics, but do not depend upon ρ or $\Delta\rho$. We have evaluated $\lambda^{(2)}$ for specific second-order processes, viz., the sequential p-wave scattering of the pion from clusters of two nucleons in the nucleus. We include the effect of both long- and short-range correlations and the spin dependence of the pion-nucleon scattering amplitude. In cases where comparison is possible, namely for the isoscalar and isovector potential, our results are similar to those found by others. However, the details of the isotensor potential, as well as the simple universal form for the overall isospin structure of U we have found, are completely new.

In our numerical calculations we found a large correction to the isotensor potential arising from nonanalog intermediate states in sequential scattering of the pion from two nucleons. The significance of this correction for the isospin dependence of the double-charge-exchange cross section will be evaluated in a subsequent paper and shown to resolve a puzzle observed in the systematics of the measured forward cross sections.

We found that the λ_i have a weak residual dependence on R and A, but are essentially independent of $N-Z$. Additional R dependence may arise from third- and higher-order terms in U which could be taken into account in our results as renormalizations of the coefficients λ_i. One example of these renormalizations is our use of the optical potential damping on the pion propagation between $\vec{r}_1$ and $\vec{r}_2$ in the evaluation of Fig. 1, which means that our results actually go beyond a strict second-order calculation. In any case, for scattering near the (3,3) resonance, the pion elastic and charge exchange scattering is dominated by interactions with the nucleus in the surface near $R=\bar{R}$, where

$$\rho(\bar{R})/\rho(0)\approx 0.1 .$$

Thus, in practice it should be a good approximation to regard $\lambda_i(R)$ as constant in Eq. (6.4) and evaluate them at $R=\bar{R}$.

A major correction to be applied to our result is pion scattering from the spin density of the valence nucleons. We have explicitly ignored these effects, although our methods are sufficiently general to allow their calculation. However, if the theory is applied for medium to heavy $J=0$ spherical nuclei, we estimate that the contribution of the spin density is small. We can make a straightforward calculation of the relative contribution of these terms by first noting that in the static approximation the terms linear in the spin density average to zero, and that the leading nonvanishing contributions come in quadratically, from exchange diagrams. If we write the density matrix as

$$\rho(\sigma_1\vec{r}_1;\sigma_2\vec{r}_2)=\rho(\vec{r}_1,\vec{r}_2)\delta_{\sigma_1,\sigma_2} +(\sigma_1\,|\,\vec{\sigma}\,|\,\sigma_2)\cdot\vec{\rho}(\vec{r}_1,\vec{r}_2) , \quad (7.4)$$

where the spin density matrix is[26]

$$\vec{\rho}(\vec{r}_1,\vec{r}_2)=\frac{\pm i\vec{r}_1\times\vec{r}_2}{4\pi r_1{}^2 r_2{}^2}R_{nlj}(r_1)R_{nlj}(r_2) \times P_l'(\vec{e}_{r_1}\cdot\vec{e}_{r_2}) \quad (7.5)$$

for an orbit ϕ_{nlj} $(j=l\pm\frac{1}{2})$,

$$\phi_{nlj}=\frac{R_{nlj}}{r}(r)\{Y_{lr}\chi_{1/2}\}_{jm} , \quad (7.6)$$

we find that the average of two powers of $\vec{\rho}$, the spin-dependent density matrix, is proportional to (see Table II)

$$\frac{\langle r^2\rangle}{\bar{R}^2}\simeq\frac{1}{\bar{R}}(R \text{ in fm}) , \quad (7.7)$$

where r and R are the quantities defined in Eqs. (4.4) and (4.5). The proportionality constants are approximately the same as those in the expression for the spin-independent piece of the density matrix, so Eq. (7.7) is also a measure of the relative importance of the spin-independent to spin-dependent density matrix. For a light nucleus, this term may be of order unity, but for a medium or heavy nucleus it is relatively small. If we include nucleon motion, then there may be induced current effects which occur linearly in the spin density. These are suppressed by factors of $\omega_\pi/(\omega_\pi+m_n)$, but have been conjectured to contribute appreciably to charge exchange.[31] For nonspherical nuclei the situation is not clear, and an explicit calculation of the spin density terms would be a useful exercise in all cases.

