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Field theoretic aspects of meson-nucleus scattering

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Field theoretic aspects of the pion field interacting with a nucleus are investigated. The quantum field nature of the pion is shown to be most simply and fundamentally incorporated into multiple scattering theory through the Klein-Gordon equation. This is accomplished so that crossing symmetry of the scattering amplitude is manifest and can be maintained without recourse to nonlinear equations. We give specific examples of calculating the first and second order optical potential in the static approximation. Our approach provides a clean separation between the energy and momentum dependence of the pion-nucleon amplitude and does not have the strong momentum cutoffs or the spurious reactive content of other approaches. An off-shell extrapolation of the pion-nucleon amplitude in terms of the four-momenta is required for the evaluation of second and higher order contributions to the optical potential. We discuss the conditions under which our static field theory can be regarded as a limit of a theory which allows for nucleon recoil.

[NUCLEAR REACTIONS Pion-nucleus elastic scattering, optical potential, field theoretic effects, crossing symmetry, off-shell behavior of two-body amplitudes.]

I. INTRODUCTION

The advent of pion factories has led to a renewed interest in models of low energy pion-nucleon scattering and in the problems of embedding this interaction into a multiple scattering theory. Most work in this direction has relied on traditional approaches[1] which postulate that the underlying dynamics can be described in terms of two-body potentials[2] or simple isobar models.[3]

However, when the pion is properly treated as a quantum field new issues arise which have no counterpart in potential model descriptions. One source of these differences is that the number of pions is not conserved: Pions are created and destroyed, both physically and virtually, with the pion antiparticles being described as pion particles propagating backward in time.[4] Furthermore, an additional symmetry arises which is not present in potential theory: crossing symmetry. Thus, one of the major challenges of medium energy physics remains largely unsettled, namely to find an alternative formulation of multiple scattering theory which is based on field theoretical ideas and which can be applied to microscopic calculations of pion-nucleus scattering.

There have been several attempts to formulate a field theoretical approach to pion-nucleus scattering, but none has achieved general acceptance. Most of these utilize methods and concepts which have proved useful in nonrelativistic studies, especially the fixed pion number expansion (FPNE) in which diagrams are first time ordered and then selectively summed, with multipion intermediate states being eliminated as a first approximation. Especially for low mass particles such as the pion, this approximation scheme is not well justified because it can affect the analytical structure of the amplitude, altering the geometry[5] of the optical potential U and leading to spurious reactive content[6] in higher levels of approximation. The introduction of time-ordered diagrams has the further drawback that involved graph summation arguments would be needed in order to recover the simple underlying structure which does not have any of the difficulties of the FPNE. The complications of the FPNE also obscure the problem of matching the energy and momentum variables in time-ordered diagrams with those of the off-shell field theoretical amplitude. Speculation on how to disentangle these variables has added a new dimension to the long standing controversy on the range of the pion-nucleon vertex and has led to an unphysical prescription for embedding the pion-nucleon amplitude in the pion-deuteron scattering problem.

In this paper we investigate a prototype field theory based on the fixed scatterer approximation. We work with time-dependent methods, which simplify the analysis and enable us to avoid the FPNE and its objectional consequences. We explicate the interplay between crossing symmetry, the off-shell variables which characterize the pion-nucleon amplitude, and the structure of higher order multiple scattering terms. We expect that the experience gained here will facilitate the adoption of correct approximation schemes within more comprehensive theoretical frameworks.

We begin in Sec. II by defining U as the proper self-energy of the Dyson equation for the Green's function, which leads us to a derivation of U appropriate to the Klein-Gordon equation. Our use of time-dependent methods enables us to sum naturally all relative time orderings of diagrams of a given topology, thus simplifying the problem of maintaining crossing symmetry and imposing proper boundary conditions on the pion field. Next, we show how our definition of U leads to a result which is manifestly crossing symmetric and which produces an explicitly crossing symmetric T matrix. Cammarata and Banerjee[8] have also shown that the optical potential defined as the self-energy of the Klein-Gordon propagator leads to a crossing symmetric T matrix, but their approach is different from ours in practice, because they follow the convention of low energy nuclear physics in which diagrams are time ordered. Crossing symmetry is also incorporated in the work of Celenza, Liu, and Shakin,[9] and Siciliano and Thaler,[10] where it is accomplished by solving a nonlinear integral equation. In Sec. III we define the field theoretical model with which we illustrate these ideas. For definiteness we expand U following the precepts of the spectator expansion of Refs 11 and 12. According to this, the lowest order U bears a definite relationship to free pion-nucleon scattering, discussed in Sec. IV, and the second order potential to pion-deuteron scattering and scattering of a pion from two free nucleons, discussed in Sec. V. We examine, in Sec. V, coupled integral equations which express a relationship between the pion-nucleon and pion two-nucleon scattering. The driving term requires an off shell extrapolation of the free pion nucleon amplitude which depends on all four components of the momentum variables. The equations have the virtue that they include the crossed pion-nucleon amplitude and the proper boundary conditions for the pion field, i.e., the possibility of the pion propagating backward in time as an antiparticle is allowed everywhere, in addition to the possibility of propagating forward in time as a particle. The difficulties found by Myhrer and Thomas[13] do not occur when the off-shell amplitude is embedded in the pion-deuteron problem according to our theory. The resulting U is of substantially shorter range than the corresponding results of Miller[14] and Miller and Henley.[15]

Finally, in Sec. VI we briefly discuss corrections to the theory which arise from field theoretical effects such as renormalization and true absorption. We observe that in more comprehensive theories the ideas we have developed in this paper should be extended in the direction advocated in Refs. 16–18, but that to do this will probably require relaxing the requirement of strict n-body unitarity for $n \geq 3$.

II. PION-NUCLEUS OPTICAL POTENTIAL

We formulate the theory of pion-nucleus scattering in terms of the pion-nucleus Green's function, defined by

$$G_{\vec{k}',\vec{k}}(t'-t)=i^{-1}\langle \psi_0 \,|\, T(a_{\vec{k}'}(t')a^{\dagger}_{\vec{k}}(t))\,|\,\psi_0\rangle \ , \tag{2.1}$$

where $|\psi_0\rangle$ is the (interacting) ground state wave vector of the target nucleus and $a^{\dagger}_{\vec{k}}(t)$ is the pion creation operator in the Heisenberg representation. We use the conventional shorthand notation in which $\vec{k}$ represents the isospin and three-momentum of the pion. The optical potential can be defined in terms of the diagrammatic expansion[7] of G. We organize diagrams into proper self-energy insertions Σ and meson propagators so that the terms have the appearance of Fig. 1. The proper self-energy Σ is defined to contain all diagrams which cannot be divided into two parts by cutting a single pion propagator, either propagating forward or backward in time. In terms of Σ and the pion propagator given by

$$P_k(t'-t)=\frac{i}{2\pi}\int_{-\infty}^{+\infty} d\omega \frac{e^{-i\omega(t'-t)}}{\omega^2-\omega_k{}^2+i\eta} \ , \tag{2.2}$$

with $\omega_k=(k^2+m_\pi{}^2)^{1/2}$, G satisfies the integral equation

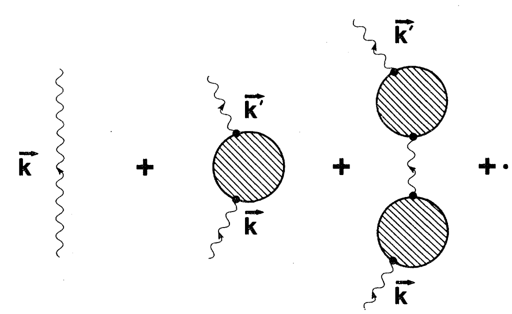

FIG. 1. The Green's function written in terms of proper self-energy insertions (shaded circles) and meson propagators (wiggly lines).

$$G_{\vec{k}'\vec{k}}(t'-t)=P_k(t'-t)(2\pi)^3\delta(\vec{k}'-k)$$
$$+\int_{-\infty}^{+\infty}dt''\int\frac{d\vec{k}}{(2\pi)^3}\int_{-\infty}^{+\infty}dt'''P_{k'}(t'-t'')\langle\vec{k}'\,|\,\Sigma(t''-t''')\,|\,\vec{k}'''\rangle G_{\vec{k}'''\vec{k}}(t'''-t)\;. \tag{2.3}$$

The scattering T matrix and pion wave function $\psi(\vec{r})$ may be calculated from Eq. (2.3). The T matrix is given by the reduction formula[19]

$$\frac{-2\pi i}{(2\omega_k)(2\omega_{k'})}\delta(\omega_{k'}-\omega_k)\langle\vec{k}'\,|\,T\,|\,\vec{k}\rangle=i\int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dt'e^{i(\omega_{k'}t'-\omega_k t)}\left[i\frac{\partial}{\partial t}+\omega_k\right]\left[i\frac{\partial}{\partial t'}-\omega_{k'}\right]G_{\vec{k}'\vec{k}}(t'-t)\;. \tag{2.4}$$

Inserting Eq. (2.3) into (2.4) we find that T satisfies the equation

$$\langle\vec{k}'\,|\,T(\omega)\,|\,\vec{k}\rangle=\langle\vec{K}'\,|\,\Sigma(\omega)\,|\,\vec{k}\rangle+\int\frac{d\vec{k}''}{(2\pi)^3}\frac{\langle\vec{k}\,|\,\Sigma(\omega)\,|\,\vec{k}''\rangle\langle\vec{k}''\,|\,T(\omega)\,|\,\vec{k}\rangle}{\omega^2-\omega_{k''}{}^2+i\eta}\;. \tag{2.5}$$

This is just the Klein-Gordon equation for the scattering from a potential Σ and thus enables us to identify the optical potential U as

$$\langle\vec{k}'\,|\,U(\omega)\,|\,\vec{k}\rangle\equiv\langle\vec{k}'\,|\,\Sigma(\omega)\,|\,\vec{k}\rangle\;. \tag{2.6}$$

The wave function $\psi(\vec{r})$ may therefore be found by solving the nonlocal Klein-Gordon equation

$$(-\nabla^2+m_\pi{}^2)\psi_{\vec{k}}(\vec{r})+\int d\vec{r}'\langle\vec{r}\,|\,U(\omega)\,|\,\vec{r}'\rangle\psi_{\vec{k}}(\vec{r}')=\omega_k{}^2\psi_{\vec{k}}(\vec{r})\;. \tag{2.7}$$

The wave function ψ has the same phase shifts as T, and the connection between ψ and the Green's function G expressed in Eqs. (2.1)–(2.7) is important for deciding how to use it in a calculation of observables other than elastic scattering, e.g., inelastic scattering.

We have just presented a formal derivation of the optical potential for the Klein-Gordon equation. The virtues of this equation stem from the fact that the boundary conditions for the pion field are automatically included in the Klein-Gordon propagator: Pions are propagated forward in time as particles and backward in time as antiparticles.[4] The standard difficulty of deciding whether the optical potential enters as a scalar, or as the fourth component if a four vector, has not arisen in our present derivation.

Assume now that we have some systematic method for evaluating Σ, so that Σ has contributions of order $1,2,\ldots,n,\ldots$. A particular choice for the meaning of "order n" will be given in the following sections. Denote by a "box"

$$\langle\vec{k}'\,|\,B_n(t'-t)\,|\,\vec{k}\rangle$$

the sum of all diagrams which contribute to Σ_n. There are then generally two distinct ways to attach the external pions to the box, shown in Fig. 2. The relationship between

$$\langle\vec{k}'\,|\,\Sigma_n(t'-t)\,|\,\vec{k}\rangle$$

and B_n is then

$$\langle\vec{k}'\,|\,\Sigma_n(t'-t)\,|\,\vec{k}\rangle=\langle\vec{k}'\,|\,B_n(t'-t)\,|\,\vec{k}\rangle+\langle-\vec{k}\,|\,B_n(t-t')\,|\,-\vec{k}'\rangle\;, \tag{2.8a}$$

where the exchange of $\vec{k}$ and $\vec{k}'$ involves an exchange of the pion isotopic spin labels. In the cases for which attaching the pion legs shown in Fig. 2 does not lead to distinct processes, an explicit factor of $\frac{1}{2}$ is required. Taking the Fourier transform of Eq. (2.8a) we have

$$\langle\vec{k}'\,|\,U(\omega)\,|\,\vec{k}\rangle=\sum_n[\langle\vec{k}'\,|\,B_n(\omega)\,|\,\vec{k}\rangle+\langle-\vec{k}\,|\,B_n(-\omega)\,|\,-\vec{k}'\rangle]\;. \tag{2.8b}$$

Note that U is unchanged by the replacement

$$\vec{k}'\to-\vec{k}\;,\quad\vec{k}\to-\vec{k}'\;,\text{ and }\omega\to-\omega\;. \tag{2.9}$$

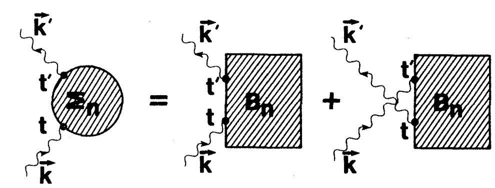

FIG. 2. Illustrating the two ways in which a given box contributes to the pion self-energy.

This is crossing symmetry. It is an immediate consequence of Eq. (2.5) that, because the Klein-Gordon propagator depends quadratically on ω, a crossing symmetric U will automatically yield a T matrix that is also crossing symmetric. This point may be found in Appendix B of Ref. 8. We shall find that the impulse approximation to the optical potential requires the folding of a crossing symmetric two-body amplitude with the target density. One sees that we are able to maintain crossing symmetry in the pion nucleus problem without recourse to nonlinear equations because we are working with crossing symmetric two-body amplitudes and because we truncate the perturbation theory in such a way as to maintain this symmetry order by order.

There have been numerous attempts to introduce crossing symmetry into the pion-nucleus optical potential. In Refs. 9 and 10 crossing symmetry is introduced as the solution of a nonlinear Low equation. By contrast, in our approach and in that of Ref. 8, crossing symmetry is guaranteed by the way we imbed the boxes into a linear scattering equation. We feel that this separation of the symmetry and dynamics is a desirable attribute of the present formulation.

III. STATIC MODEL AND CLUSTER EXPANSION

The above considerations are completely general and independent of the specific field theoretic dynamical model. Now we want to introduce the static model in order to study the structure of the boxes B_n introduced above, without becoming encumbered by technical details of nuclear structure. We are specifically interested in establishing a structural connection between the free pion-nucleon scattering amplitude and the complete multiple scattering series in a field theoretical approach.

A. The static model

The static model for a pion interacting with a single source is carefully addressed in Ref. 20, to which we refer the reader for details. Although the assumption of fixed nucleons is too restrictive for many modern applications, the theory gives a good account of pion-nucleon scattering in the (3,3) channel for pion energies less than 1.2 GeV,[21] when inelastic pion-production channels are considered, for pion-nucleon form factors

$$v(k)=\exp(-k^2/\beta^2)\ ,\quad \beta\simeq 750\ \text{MeV}\ . \tag{3.1}$$

The static model is not incompatible with the quark-bag concept. Indeed, we may add into the unperturbed Hamiltonian an excited state Δ of the nucleon and to the interaction an "effective interaction" term which describes the excitation of the elementary Δ_{33} resonance,

$$\Delta H_{\pi N\Delta}=\int d\vec{r}_\pi \int d\vec{r}\,[\phi^\dagger_{\vec{k}\alpha}(r_\pi)V_{\pi N\Delta}(\vec{r}_\pi-\vec{r})\psi^\dagger_\Delta(\vec{r})\psi_N(\vec{r})+\text{h.c.}]\ , \tag{3.2}$$

where $\phi^\dagger_{\vec{k}\alpha}$ is the pion creation operator, $\psi_i(\vec{r})$ is the creation operator for an N or Δ at point $\vec{r}$, and $V_{\pi N\Delta}$ contains the coupling constant, form factor, and transition spin and isospin operators.[22] Recent works[23] on chiral extensions of the bag model have discussed meson-nucleon effective Hamiltonians of this form. By adding more terms in this manner the theory is capable of being generalized in a straightforward fashion to include all excited nucleon and meson states and interactions among them. Strange particles may also be incorporated if the interest should arise. Our theory is therefore not meant to be a fundamental theory which predicts the effective interactions, but rather it must accept them from other more comprehensive theories. The theory may then be solved to predict scattering of mesons from a collection of scattering centers.

Application of the static model to the case of a meson field interacting with a collection of A fixed nucleons resembles very closely the model of Ref. 20. The main difference is that the form factors are replaced by

$$v(\vec{r})\rightarrow \sum_{i=1}^{A} v(\vec{r}-\vec{r}_i)\ , \tag{3.3}$$

where $\vec{r}_i$ is the position of the ith source. The scattering amplitude then becomes a function of the positions of the A nucleons. These positions are averaged over the nuclear wave function to get the physical amplitude.[24]

The main drawback of the model as a framework for calculations of pion-nucleus scattering is the lack of nucleon recoil and the resulting inability to describe true absorption of the pion. On the other hand, our static theory is an approximation to a more complicated theory which includes nucleon recoil and is therefore capable of describing nuclear dynamics as well as multiple scattering and absorption of meson projectiles. We show in some detail in Sec. VI the corresponding "model exact" theory and specify the conditions under which the correspondence becomes quantitative.

Now that we have specified the Hamiltonian, $G_{\vec{k}'\vec{k}}(t'-t)$ and U may be calculated by standard rules of time-dependent perturbation theory. Some of the details are given in Appendices A and B.

B. Cluster expansion

In this subsection we will make a definite choice of the boxes B_n. In the spirit of low-density expansions, we order the contributions to U according to the number of different nucleons involved in the interaction. Denote by $B_1, B_2, \ldots, B_n$ the sum of all valid contribution to U of $1, 2, \ldots, n, \ldots,$ nucleons, respectively. These are to be identified with the boxes of the previous section. This expansion is also known as the spectator expansion.[11,12]

In what follows it is useful to express B_n in terms of a more elementary quantity which is related to the scattering of a pion from n fixed sources at points $\vec{r}_1, \vec{r}_2, \ldots, \vec{r}_n$. It is the sum of all connected diagrams for which the last interaction of the outgoing pion and the first of the incoming pion occur at times t' and t on nucleons j and i, respectively. The first and last nucleon interactions occur, respectively, at times $(\bar{t}_1, \bar{t}'_1), (\bar{t}_2, \bar{t}'_2), \ldots, (\bar{t}_n, \bar{t}'_n)$. This quantity is denoted by

$$\langle \vec{k}' \mid B_n'^{(ji)}(t't{:}x'_1, x'_2, \ldots, x'_n; x_1, x_2 \cdots x_n) \mid \vec{k} \rangle \equiv \langle \vec{k}' \mid B_n'^{(ji)}(t't{:}\{\alpha'\alpha\bar{t}'\bar{t}\vec{r}\}) \mid \vec{k} \rangle \tag{3.4}$$

where $x_m \equiv (\alpha_m, \vec{r}_m, \bar{t}_m)$ and $x'_m \equiv (\alpha'_m, \vec{r}_m, \bar{t}'_m)$, with α_m (α'_m) the initial (final) spin and isospin of the nucleon. By definition, the nucleon times satisfy the inequality $\bar{t}'_i \geq \bar{t}_i$. In order to obtain $\langle \vec{k}' \mid B_n(t'-t) \mid \vec{k} \rangle$ from B'_n one must attach hole lines [rule (7) of Appendix B], sum over hole line labels [rule (11)], and integrate over the nucleon positions and times [rule (12)]. However, this still does not yield B_n because some of the terms contained in B_n are generated when Eq. (2.5) is solved for T. The proper way to disentangle the double counting problem is carefully discussed by Siliciano and Thaler.[12] We discuss each of these points below.

Consider first the integration over the time variables of the nucleons. Because the nucleon hole lines do not contribute time-dependent factors in the static theory, the result is very simple and expressed in terms of the Fourier transform of B'_n, which we shall call $\tilde{B}'_n$,

$$-2\pi i \delta(\omega' + \bar{\omega}'_1 + \cdots - \omega - \bar{\omega}_1 - \bar{\omega}_2 - \cdots) \tilde{B}_n'^{(ji)}(\omega', \omega{:}\{\alpha'\alpha\bar{\omega}'\bar{\omega}\,\vec{r}\}) \equiv (-i)^{2(n+1)} \int dt' \int dt \cdots e^{i(\omega' t' + \bar{\omega}_1 \bar{t}'_1 + \cdots)} e^{-i(\omega t + \bar{\omega}_1 \bar{t}_1 + \cdots)} B_n'^{(ji)}(t't{:}\{\alpha'\alpha\bar{t}'\bar{t}\vec{r}\}) \,. \tag{3.5}$$

The delta function in Eq. (3.5) arises from the fact that B_n is unchanged by a uniform translation of each time variable by an identical amount. The integration over the time variables thus gives

$$(-i)^{2n} \int d\bar{t}'_1 \cdots \int d\bar{t}_1 \cdots B_n'^{(ji)}(t't'{:}\{\alpha'\alpha\bar{t}'\bar{t}\vec{r}\}) = i \int d\omega\, e^{-i\omega(t'-t)} T_n'^{(ji)}(\omega{:}\{\alpha\alpha'\vec{r}\}) \,, \tag{3.6a}$$

where we have defined the shorthand notation

$$T_n'^{(ji)}(\omega{:}\{\alpha\alpha'\vec{r}\}) \equiv \tilde{B}_n'^{(ji)}(\omega\omega{:}\{\alpha'\alpha 0 0 \vec{r}\}) \,, \tag{3.6b}$$

and where we have written $\{\alpha\alpha'\vec{r}\}$ to denote that the nucleon positions and initial and final spins and isospins enter explicitly into the terms in Eqs. (3.5) and (3.6).

Next consider the attachment of the hole lines to B_n. In Appendix A the perturbation series for G is studied and there it is shown that the nucleon hole lines connect to the nucleon particles to form loops of a well-defined sense. The n hole lines may also attach to B_n in all ways consistent with this principle. (Note that hole lines point both forward and backward in time in the present theory, in contrast to familiar Feynman-Goldstone diagrams where hole lines propagate only backwards.) The possibilities for $n=1$ and $n=2$ are shown in Fig. 3. Each possibility gives a different order of distributing the nucleon wave functions among the n coordinates $\vec{r}_1, \ldots, \vec{r}_n$.

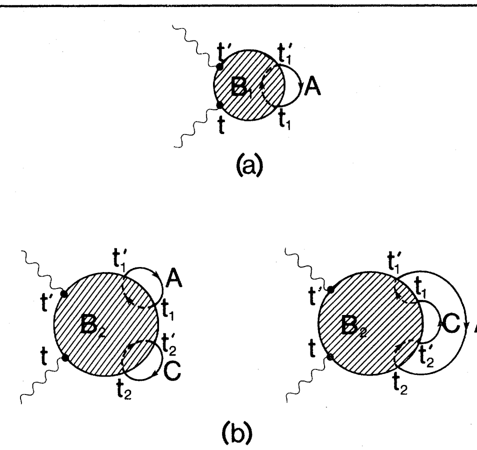

FIG. 3. The ways of attaching external nucleon hole lines to clusters of nucleons. The dotted lines indicate the distinct nucleons contributing to the cluster. The extension of the lines indicates the time evolution of the system. The sign of the cluster is $(-)^{l+n}$, where l is the number of closed loops and n the number of hole lines.

Finally, to avoid double counting, one applies the combinatorics[12] of the spectator expansion. In the case of the quantity B_2 the complete expression for B_2 in terms of B'_2 and B_1 is written out diagrammatically in Fig. 4. Note that the pion propagator in the subtracted term is a Klein-Gordon propagator.