We believe it to be very interesting to determine the coefficients in Eq. (6.4) phenomenologically in order to compare them to theoretical models of pion reaction dynamics. The potential is simple enough in form to be incorporated into the currently available coordinate and momentum space programs and it is sufficiently motivated by theory that calculations of the characteristic parameters are straightforward. The parameters to be adjusted are $\lambda_0^{(2)}$–$\lambda_3^{(2)}$; these can be determined separately for each nucleus and each energy. The quantity $\lambda_4^{(2)}$ is a correction term which should be calculated from theory even for data analysis, since it arises from the iteration of the lowest-order optical potential.

Based on our theoretical analysis, we expect to find that dependence of $\lambda_0^{(2)}$–$\lambda_3^{(2)}$ on A will be quite weak, and that there will be no dependence on the isospin of the target. However, there may be some scatter of the parameters due to shell effects not accounted for in our nuclear matter averages. Our approximations are most valid in the vicinity of the (3,3) resonance, so that these expectations may prove to be borne out to a lesser extent as energy is varied away from the resonance region. Because the (3,3) resonance is the basic building block of the $\lambda_i^{(2)}$, we expect a strong energy dependence to be found for these parameters.

The main assumption used to derive $U^{(2)}$ in the form given in Eqs. (7.1)–(7.3) is isospin invariance. We have also made the fixed scatterer approximation; corrections to Eqs. (7.1)–(7.3) arising from Fermi motion and relativistic kinematics for nucleons may be made in a straightforward fashion and will be considered in a subsequent paper[32] for the case of zero-ranged pion-nucleon form factors.

An important question is how to account for isospin breaking in our framework. Miller and Spencer[10] have shown that the isospin breaking effects in the reaction theory are small, but Auerbach[33] has emphasized that isospin breaking in the nuclear wave functions requires $\Delta\rho$ to be the valence neutron density ρ_{vn}, rather than $(\rho_n-\rho_p)$. Of course, if $\Delta\rho$ is identified with ρ_{vn}, then there is some mistake made in elastic scattering (and hence the distortions included implicitly in the coupled channels) since the pion elastically scatters with $\rho_n-\rho_p$ in the isovector potential. Thus, whether we identify $\Delta\rho$ with $\rho_n-\rho_p$ or ρ_{vn} in Eq. (3.7), an isospin breaking correction ΔU must be added to the theory to correct the mistake, ΔU being of course different depending upon the identification. The preferred prescription for $\Delta\rho$ is the one which leads to the smallest ΔU, since one would like to be able to handle ΔU perturbatively. We believe that the best prescription is to take $\Delta\rho=\rho_{vn}$, because ΔU would then correct for a small percentage error in the distorting potential as opposed to a relatively large error in the transition densities for charge exchange.

Various minor approximations were needed to achieve the fairly simple form of Eqs. (7.1)–(7.3). These incorporate a truncated density matrix expansion and the assumption of forward scattering of the pion from the two-nucleon clusters. The version of the local density approximation that we have used gives special consideration to the nuclear surface, which dominates pion scattering close to the (3,3) resonance. As the pion energy is varied away from resonance, especially to lower energies, these approximations must be reexamined. The sensitivity of the cross sections to the parameters $\lambda_0^{(2)}$–$\lambda_4^{(2)}$ which we find in our subsequent paper[32] lends support to the long expressed hope that charge exchange reactions will lead to new insights into nuclear structure and reaction dynamics. On the basis of our analysis, we propose that the parameters $\lambda_0^{(2)}$–$\lambda_3^{(2)}$ should be the object of both phenomenological parametrizations of the elastic and single- and double-charge-exchange data and evaluations of theoretical models of nuclear structure and reaction dynamics.

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APPENDIX

Here we list some tensor product identities that we have used in obtaining the operators $\hat{\Lambda}_0^{(1)}$ and $\hat{\Lambda}_2^{(1)}$. We also display these operators, however, we keep only their spin dependence that is *even* in the number of Pauli spinors, because odd spin dependences vanish upon spin averaging. The notation we adopt is that of Ref. 25, where $T_\mu^{(m)}$ denotes an irreducible spherical tensor of rank m and projection μ. The tensor product of $T_{\mu_1}^{(m_1)}$ and $T_{\mu_2}^{(m_2)}$ is denoted as

$$T_\mu^{(m)}=\sum_{\mu_1\mu_2}(m_1\mu_1,m_2\mu_2\,|\,m\mu)T_{\mu_1}^{(m_1)}T_{\mu_2}^{(m_2)}\equiv[T^{(m_1)}\times T^{(m_2)}]_\mu^{(m)}\ . \tag{A1}$$