Finally, we want to consider the momentum dependence of a contribution to the boxes B'_n. In the static theory *the momentum dependence is determined only by the momentum dependence of the vertices at which the external pion attaches to the box.* This is a consequence of the fact that there are no momentum conserving delta functions to transfer knowledge of the external momenta into the box. Thus, we write for $B_n'^{(ji)}$

$$\langle \vec{k}' \mid \tilde{B}_n'^{(ji)}(\omega',\omega{:}\{\alpha'\alpha\bar{\omega}'\bar{\omega}\,\vec{r}'\} \mid \vec{k}\rangle = v(k')v(k)e^{i\vec{k}\cdot\vec{r}_i}e^{-i\vec{k}'\cdot\vec{r}_j}\vec{k}'\cdot\overleftrightarrow{B}'^{(ji)}(\omega'\omega{:}\{\alpha'\alpha\bar{\omega}'\bar{\omega}\,\vec{r}\})\cdot\vec{k}\;, \tag{3.7}$$

where the tensor $\overleftrightarrow{B}'^{(ji)}$ is independent of $\vec{k}$ and $\vec{k}'$. (Recall that $\vec{k}$ and $\vec{k}'$ represent momentum and isospin variables of the pion. Thus, $\overleftrightarrow{B}$ is a tensor with respect to the contraction on the momentum and isospin of the pion.) Similar separations of momentum and energy hold for $T'^{(ji)}$.

C. Discussion

The simple form of the result in Eq. (3.7) occurs because of the complete separation of the external pion propagators from the boxes that constitute Σ in Eq. (2.3). It is possible to make this separation because we treat the forward and backward propagation of the pion on the same footing. The simplicity is lost in essentially all other theoretical approaches, which have tended to make an ordering in terms of the number of pions present during any given interval of time[25] (FPNE). This point is elucidated in Sec. V and Appendix D.

The details of the momentum dependence of the optical potential has been the object of much controversy. In our static theory this dependence is universal, i.e., the same for all boxes, and is the same as that given in Eq. (3.7) with $v(k)$ given in Eq. (3.1). In contrast, the static theory of Miller[14] and Miller and Henley[15] gives rise to a coupling with our $v(k)$ replaced by

$$v(k) \to \frac{m_\pi}{\omega_k} v(k)\;. \tag{3.8}$$

This form factor corresponds to a very rapid cutoff in momentum which affects the geometry[5] of U and severely damps out many of the higher order contributions in the multiple scattering theory.[14] The origin of the extra factor of m_π/ω_k is partly the well-known relativistic phase space factor which arises when the pions propagating forward in time are projected out of the Klein-Gordon propagator and partly an additional momentum dependence induced by the truncation of perturbation theory in terms of time-ordered diagrams. In Appendix C we review the origin of the cutoffs proposed in Refs. 14 and 15 and demonstrate how they disappear in deriving our results.

Of the many theoretical attempts to establish a framework for systematically evaluating the multibody cluster contributions to the optical potential, we have chosen to illustrate our ideas with the spectator expansion of Refs. 11 and 12. Our work is, however, different from Ref. 10, where the scattering from successively larger clusters is always reevaluated from scratch in terms of the underlying Lagrangian. Here, we proceed as far as possible by building up the higher order clusters in terms of an off shell pion-nucleon amplitude, including the crossed piece. Our use of the spectator expansion is

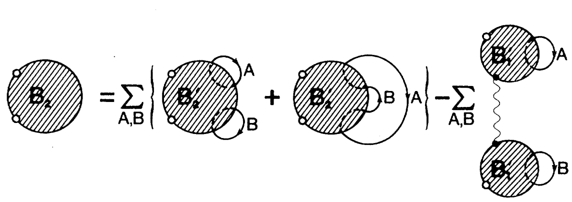

FIG. 4. The second order term in the spectator expansion. Note that the intermediate pion propagator in the subtracted term is the Klein-Gordon propagator.

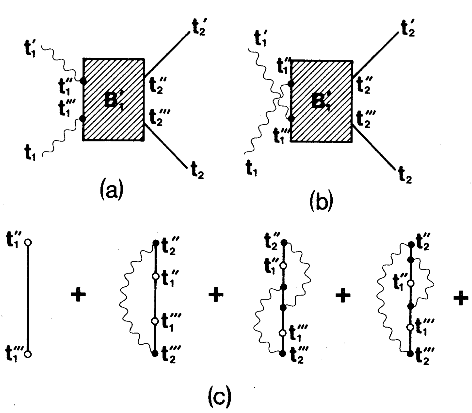

FIG. 5. Illustrating contributions to pion-nucleon scattering. (a) and (b) are the direct and cross pieces of the two-body Green's function given in terms of the box B'_1. (c) illustrates terms which appear in B'_1.

made only for purposes of illustration. Other systematic expansions are possible in the field theoretical context. We mention here that the self-consistent theory proposed in Ref. 26 is a general idea[27] and the rate of convergence of the cluster expansion could be enhanced by treating the spectator expansion self-consistently.

IV. LOWEST ORDER OPTICAL POTENTIAL AND PION-NUCLEON SCATTERING

A. Pion-nucleon scattering

Pion-nucleon scattering may be evaluated in terms of the two-body Green's function

$$G_{i'i}(x_1',x_2';x_1,x_2)\ ,$$

which gives the amplitude to remove a pion at point $x_1'=(\vec{x}_1',t_1')$ and a nucleon at point $x_2'=(\vec{r},t_2')$ if they are inserted in the medium, respectively, at points $x_1=(\vec{x}_1,t_1)$ and $x_2=(\vec{r},t_2)$. The nucleon carries a spin-isospin index i and i'. Because we work in the static theory, there is only one position variable characterizing the nucleon, but there are two time variables. Once G is known, the scattering T matrix may be calculated from it by the reduction formula

$$\begin{aligned}(2\pi)^3\delta(\vec{k}'+\vec{p}\,'-\vec{k}-\vec{p})\delta(\omega_k'-\omega_k)(2\omega_k')^{-1}(2\omega_k)^{-1}\langle \vec{k}'\beta'\,|\,T_{\pi N}(\omega_k)\,|\,\vec{k}\alpha\rangle \\ =\int d^4x_1'\int dt_2'\int d^4x_1\int dt_2 e^{ik'\cdot x_1'}e^{i\vec{p}\cdot x_2'}e^{-ik\cdot x_1}e^{-ip\cdot x_2} \\ \times\left[i\frac{\partial}{\partial t_1'}-\omega_k'\right]\left[i\frac{\partial}{\partial t_2'}\right]\left[i\frac{\partial}{\partial t_2'}+\omega_k\right]\left[i\frac{\partial}{\partial t_2}\right]G_{\beta'\alpha}(x_1',x_2';x_1,x_2)\ .\end{aligned} \tag{4.1}$$

The diagrams which contribute to G have the form shown in Figs. 5(a) and 5(b). The quantity B_1' is the same box as defined in Eq. (3.4). The terms which contribute to B_1' are illustrated in Fig. 5(c): They consist of mesons emitted and absorbed in all possible ways on a nucleon. The rules for evaluating the diagrams are given in Appendix B.

For the purposes of this work, we shall assume that the summation of all these diagrams has been performed (at least under some suitable set of approximations). The reader is referred to Refs. 21 and 28 for detailed discussions of the present status of these pion-nucleon models.

Applying Eq. (4.1) to G given by Figs. 5(a) and (b) we easily find

$$\begin{aligned}(2\pi)^3\delta(\vec{k}'+\vec{p}\,'-\vec{k}-\vec{p})\langle \vec{k}'\beta'\,|\,T_{\pi N}(\omega_k)\,|\,\vec{k}\alpha\rangle \\ =\int d\vec{r}\,e^{i\vec{r}\cdot(\vec{p}-\vec{p}\,')}[\langle \vec{k}'\,|\,T_1'(\omega_k:\beta'\alpha\vec{r})\,|\,\vec{k}\rangle+\langle -\vec{k}\,|\,T_1'(-\omega_k:\beta'\alpha\vec{r})\,|\,-\vec{k}'\rangle]\ ,\end{aligned} \tag{4.2}$$

where T_1' is the Fourier transform function defined in Eq. (3.6). Now utilizing Eq. (3.7) we find

$$\langle \vec{k}'\beta'\,|\,T_{\pi N}(\omega)\,|\,\vec{k}\alpha\rangle=v(k)v(k')[\vec{k}'\cdot\overleftrightarrow{T}_1'(\omega:\beta'\alpha)\cdot\vec{k}+\vec{k}\cdot\overleftrightarrow{T}_1'(-\omega;\beta'\alpha)\cdot\vec{k}']\ , \tag{4.3}$$

where, for the case of one nucleon, the B_n' in Eq. (3.7) has no dependence on $\vec{r}$. Equation (4.3) is the desired relation between $T_{\pi N}$ and B_1'. [See Eq. (3.6) for relation between B_1' and T_1']. Note that momentum conservation arises because the initial and final nucleons are plane waves.

B. Lowest order optical potential

To obtain the lowest order optical potential we must first add the hole-line contribution as discussed in Sec. III and sum over hole states,

$$\begin{aligned}\langle \vec{k}'\,|\,B_1(\omega)\,|\,\vec{k}\rangle &=\sum_{A\alpha\beta}\int d\vec{r}\langle\psi_A\,|\,\vec{r}\beta'\rangle\langle \vec{k}'\beta'\,|\,T_1'(\omega:\vec{r})\,|\,\vec{k}\alpha\rangle\langle\vec{r}\alpha\,|\,\psi_A\rangle \\ &=v(k)v(k')\sum_\alpha \vec{k}'\cdot\overleftrightarrow{T}_1'(\omega:\alpha\alpha)\cdot\vec{k}\int\rho_\alpha(r)e^{-i(\vec{k}'\cdot\vec{k})\cdot\vec{r}}d\vec{r}\ ,\end{aligned} \tag{4.4}$$

where ρ_α is the ground state density of nucleons of type α (e.g., neutrons with spin up). Now, using Eq. (2.8b) we find for the lowest order optical potential $U^{(1)}$

$$\langle \vec{k}'\,|\,U^{(1)}(\omega)\,|\,\vec{k}\rangle=v(k)v(k')\sum_\alpha\rho_\alpha(\vec{k}'-\vec{k})[\vec{k}'\cdot\overleftrightarrow{T}_1'(\omega:\alpha\alpha)\cdot\vec{k}+\vec{k}\cdot\overleftrightarrow{T}_1'(-\omega:\alpha\alpha)\cdot\vec{k}']\ . \tag{4.5}$$

Inserting Eq. (4.3) into Eq. (4.5) we find

$$\langle \vec{k}' \mid U^{(1)}(\omega) \mid \vec{k} \rangle = \sum_{\alpha} \rho_\alpha(\vec{k}'-\vec{k}) \langle \vec{k}'\alpha \mid T_{\pi N}(\omega) \mid \vec{k}\alpha \rangle \ . \tag{4.6}$$

C. Discussion

We found that the lowest order optical potential is determined by the free off-shell pion-nucleon scattering amplitude, the relationship being given in Eq. (4.6). The resulting Klein-Gordon theory is manifestly crossing symmetric. We emphasize that the range of the pion-nucleon scattering amplitude entering into $U^{(1)}$ is determined by the pion-nucleon vertex, which in field theoretical models[21] is of shorter range than potential models. We showed, in Ref. 6, that this difference shows up in the pion-nucleus angular distributions. The FPNE gives rise to damping factors [see Eq. (3.8) and the ensuing discussion] which looks identical to the extra damping introduced by potential models, and the results of Ref. 6 are thus indicative of the differences between the FPNE and our Klein-Gordon theory. We note in passing that our leading approximation is essentially the same as the leading term in the model of Brown and Weise.[30]

Although the result in Eq. (4.6) is simple, the basic amplitude which determines $T_{\pi N}$ and $U^{(1)}$ is the box B_1' of Eq. (3.5) which depends on *three* energy variables. The reason the amplitude depends on three energy variables lies in the field theoretical nature of the problem. In order to be able to embed this amplitude in the higher order multiple scattering series, we will require knowledge of the additional energy dependence.

Note also that the scattering amplitude entering into the lowest order optical potential is the free pion-nucleon amplitude with no Pauli restrictions on the intermediate states. The Pauli principle, is, rather, imposed only in the higher order U (see the discussion in Appendix B). In this regard, our results at least superficially resemble those of Dover and Lemmer[29] who have shown that omitting Pauli restrictions on intermediate states of T is probably a better approximation than including them.

Finally, consider how one might solve the theory to obtain B_1' of Eq. (3.7). In principle, one should obtain B_1' from the Bethe-Salpeter equation. For the purpose of determining the lowest order optical potential, the dependence of T_1 on only one energy variable is required. The theory of Chew and Low[31] is then easily applied in which case the energy dependence of T_1 is determined by a dispersion relation similar to that satisfied by $h_\alpha(\omega)$ in Ref. 31. The extension of this work to include a delta state of the nucleon quark bag[23a] could also be applied, although it would have to be extended to include crossing symmetry. The approach of Chew[32] makes some time ordering approximations and the resulting amplitude is thus *not* simply related to B_1' (see Appendix D for more discussion). If one accepts the approximate crossing symmetric solution of Ref. 21 as adequate, it may be employed directly in Eq. (4.6).

V. PION-TWO NUCLEON SCATTERING

For the cluster expansion we are examining, there is a close relationship between pion-deuteron scattering and the second order optical potential. In this section we want to show how to calculate these quantities and to relate them to the pion-nucleon box B_1 discussed in Secs. III and IV. This, in fact, constitutes only a partial solution to pion-two-nucleon scattering, and corrections are discussed in Sec. VI. In Sec. V B we compare our results to previous work, in which a similar objective had met some difficulties.

A. Pion-deuteron scattering and $U^{(2)}$

In this subsection we consider both the problem of pion deuteron scattering and the second order optical potential. In the spectator expansion and the fixed scatterer approximation, these problems are closely related. Both require the solution of a pion scattering from two nucleons located at $\vec{r}_1$ and $\vec{r}_2$. Below, we derive explicit formulas for the πD problem, and then utilize results derived in Sec. III which render these formulas applicable to the calculation of $U^{(2)}$.

In order to calculate pion-deuteron scattering in the fixed scatter model one may proceed in analogy to pion-nucleon scattering as developed in Eqs. (4.1) and (4.2) and Fig. 5. However, now the wave function for the system becomes (ignoring spin and isospin for simplicity)

$$\psi(\vec{r}_1,\vec{r}_2) = e^{i(\vec{p}_1+\vec{p}_2)\cdot\vec{R}} \psi_D(\vec{r}) \ , \tag{5.1a}$$

where

$$\vec{R} = \frac{\vec{r}_1+\vec{r}_2}{2}, \quad \vec{r} = \vec{r}_1 - \vec{r}_2 \ . \tag{5.1b}$$

We find

$$(2\pi)^3\delta(\vec{p}\,'-\vec{p})\langle \vec{k}\,'\beta_1'\beta_2' \mid T_{\pi D}(\omega_k) \mid \vec{k}\alpha_1\alpha_2\rangle$$
$$= \int d\vec{R}\, d\vec{r}\, e^{i(\vec{p}_1'+\vec{p}_2')\cdot\vec{R}}\psi_D^*(r)\Big\{ \sum_i \langle \vec{k}\,'\beta_i' \mid T_{\pi N}(\omega_k) \mid \vec{k}\alpha_i\rangle e^{i\vec{r}_i\cdot(\vec{k}-\vec{k}\,')}$$
$$+ \sum_{ij} [\langle \vec{k}\,' \mid T_2'^{(ji)}(\omega_k:\vec{r}_1\vec{r}_2;\beta_1'\beta_2'\alpha_1\alpha_2) \mid \vec{k}\rangle$$
$$+\langle -\vec{k} \mid T_2'^{(ji)}(-\omega_k:\vec{r}_1\vec{r}_2;\beta_1'\beta_2'\alpha_1\alpha_2) \mid \vec{k}\rangle]\Big\}\psi_D(r)e^{i(\vec{p}_1+\vec{p}_2)\cdot\vec{R}} \,. \tag{5.2}$$

The pion-nucleon scattering amplitude is added explicitly in Eq. (5.2) because $T_2'^{(ji)}$ has been defined to be the connected part of the amplitude. Now we would like to use the relationship in Eq. (3.7). Note that there are now four amplitudes $T_2'^{(ji)}(\omega;\beta_1'\beta_2'\alpha_1\alpha_2)$ corresponding to the four pairs of (ij): (1,1), (1,2), (2,1), and (2,2). It is clear from the translational invariance of the theory that $\overleftrightarrow{T}_2'$ can only depend on $\vec{r}_1$ and $\vec{r}_2$ through $\vec{r}$. Putting Eq. (3.7) into Eq. (5.2) we see that the integrals over $\vec{r}_1$ and $\vec{r}_2$ may be written

$$\int d\vec{r} \int d\vec{R}\, e^{i\vec{R}\cdot(\vec{p}_1'+\vec{p}_2')}\psi_D^*(r)\Big\{ \sum_i \langle \vec{k}\,'\beta_i' \mid T_{\pi N}(\omega_k) \mid \vec{k}\alpha_i\rangle e^{i(\vec{k}-\vec{k}\,')\cdot(\vec{R}+\epsilon_i\vec{r}/2)}$$
$$+ \sum_{ij} [\,\vec{k}\cdot\overleftrightarrow{T}_2'^{(ji)}(\omega_k;\beta_1'\beta_2'\alpha_1\alpha_2;\vec{r})\cdot\vec{k}+\vec{k}\cdot\overleftrightarrow{T}_2'^{(ji)}(-\omega_k;\beta_1'\beta_2'\alpha_1\alpha_2;\vec{r})\cdot\vec{k}\,']$$
$$\times v(k')v(k)e^{i\vec{k}\cdot(\vec{R}+\epsilon_i\vec{r}/2)}e^{-i\vec{k}\,'\cdot(\vec{R}+\epsilon_j\vec{r}/2)}\Big\}\psi_D(r)e^{i\vec{R}\cdot(\vec{p}_1+\vec{p}_2)} \,, \tag{5.3}$$

or

$$\langle \vec{k}\,'\beta_1'\beta_2' \mid T_{\pi D}(\omega_k) \mid \vec{k}\alpha_1\alpha_2\rangle = \sum_i \int d\vec{r}\,\psi_D^*(r)\langle \vec{k}\,'\beta_i' \mid T_{\pi D}(\omega_k) \mid \vec{k}\alpha_i\rangle\psi_D(r)e^{i\epsilon_i\vec{r}\cdot\vec{k}/2}$$
$$+v(k')v(k)\sum_{ij}\int d\vec{r}\,\psi_D^*(r)[\,\vec{k}'\cdot\overleftrightarrow{T}_2'^{(ji)}(\omega_k:\beta_1'\beta_2'\alpha_1\alpha_2;\vec{r})\cdot\vec{k}$$
$$+\vec{k}\cdot\overleftrightarrow{T}_2'^{(ji)}(-\omega_k:\beta_1'\beta_2'\alpha_1\alpha_2)\cdot\vec{k}\,']$$
$$\times\psi_D(r)e^{i\vec{r}\cdot(\epsilon_i\vec{k}-\epsilon_j\vec{k}\,')/2} \,, \tag{5.4}$$

where $\epsilon_i=+1$ if $i=1$ and -1 if $i=2$.

To obtain the quantity $T_2'^{(ji)}$ in Eq. (5.2) we must derive integral equations. These equations will necessarily couple the quantities $B_2'^{(ji)}(t't:\{\alpha'\alpha\bar{t}'\bar{t}\vec{r}\})$ of Eq. (3.4). Let us begin by defining the driving term D, shown in Fig. 6(a). It is related to the box $B_1'(t't;\alpha'\alpha\bar{t}'\bar{t}\vec{r})$ of Eq. (3.6):

$$\langle \vec{k}\,' \mid D(t't:\{\alpha'\alpha\bar{t}'\bar{t}\vec{r}\}) \mid \vec{k}\rangle = \langle \vec{k}\,' \mid B_1'(t't:\alpha'\alpha\bar{t}'\bar{t}\vec{r}) \mid \vec{k}\rangle$$
$$+\langle -\vec{k} \mid B_1'(tt':\alpha'\alpha\bar{t}'\bar{t}\vec{r}) \mid -\vec{k}\,'\rangle \,, \tag{5.5}$$

which has the Fourier transform

$$(-2\pi i)\delta(\omega'+\omega-\bar{\omega}'-\bar{\omega})\langle \vec{k}\,' \mid \tilde{D}(\omega'\omega:\alpha'\alpha\bar{\omega}'\bar{\omega}\,\vec{r}) \mid \vec{k}\rangle$$
$$=(-2\pi i)\delta(\omega'+\omega-\bar{\omega}'-\bar{\omega})[\langle \vec{k}\,' \mid \tilde{B}_1'(\omega'\omega:\alpha'\alpha\bar{\omega}'\bar{\omega}\,\vec{r}) \mid \vec{k}\rangle$$
$$+\langle -\vec{k} \mid \tilde{B}_1'(-\omega-\omega':\alpha'\alpha\bar{\omega}'\bar{\omega}\vec{r}) \mid -\vec{k}\,'\rangle] \,. \tag{5.6}$$

The integral equations are required to couple the successive scatterings of the pion from nucleon i to nucleon j through D. The intermediate pions will be allowed to propagate both forward and backward in time. The coupled integral equations shown in Figs. 6(b) and (c) accomplish this goal (a similar development holds for $B^{(12)}$ and $B^{(11)}$).

In terms of equations, Figs. 6(b) and (c) read

$$\langle \vec{k}' | \tilde{B}_2'^{(21)}(\omega'\omega:\bar\omega_1'\bar\omega_1;\bar\omega_2'\bar\omega_2:\vec{r}_1\vec{r}_2) | \vec{k} \rangle$$
$$= \int \frac{d^4k''}{(2\pi)^4} e^{i\vec{k}''\cdot(\vec{r}_1-\vec{r}_2)} \langle \vec{k}' | \tilde{D}^{(1)}(\omega'\omega'':\bar\omega_1'\bar\omega_1:\vec{r}_1) | \vec{k}'' \rangle \frac{1}{\omega''^2-\omega_k''^2+i\eta} \langle \vec{k}'' | \tilde{D}^{(2)}(\omega''\omega:\bar\omega_2'\bar\omega_2;\vec{r}) | \vec{k} \rangle$$
$$+ i \int \frac{d^4k''}{(2\pi)^4} \langle \vec{k}' | \tilde{D}^{(1)}(\omega'\omega'':\bar\omega_1', W-\omega''-\bar\omega_2';\vec{r}_1) | \vec{k}'' \rangle \frac{1}{\omega''^2-\omega_k''^2+i\eta}$$
$$\times e^{i\vec{k}\cdot(\vec{r}_1-\vec{r}_2)} \frac{1}{W-\omega''-\bar\omega_2'+i\eta} \langle \vec{k}'' | \tilde{B}_2'^{(22)}(\omega''\omega:W-\omega''-\bar\omega_2,\bar\omega_1;\bar\omega_2'\bar\omega_2;\vec{r}_1\vec{r}_2) | \vec{k} \rangle , \tag{5.7a}$$