Denoting unit vectors by $\vec{e}$ and arbitrary vectors by $\vec{A}$ and $\vec{B}$, we list some familiar tensor products

$$[\vec{A}\times\vec{B}]^{(0)}=\frac{-1}{\sqrt{3}}\vec{A}\cdot\vec{B}\ , \tag{A2}$$

$$[\vec{A}\times\vec{B}]^{(1)}=\frac{i}{\sqrt{2}}\vec{A}\times\vec{B}\ , \tag{A3}$$

$$[\vec{A}\times\vec{B}]^{(2)}\cdot[\vec{e}_r\times\vec{e}_r]^{(2)}=\vec{A}\cdot\vec{e}_r\vec{B}\cdot\vec{e}_r-\tfrac{1}{3}\vec{A}\cdot\vec{B}\ , \tag{A4}$$

$$[[\vec{e}_r\times\vec{e}_r]^{(2)}\times\vec{A}]^{(1)}=\left[\frac{3}{5}\right]^{1/2}\left[\frac{\vec{A}}{3}-\vec{e}_r(\vec{e}_r\cdot\vec{A})\right]\ , \tag{A5}$$

$$[[\vec{e}_r\times\vec{e}_r]^{(2)}\times\vec{A}]^{(1)}\cdot[[\vec{e}_r\times\vec{e}_r]^{(2)}\times\vec{B}]^{(1)}=\tfrac{1}{5}\left[\vec{e}_r\cdot\vec{A}\vec{e}_r\cdot\vec{B}+\frac{\vec{A}\cdot\vec{B}}{3}\right]\ , \tag{A6}$$

where Eqs. (A4) and (A6) are valid provided $[\vec{B},\vec{e}_r]=0$. We also have basic relations involving spherical harmonics and Pauli spinors,

$$Y_{1\mu}(\vec{e})=\left[\frac{3}{4\pi}\right]^{1/2}(\vec{e})^{(1)}_{\mu}\ , \tag{A7}$$

$$Y_{2\mu}(\vec{e})=\left[\frac{5}{2}\right]^{1/2}\left[\frac{3}{4\pi}\right]^{1/2}[\vec{e}\times\vec{e}]^{(2)}_{\mu}\ , \tag{A8}$$

$$\sum_{\mu\nu}Y_{1\mu}(\vec{e}_{k'})(2m,1\mu\mid 1\nu)Y^{*}_{1\nu}(\vec{e}_k)=-\left[\frac{3}{5}\right]^{1/2}\left[\frac{3}{4\pi}\right][\vec{e}_{k'}\times\vec{e}_k]^{(2)*}_{m}\ , \tag{A9}$$

$$\sum_{\mu\nu}Y_{1\mu}(\vec{e}_k)\sigma_\nu(1\mu,1\nu\mid 1m)\left[\frac{3}{4\pi}\right]^{1/2}[\vec{e}_k\times\vec{\sigma}]^{(1)}_{m}\ . \tag{A10}$$

From the above equations, we obtain

$$\sum Y_{1\mu}(\vec{e}_{k'})(2M,1\mu\mid 1\nu)Y_{2M}(\vec{e}_r)Y^{*}_{1\nu}(\vec{e}_k)=\frac{3}{4\pi}\frac{1}{\sqrt{8\pi}}(\vec{e}_{k'}\cdot\vec{e}_k-3\vec{e}_{k'}\cdot\vec{e}_r\vec{e}_k\cdot\vec{e}_r)\ , \tag{A11}$$

$$\sum Y_{1\mu'}(\vec{e}_{k'})\sigma_{\nu'}(1)(1\mu',1\nu'\mid 1M)(1M,1\nu\mid 1\mu)\sigma_\nu(2)Y^{*}_{1\mu}(\vec{e}_k)=\frac{3}{8\pi}(\vec{e}_{k'}\times\vec{\sigma}_1)\cdot(\vec{e}_k\times\vec{\sigma}_2)\ , \tag{A12}$$

$$\begin{aligned}\sum Y_{1\mu'}(\vec{e}_{k'})\sigma_{\nu'}(1)(1\mu',1\nu'\mid 1m')(1m',1\nu\mid 1m)\sigma_\nu(2)Y_{2M}(\vec{e}_r)(2M,1m\mid 1\mu)Y^{*}_{1\mu}(\vec{e}_k)\\ =\frac{3}{(8\pi)^{3/2}}\{\vec{\sigma}_1\cdot\vec{\sigma}_2[\vec{e}_{k'}\cdot\vec{e}_k-3\vec{e}_{k'}\cdot\vec{e}_r\vec{e}_k\cdot\vec{e}_r]-\vec{e}_{k'}\cdot\vec{\sigma}_2[\vec{e}_k\cdot\vec{\sigma}_1-3\vec{e}_k\cdot\vec{e}_r\vec{e}_r\cdot\vec{\sigma}_1]\}\ ,\end{aligned} \tag{A13}$$