and

$$\langle \vec{k}' | \tilde{B}_2'^{(22)}(\omega'\omega:\bar\omega_1'\bar\omega_1;\bar\omega_2'\bar\omega_2:\vec{r}_1\vec{r}_2) | \vec{k} \rangle$$
$$= i \int \frac{d^4k''}{(2\pi)^4} e^{i\vec{k}''\cdot(\vec{r}_2-\vec{r}_1)} \langle \vec{k}' | \tilde{D}^{(2)}(\omega'\omega'':\bar\omega_2', W-\omega''-\bar\omega_1') | \vec{k}'' \rangle$$
$$\times \frac{1}{\omega''^2-\omega_k''^2+i\eta} \frac{1}{W-\omega''-\bar\omega_1'+i\eta} \langle \vec{k}'' | \tilde{B}_2'^{(21)}(\omega''\omega:\bar\omega_1'\bar\omega_1;W-\omega''-\bar\omega_1',\bar\omega_2:\vec{r}_1\vec{r}_2) | \vec{k} \rangle , \tag{5.7b}$$

where the diagrams have been Fourier transformed to energy variables. We have used W to denote the total conserved energy,

$$W=\omega+\bar\omega_1+\bar\omega_2=\omega'+\bar\omega_1'+\bar\omega_2' .$$

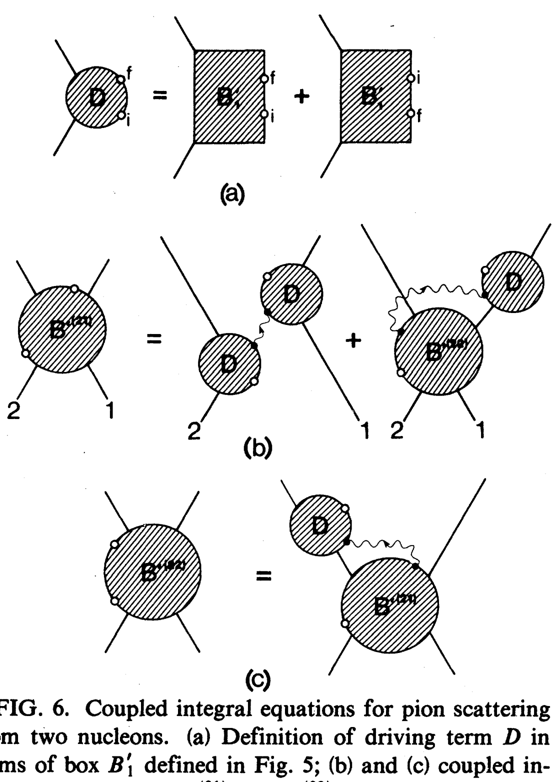

FIG. 6. Coupled integral equations for pion scattering from two nucleons. (a) Definition of driving term D in terms of box B_1' defined in Fig. 5; (b) and (c) coupled integral equations for $B'^{(21)}$ and $B'^{(22)}$.

Note that $\tilde{B}^{(ij)}$ is needed only for all $\bar\omega=0$ [see Eq. (3.6)]. The variables $\bar\omega_1$ and $\bar\omega_2$ are not involved in the integration in Eq. (5.7) and may thus immediately be set to zero, in which case $W=\omega$. Note that in order to solve these equations, the dependence of B_1 on *all* its variables must be known [see Eq. (5.6)], unlike the case of pion-nucleon scattering and the lowest order optical potential. As we have remarked, the dependence on the momentum of the $\tilde{B}_2$ and $\tilde{D}$ are known *a priori*, so that Eq. (5.7) is solved to determine the functional dependence on ω', $\bar\omega_1'$, and $\bar\omega_2'$. Thus, the integrations over $\vec{k}''$ may be performed, and the equations are actually one-dimensional integral equations. To obtain $T_{\pi D}$ from the solution of Eq. (5.7), use Eq. (3.6) to find $T_n'^{(ji)}$, and finally use Eq. (5.4).

To calculate the second order cluster correction to the spectator expansion of the optical potential one uses the solution of Eq. (5.7) to evaluate $\tilde{B}^{(ji)}$. To calculate $T'^{(ji)}$, one uses the relationship in Eq. (3.6). The correction shown in Fig. 4 is easily applied once Eq. (5.7) is solved, because the subtracted term is just the first integral on the right-hand side of Eq. (5.7). The addition of hole lines and the calculation of the sign of the term proceeds in accordance with the general discussion of Sec. III. Note that the solution for $U^{(2)}$ requires all the πNN amplitudes, $T^{(ij)}$, not just $T_{\pi D}$.

B. Discussion

We have derived integral equations for ladders of pion-nucleon scattering from two nucleons. Both the direct and crossed pieces of the pion-nucleon amplitude are contained in the theory, including the nucleon (P_{11}) pole. All topologically equivalent diagrams are included in the solution, including mesons propagating backward in time. The formulation proposed here differs from previous work in that the crossed pion-nucleon amplitudes and the backward time propagation of pions are explicitly considered in accordance with the requirements of the field theoretic character of the pion.

It is clear from the development of this paper that for the purposes of building up multiple scattering in field theoretical contexts, the required off-shell dependence of the elementary scattering amplitude is different from that needed in nonrelativistic approaches. In particular, the dependence of the pion-nucleon scattering amplitude on three independent energy variables must be known in order to solve the integral equation in Eq. (5.7). Most theories, including the Chew-Low theory, only provide the dependence on one of these variables. Correct microscopic approaches need to know the full energy dependence and little theoretical work has been devoted to the determination of the completely off-shell Bethe-Salpeter amplitude. In the simple isobar models,[3] however, the dependence is trivial because the $\pi N \Delta$ vertex compels the box

$$\langle \vec{k}\,' \,|\, B_1(t't;\bar{t}'\bar{t}) \,|\, \vec{k} \rangle$$

to contain the two delta-functions $\delta(t'-\bar{t}')$ and $\delta(t-t')$. Consequently, B_1 is a function of only one independent ω variable. If one were to add a Chew-Low component to the simple delta model by including a πNN vertex as proposed in Ref. 23a, these additional delta functions in time would be lost and the theory would again require the more general amplitude as a function of three energies.

All previous work on pion-deuteron scattering has involved the FPNE (Refs. 24, 13, 16, and 17). In order to compare our work to these, we derive in Appendix D integral equations assuming that the boxes never overlap and that pions propagate only forward in time. Our results differ significantly from previous work, especially in the case of Myhrer and Thomas.[13] Their theory is different from ours in two important respects. The first is that the solution of their theory entails integrations which encounter the pole of the pion-nucleon amplitude in unphysical regions. We show in Appendix D, as a special case of our theory, how to match the energy and momentum variables in the FPNE with those of the field theoretical amplitude and demonstrate thereby that no spurious poles of the type encountered in Ref. 13 are found in our theory.

The second difference between Ref. 13 and the present theory is in the off-shell momentum dependence of the pion-nucleon amplitude. In the work of Myhrer and Thomas it is argued that the momentum dependence of the pion-nucleon amplitude should have a cutoff in momentum space characteristic of the pion mass rather than of the pion-nucleon form factor. As discussed in Appendix D in some detail, this is caused by the FPNE, and when the boundary conditions of the pion field are treated according to our approach the range of the amplitude is given by the pion-nucleon vertex function.

VI. OTHER CORRECTIONS

There are, of course, many corrections to the theory presented in this paper. These corrections fit into two categories. The first category consists of corrections which are not specifically related to the quantum field character of the pion. It would include effects of nucleon recoil, for example. Recoil must be included in all quantitative work, and because of its importance we will address this specifically in a subsequent publication.

A second example of this category is the subject of medium corrections. We have used the spectator expansion of Refs. 11 and 12 for the development of this paper, but there are good reasons to believe that the spectator expansion may not converge sufficiently rapidly to give the optical potential. An alternative which might work is the self-consistent expansion,[26] which although originally proposed for potential theories can be extended as well to field theories.[27]

The second category of correction to our theory consists of specifically quantum field theoretic effects. This includes (1) renormalization, (2) true absorption, (3) double counting with the nuclear force, and (4) intrinsic m-pion ($m > 2$) processes. Many papers have been devoted to (1)—(3) and we will only mention these subjects. It is probably worth reiterating at this point that our main interest has been the role of crossing symmetry, boundary conditions on the pion field, and the correct manner of defining and embedding the off-shell pion-nucleus scattering amplitude into the multiple scattering theory, which have been much less carefully treated in the literature. For the purposes of building a complete theory one must bring together all of the above considerations.

Of the above mentioned quantum field theoretical corrections, multipion processes can be expected to constitute a significant quantitative correction to multiple scattering of pions and will play an increasingly important role in pion physics. An example of an intrinsic three-pion process is shown in Fig. 7(a). One could define a new type of box which sums all

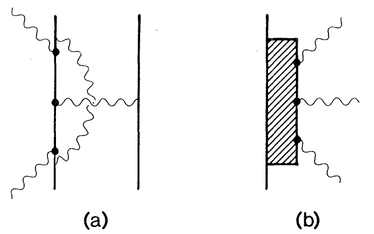

FIG. 7. Examples of m-pion box for $m=3$.

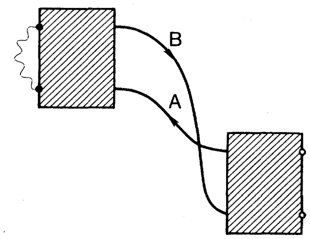

FIG. 8. A term requiring renormalization treatment and not encountered in the pion-deuteron problem.

m-pion processes, as shown in Fig. 7(b) for $m=3$. This correction would give, for example, the interaction of the isobar with other nucleons in the medium and would also be required for discussions of two-pion production processes.

Questions of true absorption in pion-deuteron scattering have been discussed in Refs. 13 and 16–18, and in pion-nucleus scattering in Refs. 33–35. With the extension of our present formulation to include nucleon recoil, true absorption will naturally occur by virtue of our inclusion of the P_{11} hole in $T_{\pi N}$. However, other mechanisms which lead to true absoprtion must be added.[33] We refer the reader to the literature for a discussion of the current status of this problem.

A very thorough discussion of overcounting corrections, renormalization, and absorption in pion-deuteron scattering has been given by Avishai and Mizutani,[17] but we believe that future formulations should combine the considerations of Ref. 17 with a careful treatment of crossing symmetry and the boundary conditions on the pion field. Note that when the pion scatters in the nuclear medium new types of renormalization arise involving hole lines. One such term involving two hole lines is shown in Fig. 8.

By employing approximations which allow arbitrary numbers of pions in intermediate states, we are forced to sacrifice a principle which is basic to many pion-few nucleon scattering theories,[17,18] namely manifest n-body unitary $n \geq 3$. While unitary is an important concept, we feel that insisting that approximations display manifest unitary properties is unduly restrictive, especially when large numbers of inelastic channels are possible. What is important, rather, is that the theory *before* approximations be unitary and that the approximations capture the essential quantitative physics.

Next we consider a generalization of the static theory of Sec. III, which is equivalent to it in a certain qualified sense to be discussed. In place of the Hamiltonian H described there, we take

$$H=H_{0B}+H_{0M}+H'-u \ , \tag{6.1}$$

where

$$H_{0B}=\sum_k (k^2+m^2)^{1/2} b_k^\dagger b_k + u \ . \tag{6.2}$$

In these expressions, u is a one-body mean field introduced in order to define a basis for perturbation theory. The quantity b_k is an annihilation operator for nucleon of momentum k and mass m (if we wish, excited states of nucleons could also be introduced in the sum over quantum numbers). The operator H_{0M} is the free Hamiltonian for the meson; and H' is the meson-baryon coupling described in Sec. III, but generalized in a straightforward manner to account for recoil of the scattering center, i.e., if we represent by $V(k)a_k^\dagger$ a term in the interaction of Sec. III describing the creation of a meson of momentum k, then a possible generalization is

$$v(k)a^\dagger_{\vec{k}} \to \langle \vec{p}\,'\vec{k} \,|\, v \,|\, \vec{p}\,\rangle a^\dagger_{\vec{k}} b^\dagger_{\vec{p}\,'} b_{\vec{p}} \ , \tag{6.3}$$

$$\langle \vec{p}\,'\vec{k} \,|\, v \,|\, \vec{p}\,\rangle = (2\pi)^3 \delta(\vec{p}-\vec{p}\,'-\vec{k}) v \left[\vec{k} - \frac{\mu}{m_B}\vec{p}\,' \right] . \tag{6.4}$$

Of course, the questions of mass and coupling constant renormalization must be dealt with in order to have a well-defined theory, but these can be handled in a well-known manner.

This generalization is not a local field theory, nor is it Lorentz invariant. It also does not contain explicit antinucleon degrees of freedom and may not be appealing for other reasons as well. However, for the purposes of low-energy nuclear physics, these characteristics may not constitute a drawback. It is clear that this Hamiltonian contains many of the necessary ingredients. One requires that an adequate theory must include the observable "collective" mesonic and baryonic degrees of freedom and interactions among them. To describe true absorption of mesons, recoil of the baryons must also be allowed. What is of ultimate significance is that a systematic, quantitative method be found for deriving this "effective interaction theory" from more fundamental theory (an example of which in other contexts is the folded diagram theory[36]), which one believes must be expressed in terms of confined, interacting quark and gluon fields. We next indicate a possible relationship between the fixed scatterer theory of Sec. III and the one described in Eq. (6.1).

We write an arbitrary diagram for a Green's function or the time evolution operator in time-ordered perturbation theory considering $H'-u$ as the perturbation and the "free" propagator

$$G_0(\omega)=(\omega-H_{0M}-H_{0B}+i\eta)^{-1} . \tag{6.5}$$

If we assume that all the eigenvalues of H_{0B} can be neglected relative to the other energies occurring during the same time interval, then H_{0B} in the denominator can be replaced by an appropriate constant $\langle H_{0M}\rangle$. The sum over all intermediate nuclear states may be performed using closure with the result that the nucleon positions *over the time interval in question* are fixed. This argument is the same as that given in Ref. 24, and the interested reader may refer to this paper for a more explicit argument.

However, at each action of H', one has a matrix element of the form [use Fourier transform of Eq. (6.4) to coordinate space]

$$\langle \vec{r}\,'_B\vec{r}_m \mid v \mid \vec{r}_B\rangle = v(\vec{r}\,'_B-\vec{r}_m)\delta\left[\frac{m_B\vec{r}\,'_B+\mu\vec{r}_m}{m_B+\mu}-\vec{r}_B\right], \tag{6.6}$$

where the delta function conserves the center-of-mass position and $\vec{r}\,'_B$ and $r_{\vec{B}}$ are the final and initial position of the struck baryon. In order that our static result follow, it is necessary to argue that $\vec{r}\,'_B=r_{\vec{B}}$, *in addition* to the closure argument. This will be done if either of two conditions are satisfied[37]:

$$\vec{r}\,'_B=\vec{r}_m \tag{6.7}$$

or

$$m_B \gg \mu . \tag{6.8}$$

Condition (6.7) will be satisfied only if the form factors are short ranged. The extent to which v is short ranged is a subject of much controversy, and we must therefore leave our analysis of the connection somewhat unsettled on this point. The question of the range of the effective hadron-nucleon interaction is one of the central dynamical questions which must be answered before firmer theoretical predictions of multiple scattering can be made.

If closure and one of the two conditions in Eqs. (6.7) and (6.8) are satisfied, then we have shown that the nucleon positions remain fixed throughout the time-ordered diagram. The use of a particular initial or final state wave function is determined by the transition to be described. If the diagrams correspond to matrix elements of the time-evolution operator, then usual adiabatic switching arguments show that the initial and final states are the unperturbed configurations of $H_{0B}+H_{0M}$ which evolve into the exact final states to be described. For the case of elastic scattering, we may work with Green's function and the initial and final states are the unperturbed ground state configuration, based on similar considerations. Although the proof of the correspondence we have given relies on time-ordered diagrams, recall that the theory should be solved in terms of Feynman diagrams, which sum all allowed time orderings.

The reader may strongly object to the use of closure following Eq. (6.5), because there are certain intermediate states which obviously violate the condition stated, namely intermediate states having no mesons and having the same energy as the initial state. Such states correspond to true absorption and are, indeed, the states which require us to introduce nucleon recoil. We can only reply that closure should be applied to the subset of diagrams in which there are no time intervals containing only one-shell nucleons. How restrictive is this in practice? For pions, true absorption is known to account for as much as half of the reaction cross section in the region of the (3.3) resonance[38] (except at very low energies, where it accounts for all of it). However, as the energy increases away from the resonance, the true absorption cross section appears to be falling.[38] So, closure may be a useful approximation only at relatively high pion energies. On the other hand, there is some possibility that it is also useful at lower energies, due to the observation that true absorption seems to occur on clusters containing as many as five nucleons.[39] If this is true, closure may break down only in relatively high orders of the spectator expansion for the optical potential, so that for elastic scattering omission of true absorption may not be a major quantitative neglect.

Note that the subtraction of the mean field u remains as a perturbation in the static limit. Exactly how one chooses u has been one of the central theoretical issues of microscopic nuclear physics. For the lowest eigenstates of H_{0B}, Brueckner theory[40] specifies u as an appropriate average of the G matrix with self-consistent nuclear wave functions, which is in turn derived from nucleon-nucleon two-body potentials, and in meson mean field theories[41] it is the self-consistent Hartree potential generated from various meson interactions with the medium. In any case, u should be designed to cancel or partially cancel some of the important diagrams in the meson theory, and by a sufficiently careful choice of u one may in principle avoid the problems of double counting physics in the wave function $|\psi_0\rangle$ of Eq. (A2) with physics in the multiple scattering of the fixed scatterer theory. We have ignored u in this paper for simplicity of discussion. This is approximately all right for pions in most nuclei, since in lowest order the pionic contribution to u from closed shells is zero. However, in quantitative approaches to pion-nucleus physics, this interaction must be carefully defined and evaluated. For scattering, we shall need to specify u for particle states as well; there is a great deal of freedom in how this is done. Presumably, this freedom can be utilized to maximum convergence of the multiple scattering series. This freedom has not been exploited in previous microscopic approaches to the extent that is possible. Among other things, this observation would imply that, for better or worse, there is not a clear theoretical separation between the physics of the lowest and higher orders of the optical potential.

VII. SUMMARY AND CONCLUSION

We have addressed in this paper the structure of the multiple scattering theory for pion-nucleus elastic scattering when the pion is properly described in a quantum field theory. We have examined, in a general context, the relationship between the optical potential U and the pion-nucleus T matrix. In a specific model we have explored the problems associated with embedding the free pion-nucleon scattering amplitude in the optical potential.

For both aspects of the problem we have been guided by the requirement that the proper boundary conditions be imposed on the pion field at the most fundamental level, i.e., we require that the pion field always propagate forward in time as a particle and backward in time as an antiparticle.[4] We found that this condition may be applied in such a way that the optical potential and pion-nucleus T matrix are manifestly crossing symmetric, and so the spurious reactive content,[6] which would occur in higher order in the fixed pion number expansions, is absent. A further consequence is the absence of the enhanced damping which occurs in the FPNE and can affect the geometry and suppress higher order terms of U. The final form of the theory which accomplishes this is quite simple, namely the Klein-Gordon equation with U entering additively to the square of the pion mass.

We have found that the problem of embedding the free pion-nucleon ampitude into the higher order cluster corrections of U, consistent with the boundary conditions on the pion field, becomes formally simpler if we express the results in terms of an extrapolation of the free pion-nucleus amplitude off-shell in the four-momentum characterizing the initial and final state. This amplitude depends on three independent energy variables instead of the single energy variable which occurs in potential theory. By carefully considering the relationship between these additional variables and those of the multiple scattering diagrams, we have been able to avoid the spurious analytical behavior encountered in some previous work.

Alternative approaches to the theory of meson-nucleus scattering include those based on the fixed pion number expansions.[25] We have emphasized that it is difficult to take account of crossing symmetry and to impose the proper boundary conditions on the pion field in these approaches, whereas it is straightforward to achieve this *at every order of approximation* for U in the Klein-Gordon equation. As far as one can ascertain, these field theoretical properties are of fundamental significance in nature and therefore we conclude that: (1) the approaches mentioned above are unacceptable alternatives and (2) the Klein-Gordon optical model theory, examined here in the static limit, is to be preferred as a framework for theoretical and phenomenological studies.

For realistic calculations of pion-nucleus scattering one should replace the static theory, which was used in Secs. III–V for illustrative purposes, by one which includes nucleon recoil. We plan to treat this in a subsequent paper. Other details of the theory will have to change as more is learned about the nature of the interaction and many-body theory. For example, one may wish to replace the spectator expansion by one which respects other principles such as that of self-consistency.[26,27] What we expect to remain unchanged, however, is the relationship between U and the scattering amplitude through the Klein-Gordon equation, the necessity for a broader definition for the off-shell pion-nucleon scattering amplitude, and the inadequacy of the fixed pion number expansion and the relativistic Schrödinger equation for handling efficiently theoretic aspects of the pion-nucleus scattering problem.

APPENDIX A: SCATTERING FROM *N* FIXED CENTERS IN MESON FIELD THEORY

The model is a straightforward generalization of the static theory of Ref. 20 to the case of N scattering centers. Because there are now many nucleons, derivation of the theory consists of two distinct steps. The first is to utilize the scattering theory to specify the meson Green's function

$$G_{\vec{k}',\vec{k}}(t't;\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N) \tag{A1}$$

for the scattering from nucleons located at positions $(\vec{r}_1,\ldots,\vec{r}_N)$ (we suppress spin and isospin labels for simplicity), where $\vec{k}'$ and $\vec{k}$ are the final (initial) meson momenta and t' and t, respectively, are the times at which the meson is removed and inserted. The second problem is to express in a convenient form the average of Eq. (A1) over the nucleon ground state $|\psi_0\rangle$ to obtain $G_{\vec{k}',\vec{k}}$ of Eq. (2.1), i.e.,

$$G_{\vec{k}',\vec{k}} \equiv \langle\psi_0|G_{\vec{k}',\vec{k}}(\omega;\vec{r}_1,\vec{r}_2\cdots\vec{r}_N)|\psi_0\rangle/\langle\psi_0|\psi_0\rangle\ . \tag{A2}$$

The multiple scattering expansion of the quantity in Eq. (A1) follows from application of standard ideas.[42] We represent terms in Eq. (A1) as diagrams of time-dependent perturbation theory. By convention, time runs upward. Diagrams of Eq. (A1) consists of N nucleons (solid lines) with arrows pointing upward and some number of meson lines (wiggly lines). The nucleon lines extend from $-\infty$ to $+\infty$. A meson must originate at t and terminate either at time t' [Fig. 9(a)] or on one of the nucleons [Fig. 9(b)]. In the latter case, a different meson line must terminate at t'. Each nucleon line has a coordinate associated with it $(\vec{r}_1,\vec{r}_2,\ldots,\vec{r}_N)$.

Meson lines originate and terminate on nucleons at vertices. Each vertex has a time associated with it; all times, except for t and t', are eventually integrated. The rules for constructing the amplitude from the diagrams are given in Appendix B.

The expectation value of $G_{\vec{k}',\vec{k}}$ over the nuclear wave function can be expressed diagrammatically. Assume for simplicity that the nuclear wave function consists of an antisymmetrized product of single particle orbitals. Denote by upper case letters $A,B,\ldots,$ the normally occupied states. After integration, Eq. (A2) would resemble the diagrams in Fig. 9, except that the initial and final legs would be labeled by $A,B,\ldots,$ corresponding to wave functions associated with the lines.

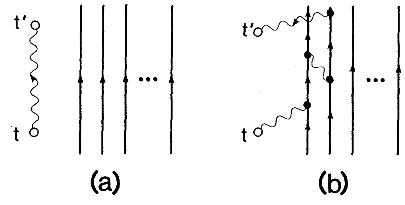

FIG. 9. Illustrating diagrams contributing to $G_{\vec{k}',\vec{k}}$ of Eq. (A1).