$$\begin{aligned}\sum Y_{1\mu'}(\vec{e}_{k'})\sigma_{\nu'}(1)(1\mu',1\nu'\mid 1m')Y_{2M}(\vec{e}_r)(2M,1m'\mid 1m)\sigma_\nu(2)(1m,1\nu\mid 1\mu)Y^{*}_{1\mu}(\vec{e}_k)\\ =\frac{3}{(8\pi)^{3/2}}\{\vec{\sigma}_1\cdot\vec{\sigma}_2\vec{e}_{k'}\cdot\vec{e}_k-\vec{e}_{k'}\cdot\vec{\sigma}_2\vec{e}_k\cdot\vec{\sigma}_1-(\vec{e}_r\times\vec{e}_{k'})\cdot\vec{\sigma}_1(\vec{e}_r\times\vec{e}_k)\cdot\vec{\sigma}_2\}\ ,\end{aligned} \tag{A14}$$

where the sums in these expressions run over all the projections. Now following the discussion of Sec. V, we use Eqs. (5.4b)–(5.9) in Eq. (5.1) and (5.10) to obtain the operators in Fig. 1 *even* in numbers of spinors,

$$\begin{aligned}\hat{\Lambda}_0=&\left[\lambda_{00}{}^2+\frac{\lambda_{00}\lambda_{01}}{2}\vec{\phi}_1(\vec{\tau}_1+\vec{\tau}_2)+\frac{\lambda_{01}{}^2}{4}\hat{\tau}_{12}\right]\vec{e}_{k'}\cdot\vec{e}_k\\ &+\left[\lambda_{10}{}^2+\frac{\lambda_{10}\lambda_{11}}{2}\vec{\phi}_1(\vec{\tau}_1+\vec{\tau}_2)+\frac{\lambda_{11}{}^2}{4}\hat{\tau}_{12}\right]\vec{e}_{k'}\cdot\vec{e}_k\vec{\sigma}_1\cdot\vec{\sigma}_2\\ &-\left[\lambda_{10}{}^2+\frac{\lambda_{10}\lambda_{11}}{2}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2)\right]\sigma_{12}(\vec{k}',\vec{k})-\frac{\lambda_{11}{}^2}{4}\hat{\Sigma}_{12}(\vec{k},\vec{k}')\ ,\end{aligned} \tag{A15}$$

and

$$\hat{\Lambda}_2=\hat{\Lambda}_0-3\left[\lambda_{00}{}^2+\frac{\lambda_{00}\lambda_{01}}{2}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2)+\frac{\lambda_{01}{}^2}{4}\tau_{12}\right]\vec{e}_{k'}\cdot\vec{e}_r\vec{e}_k\cdot\vec{e}_r$$

$$-3\left[\lambda_{10}{}^2+\frac{\lambda_{10}\lambda_{11}}{2}\vec{\phi}\cdot(\vec{\tau}_1+\vec{\tau}_2)\right]\sigma_{12}(\vec{k}'\times\vec{r},\vec{k}\times\vec{r})-\frac{3\lambda_{11}{}^2}{4}\hat{\Sigma}_{12}(\vec{k}'\times\vec{r},\vec{k}\times\vec{r}) , \quad (A16)$$

where

$$\sigma_{12}(\vec{p},\vec{q})\equiv\tfrac{1}{2}(\vec{\sigma}_1\cdot\vec{e}_p\vec{\sigma}_2\cdot\vec{e}_q+\vec{\sigma}_2\cdot\vec{e}_p\vec{\sigma}_1\cdot\vec{e}_q) , \quad (A17)$$

$$\hat{\Sigma}_{12}(\vec{p},\vec{q})\equiv\tfrac{1}{2}(\vec{\phi}\cdot\vec{\tau}_1\vec{\phi}\cdot\vec{\tau}_2\vec{\sigma}_1\cdot\vec{e}_p\vec{\sigma}_2\cdot\vec{e}_q+\vec{\phi}\cdot\vec{\tau}_2\vec{\phi}\cdot\vec{\tau}_1\vec{\sigma}_2\cdot\vec{e}_p\vec{\sigma}_1\cdot\vec{e}_q) . \quad (A18)$$

The definition of τ_{12} is given by Eq. (3.9).

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