It is superfluous to have N nucleon lines in each diagram, so we write lines for nucleons only when there is at least one interaction on the line in question. We further connect the initial and final lines which have the same label $A,B,\ldots,$ without removing the arrows, to form continuous loops. This gives the diagrams the appearance of Feynman-Goldstone diagrams (they are not, though), and we easily keep track of the sign by counting hole lines (h) and closed loops (l)

$$\text{sign} = (-)^{h+l}\ . \tag{A3}$$

Hole lines may point upward or downward (they point only downward in Feynman-Goldstone diagrams). The change of notation has another desirable effect: We no longer have to keep track of the Pauli principle. As long as we draw all diagrams having any number of hole lines and closed loops of a consistent sense of direction (clockwise or counterclockwise) and assign the sign given in Eq. (A3), the Pauli violating terms completely cancel.

The resulting diagrams will have some linked and some unlinked terms. The disconnected pieces (vacuum fluctuations) of these unlinked diagrams may be summed to give an unobservable phase common to each diagram. For all practical purposes only the linked diagrams need to be evaluated.[43]

Let us now illustrate the new diagrams. As stated in Sec. III, the various terms in G may be expressed in terms of boxes which represent certain selective summations of perturbation diagrams. Various valid contributions involving iterations of the box B_1 of Sec. III are shown in Fig. 10. The initial and final pion lines are not shown, but the point of attachment is represented by a small circle. Diagrams (10a), (10c), and (10e) consist of nucleon lines which form a continuous circuit with a well-defined clockwise or counterclockwise sense.

Figure 10(a) is a piece of the lowest order optical potential (see Sec. IV); according to the rules in Appendix B the index A is to be summed over all hole labels. Before the change of notation to hole lines it had the appearance of Fig. 10(b). Figure 10(c) is an iteration of Fig. 10(a). It may be summed independently over A and B. Before change of notation it looked like Fig. 10(d). Note that when $A=B$ in Fig. 10(d) the Pauli principle is violated because two nucleons are in the same initial (and final) state. However, the Pauli violating term is identically canceled by the piece of the valid diagram in Fig. 10(e) for which $A=B$. The original diagram corresponding to this is shown in Fig. 10(f), which is the exchange

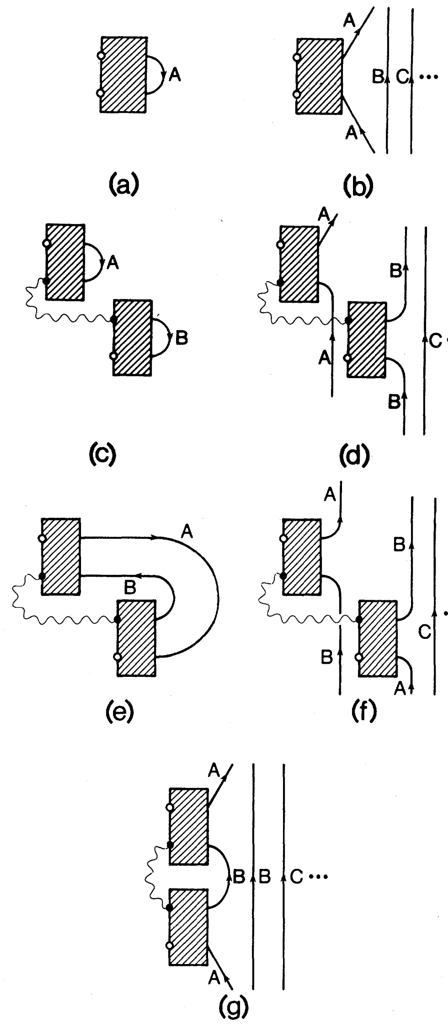

FIG. 10. Illustrating diagrams of $G_{\vec{k}'\vec{k}}$ before and after change of notation to hole lines. The shaded rectangle is the box B_1' of Sec. III.

of Fig. 10(d). Note that Fig. 10(f) appears to violate the Pauli principal even when $A \neq B$ in that it has two intermediate nucleons in state B. The function of these terms is to cancel the Pauli violating terms included within the box of Fig. 10(a). This may be clearer by referring to Fig. 10(g), which has the same value as a contribution to Fig. 10(a) but is written to look like a piece of Fig. 10(b). This discussion illustrates that the exclusion principle is maintained here not by an explicit condition imposed on the lines intermediate to the box, but through perturbative corrections which have the same structure as the other second order terms. If $|\psi_0\rangle$ contains short ranged correlations corresponding to the Jastrow-type, then other diagrams could be identified for a systematic evaluation of the scattering.[44]

APPENDIX B: RULES FOR EVALUATING DIAGRAMS

The rules for evaluating the diagrams which appear in this paper are given in this appendix. The enumeration of the diagrams was given in Appendix A.

(1). Each meson particle line, Fig. 11(a), is assigned the value

$$\frac{e^{-i\omega_k(t'-t)}}{2\omega_k}\theta(t'-t)=\frac{i}{2\omega_k}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t'-t)}}{\omega-\omega_k+i\eta}\,, \tag{B1}$$

$$\omega_k=(k^2+m_\pi{}^2)^{1/2}\,.$$

Meson particle lines always point upward. The direction of the line also denotes the direction of flow of momentum.

(2). Each meson antiparticle, Fig. 11(b), is assigned the value

$$\frac{e^{-i\omega_k(t-t')}}{2\omega_k}\theta(t-t')=\frac{-i}{2\omega_k}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t'-t)}}{\omega+\omega_k-i\eta}\,. \tag{B2}$$

Meson antiparticle lines always point downward.

(3). Each nucleon particle line, Fig. 11(c), is assigned the value

$$\theta(t-t')=i\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t-t')}}{\omega+i\eta}\,. \tag{B3}$$

(4). Each meson-nucleon vertex, Fig. 11(c) is assigned a matrix element

$$\langle\alpha\vec{k}\,|\,V_{\pi N}\,|\,\beta\rangle=\frac{f_\pi}{m_\pi}v(k)\vec{\sigma}\cdot\vec{k}\tau_k\,. \tag{B4}$$

If one includes the Δ as a physical particle, then one must add:

(5). Each delta-particle line, Fig. 11(e), is assigned the value

$$e^{-im_\Delta(t-t')}\theta(t-t')=i\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega(t'-t)}}{\omega-m_\Delta+i\eta}\,, \tag{B5}$$

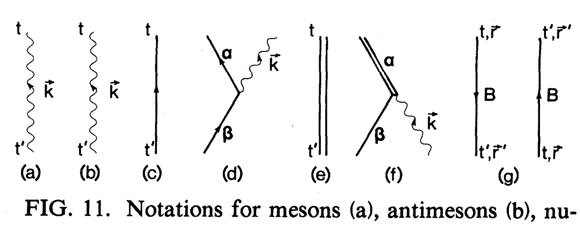

FIG. 11. Notations for mesons (a), antimesons (b), nucleons (c), meson-nucleon interaction (d), isobar Δ_{33} (e), isobar-pion-nucleon interaction (f), and nucleon hole line (g).

where m_Δ is the mass of the isobar relative to the nucleon.

(6). Each $\pi N \rightarrow \Delta$ interaction, Fig. 11(f), is assigned a matrix element

$$\langle \alpha \mid V_{\pi N \Delta} \mid \beta \vec{k} \rangle = \frac{f_{\pi N \Delta}}{m_\pi} v(k) \vec{S} \cdot \vec{k} \, \vec{T} ,$$

where $\vec{S}$ and $\vec{T}$ are[3] transition spin and isospin operators.

(7). Each nucleon hole line, Fig. 11(g), is assigned a value

$$\psi_B^*(\vec{r}\,')\psi_B(\vec{r}) ,$$

where ψ_B is the single-particle wave function for a nucleon of quantum number B. The coordinates $\vec{r}\,'$ and $\vec{r}$ are, respectively, the coordinates of the nucleon where the hole line originates (terminates). There is no time-dependent factor for the hole line, which may point either upwards or downwards as demanded by the other considerations.

(8). Each internal meson line is given a factor $e^{i \vec{k} \cdot \Delta \vec{r}}$ where $\Delta \vec{r}$ is the interval between the struck nucleons, counted in the direction of the arrow on the meson line, and $\vec{k}$ is the meson momentum.

(10). The diagram is assigned a sign $(-)^{l+h}$ where l is the number of closed loops and h the number of nucleon hole lines.

(11). Sum the nucleon hole line labels over all normally occupied states in $\mid \psi_0 \rangle$.

(12). All meson momenta, nucleon positions, and times are integrated independently

$$\int \frac{d\vec{k}}{(2\pi)^3} , \quad \int d\vec{r} , \text{ and } -i \int_{-\infty}^{+\infty} dt .$$

APPENDIX C: COMPARISON TO MILLER AND HENLEY

An alternate rearrangement of the perturbation series which we are examining has been proposed by Miller[14] and Miller and Henley.[15] In this appendix we will examine the relationship between our approach and theirs and, in particular, demonstrate why we do not obtain the low momentum cutoffs which they find. Much of this discussion can be found in Ref. 15.

We have arranged the perturbation theory according to the number of active nucleons and use no other criteria for ordering the series. In Refs. 14 and 15, the perturbation theory is ordered according to the number of pion-nucleon interactions minus the number of singular energy denominators, an arrangement first proposed by Chew.[32] In order to demonstrate the difference between these approaches, we will examine the diagrams of Fig. 12, just as was done in Ref. 15. All of these diagrams are strictly time ordered; all relative time orderings are to be maintained.

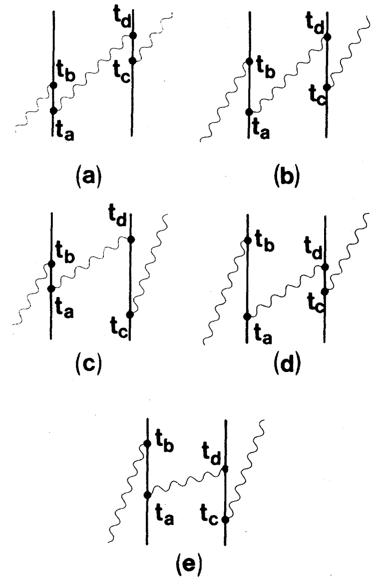

FIG. 12. Various time orderings of the interaction of a pion with two nucleons.

In our approach, each of these diagrams is of the same order because each involves two active nucleons. In the approach of Ref. 15, the diagram of Fig. 12(a) is a first order diagram; it contains $(f_\pi{}^2)^2$, which would be termed second order, but because it contains a singular propagator during the time interval t_b to t_c, the order is reduced to first order. The remaining diagrams in Fig. 12 in this approach are of second order; they contain $(f_\pi{}^2)^2$ but no singular denominators, and thus remain second order. The

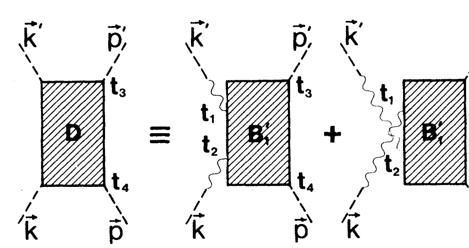

FIG. 13. Definitions of the driving term D for pion-two nucleon scattering in terms of the box B_1.

product of energy denominators associated with Fig. 12(a) is

$$12(\mathrm{a}) \Longrightarrow \frac{1}{\omega_{k''}} \frac{1}{\omega_0-\omega_k''+i\eta} \frac{1}{\omega_{k''}}\,, \tag{C1}$$

where $\omega_{k''}$ is the energy of the intermediate pion (the one which travels from t_a to t_d). The factors $(\omega_{k''})^{-1}$ provide a cutoff on the k integral and thus create an effective long range (in coordinate space) pion-nucleon interaction.

The denominator for the sum of all the diagrams pictured in Fig. 12 is

$$\begin{aligned}12(\mathrm{a})+12(\mathrm{b})+12(\mathrm{c})+12(\mathrm{d})+12(\mathrm{e}) \Longrightarrow {} & \frac{1}{\omega''}\frac{1}{\omega_0-\omega''}\frac{1}{\omega''}+\frac{1}{\omega''}\frac{1}{-\omega''-\omega_0}\frac{1}{\omega''}+\frac{1}{\omega_0}\frac{1}{-\omega''-\omega_0}\frac{1}{\omega_0''} \\ & +\frac{1}{\omega''}\frac{1}{-\omega_0-\omega''}\frac{1}{\omega_0}+\frac{1}{\omega_0}\frac{1}{-\omega''-\omega_0}\frac{1}{\omega_0} \\ & =\frac{1}{\omega_0}\frac{1}{\omega_0-\omega''}\frac{1}{\omega_0}\,.\end{aligned} \tag{C2}$$

This result is more quickly derived in Ref. 15 by noting that the sum of these diagrams is a Feynman diagram with the restriction that $t_b > t_a$ and $t_d > t_c$ (this keeps the pions crossed on each nucleon) and $t_d > t_a$ (this keeps the pion between nucleons a forward going pion). In the approach proposed here we would arrive at the result Eq. (C2). This result does not contain the cutoff factors $(\omega_{k''})^{-1}$, but rather factors of $\omega_0{}^{-1}$ which are simply the nucleon pole contribution of the crossed Born graphs.

The difference between including Fig. 12(a) alone and all of the diagrams in Fig. 12 has been shown by Henley and Miller[15] to be of the order of 20 percent. Because one can include all of these diagrams (and the additional ones with $t_d < t_a$ which converts the Schrödinger propagator to a Klein-Gordon propagator) without additional effort, we have chosen to arrange the series totally according to the number of active nucleons. We have also shown in Ref. 6 that the truncation of a perturbation series according to the number of pions present at any given time [i.e., keeping Fig. 12(a) but neglecting the rest] has the undesirable feature of containing spurious pion production at any given order, which is canceled by diagrams of higher order.

APPENDIX D: COMPARISON TO PION-DEUTERON SCATTERING IN FPNE

In Sec. V we derived equations for pion-deuteron scattering utilizing a framework which allows all pions to propagate forward and backward in time. However, all other theories of current usage[13,16–18] make the FPNE which, in effect, allows only pion propagation forward in time. In this appendix we will impose the restriction that the B_1 boxes do not overlap in time and that the pion propagators allow only forward time propagation of the pion in order to compare our theory with the standard approaches.

To impose the constraints it is useful to define a new driving term different from that of Fig. 6(a). The new driving term is shown in Fig. 13. The initial and final times, denoted, respectively, t_4 and t_3, refer to the times at which the earliest and latest interactions occur in the box B_1 of Eq. (3.4). The pion propagator extending from t_1 and t_2 to the edge of the box is included in the definition of the box. We have

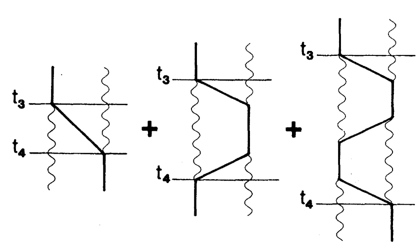

FIG. 14. The pion-nucleon scattering amplitude as given in the Chew theory.

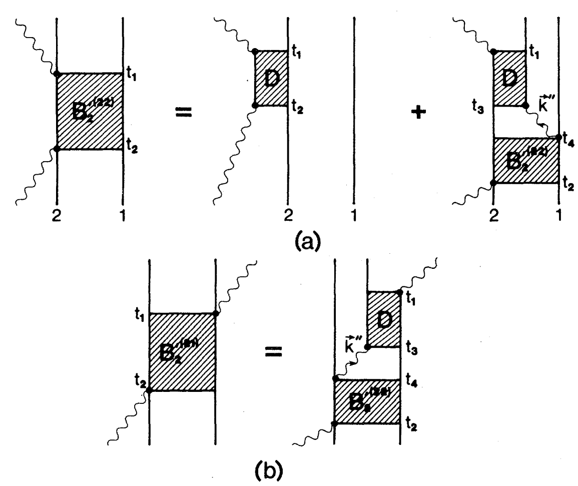

FIG. 15. Integral equations for time-ordered pion-two nucleon scattering. The time restrictions are $t_1 > t_3 > t_4 > t_2$.

$$\langle \vec{k}'\,|\,D^{(i)}(t_3,t_4)\,|\,\vec{k}\rangle$$
$$=(-i)^2\int dt_1\int dt_2\{e^{-i\omega_k'(t_3-t_1)}e^{-i\omega_k(t_2-t_4)}\langle \vec{k}'\,|\,B_1'^{(i)}(t_1t_2;t_3t_4)\,|\,\vec{k}\rangle$$
$$+e^{-i\omega_{k'}(t_3-t_2)}e^{-i\omega_k(t_1-t_4)}\langle -\vec{k}\,|\,B_1'^{(i)}(t_1,t_2;t_3t_4)\,|\,-\vec{k}'\rangle\}\ . \tag{D1}$$

In the box B_1 the nucleon times always coincide with the earliest and latest vertex of the box. Taking the Fourier transform

$$-2\pi i\delta(\omega_3-\omega_4)D^{(i)}(\omega_3)\equiv(-i)^2\int dt_3\int dt_1 e^{i\omega_3 t_3}e^{-i\omega_4 t_4}D^{(i)}(t_3,t_4)\ , \tag{D2}$$

we find

$$\langle \vec{k}'\,|\,D^{(i)}(\omega)\,|\,\vec{k}\rangle=\langle \vec{k}'\,|\,B_1'^{(i)}(\omega_k',\omega_k{:}\omega-\omega_k',\omega-\omega_k)\,|\,\vec{k}\rangle$$
$$+\langle -\vec{k}\,|\,B_1'^{(i)}(-\omega_k,-\omega_{k'}{:}\omega-\omega_k',\omega-\omega_k)\,|\,-\vec{k}'\rangle\ . \tag{D3}$$

In accordance with the considerations leading to Eq. (3.7) we write

$$\langle \vec{k}'\,|\,D^{(i)}(\omega)\,|\,\vec{k}\rangle=v(k')v(k)e^{-i\vec{r}_1\cdot(\vec{k}'-\vec{k})}$$
$$\times[\,\vec{k}'\cdot\overleftrightarrow{B}'_1(\omega_k',\omega_k{:}\omega-\omega_k',\omega-\omega_k)\cdot\vec{k}+\vec{k}'\cdot\overleftrightarrow{B}'_1(-\omega_k,-\omega_k'{:}\omega-\omega_k',\omega-\omega_k)\cdot\vec{k}]\ . \tag{D4}$$

In order to calculate the driving term in Eq. (D3) it is necessary to know B_1 as a function of all four of its energy variables. As pointed out in Sec. V there exists no theory yet for the complete functional dependence. Alternately, one could write a theory *directly* for the two quantities on the right-hand side of Eq. (D3). This could be accomplished using techniques similar to those of Chew.[32] For example, Fig. 14 represents an important sequence of diagrams contained in $\langle \vec{k}'\,|\,D^{(i)}(\omega)\,|\,\vec{k}\rangle$; the horizontal lines bound the contents of the box $D^{(i)}$ in Fig. 13. The sum of these terms may be accomplished by solving an integral equation.[32] The driving term in the equation is the first term in the sequence,

$$\frac{f_\pi^{\,2}}{m_\pi^{\,2}}\frac{\tau_{k'}\vec{\sigma}\cdot\vec{k}'\sigma\cdot\vec{k}\tau_k}{\omega-\omega_k-\omega_{k'}+i\eta}v(k')v(k)\ . \tag{D5}$$

Having defined the driving term in Fig. 15, we derive the integral equations which sum the boxes B_1' subject to the conditions discussed at the beginning of this section.

Consider first the integral equation for $T_2'^{(22)}(\omega_k{:}\vec{r}_1\vec{r}_2;\beta_1'\beta_2'\alpha_1\alpha_2)$ of Eq. (5.2). The equation for this is illustrated in Fig. 15, where it is shown that this amplitude is coupled to the amplitude $B_2'^{(21)}(\omega_k{:}\vec{r}_1\vec{r}_2;\beta_1'\beta_2'\alpha_1\alpha_2)$. Note that because part of the initial and final pion propagator is included in the driving term, Fig. 13, the external pions in Fig. 15 must be regarded as originating and terminating at the times associated with the initial and final edges of $B_2'^{(ij)}$. Similar equations couple $B_2'^{(22)}$ and $B_2'^{(12)}$.

Because we are time ordering the boxes and including pieces of the pion propagators in the box as in the case of D, the $B_2'^{(ij)}$ boxes become functions of two times, and the Fourier transform a function of one energy. With this in mind, the Fourier transform of the equations corresponding to Fig. 15 reads

$$\langle \vec{k}'\,|\,B_2'^{(22)}(\omega;\vec{r}_1\vec{r}_2)\,|\,\vec{k}\rangle=\langle \vec{k}'\,|\,D^{(2)}(\omega;\vec{r}_2)\,|\,\vec{k}\rangle$$
$$+\int\frac{d\vec{k}''}{(2\pi)^3}\langle \vec{k}'\,|\,D^{(2)}(\omega;\vec{r}_2)\,|\,\vec{k}\rangle\frac{1}{2\omega_{k''}}\frac{1}{\omega-\omega_{k''}-i\eta}\langle \vec{k}''\,|\,B_2'^{(21)}(\omega;\vec{r}_1\vec{r}_2)\,|\,\vec{k}\rangle \tag{D6a}$$

and

$$\langle \vec{k}'\,|\,B_2'^{(21)}(\omega{:}\vec{r}_1\vec{r}_2)\,|\,\vec{k}\rangle=\int\frac{d\vec{k}''}{(2\pi)^3}\langle \vec{k}'\,|\,D^{(1)}(\omega,\vec{r}_1)\,|\,\vec{k}''\rangle$$
$$+\frac{1}{2\omega_{k''}}\frac{1}{\omega-\omega_{k''}-i\eta}\langle \vec{k}''\,|\,B_2'^{(22)}(\omega;\vec{r}_1\vec{r}_2)\,|\,\vec{k}\rangle\ . \tag{D6b}$$

Equations (D6) are to be compared to Eq. (5.7). We immediately notice that in Eq. (D6) the energy and momentum variables are intricately mixed in the driving term $D^{(1)}$ [see Eq. (D4)], whereas in Eq. (5.7) the momentum dependence of the boxes arises purely through the factors of k, $v(k)$, and $\exp(i\vec{k}\cdot\vec{r})$. The same remains true of the boxes $B_2'^{(ji)}$: The factorization of energy and momentum in the optical potential discussed in Sec. III remains true only in our development, which avoids the FPNE. The additional momentum dependence of the boxes in the FPNE is the source of the additional cutoffs in momentum found in Refs. 14 and 15, discussed in more detail in Appendix C. It is also, in essence, responsible for the feeling expressed by Myhrer and Thomas[13] that the pion-nucleon T matrix should have a range longer than that of the free pion-nucleon scattering amplitude. We emphasize that the additional cutoff arises *as a result of an approximation,* namely time ordering the theory, and that the range of the momentum dependence of the fundamental pion-nucleon T matrix is *always the same* as the range of the pion-nucleon vertex in the static theory.

Finally, we note that the spurious pole contribution found by Myhrer and Thomas[13] does not occur in our theory. Although this may not be immediately obvious from Eq. (5.7), it is clear from Eqs. (D3) and (D5) that no pole contribution is encountered from the pion-nucleon T matrix in the (3-3) channel. The reason for this is that we have treated pion-nucleon and pion-deuteron scattering together in a field theory and there is no ambiguity in the way to embed the pion-nucleon scattering amplitude into the multiple scattering theory. We will further elucidate this point in the work where we extend this approach to include nucleon recoil.

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