Heavy-ion reactions with nucleon transfer using Skyrme-type potential

Ahmed Osman, M. Y. Ismail, and M. M. Osman
Physics Department, Faculty of Science, Cairo University, Cairo, Egypt
(Received 2 June 1982)

Nuclear reactions between two heavy interacting ions with nucleon transfer have been reconsidered. The direct nuclear reaction mechanism is considered. Different reaction processes are considered for single neutron or proton stripping and pickup reactions. The interacting nuclear potential of the transferred nucleon is taken to have the form of the Skyrme-type potential. With this representation for the nuclear potential of the transferred nucleon, an expression for the differential cross section is obtained by using the distorted-wave Born approximation. This expression is applied and considered for the heavy ion reactions with incident heavy ion projectiles ^{10}B, ^{13}C, ^{16}O, ^{18}O, and ^{32}S bombarding the heavy targets ^{27}Al, ^{28}Si, ^{29}Si, ^{30}Si, and ^{32}S. The energies of the incident heavy ions have values in the range between 36.0 and 100.0 MeV. Numerical calculations of the differential cross sections are carried out. The theoretically calculated angular distributions are in good agreements with the experimental measurements. Reasonable spectroscopic factors are extracted from the present calculations.

NUCLEAR REACTIONS Nucleon stripping and nucleon pickup induced by 36.0—100.0 MeV ^{10}B, ^{13}C, ^{16}O, ^{18}O, and ^{32}S on ^{27}Al, ^{28}Si, ^{29}Si, ^{30}Si, and ^{32}S; calculated $\sigma(\theta)$. Finite range DWBA calculations, extracted spectroscopic factors.

I. INTRODUCTION

Heavy ion reactions with a single nucleon transfer have been widely studied by many authors. These reactions are investigated theoretically following different approaches for stripping and pickup reaction processes. The most widely used calculations are those performed by using the finite range distorted wave Born approximation calculations.[1] These calculations with the DWBA approximation fail to predict the dependence of the observed energy on the absolute single particle transfer strength. However, using Woods-Saxon potentials helps in correcting this failure.[2]

A more complete understanding of heavy ion single nucleon transfer reactions is not only necessary in order to extract reliable spectroscopic information from such reactions, but also to form the basis for understanding more complicated heavy ion reactions. Hence, the heavy ion single nucleon transfer reactions have taken on great importance. On the other hand, steady progress has been made in recent years towards a microscopic understanding of nuclear reactions and nuclear scattering processes; that is to say, a description in terms of detailed nuclear wave functions and an effective interaction between nucleons which has some realistic basis. For example, the real part of the nucleon-nucleus interaction has been derived by folding in an effective nucleon-nucleon force with the density distribution of the target nucleus. This procedure is known as the folding model. This model has been shown to be successful in describing the real part of the nucleon-nucleus optical model potential.

One of the most interesting realistic nuclear potentials is the Skyrme-type potential.[3–8] This potential has been shown to correct the single nucleon[9] and the two-nucleon[10] transfer reactions. Also, it is shown that it improves the deuteron stripping differential cross sections as well as the polarization in the deuteron stripping reactions.[11]

In the present work, we are interested in studying the heavy ion reactions with single nucleon transfer. The aim of the present work is to extend the application of the folding model on the single nucleon transfer reactions, where the projectile nucleus is

TABLE I. Optical potential parameters.

Channel	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W_0 (MeV)	r_W (fm)	a_W (fm)	r_c (fm)
$^9Be+^{29}P$	100.000	0.989	0.700	18.030	1.198	0.786	1.250
$^{10}B+^{28}Si$	100.000	0.977	0.728	18.030	1.223	0.752	1.250
$^{12}C+^{29}Si$	112.120	1.177	0.524	10.087	1.324	0.304	1.083
$^{13}C+^{28}Si$	112.120	1.177	0.524	10.087	1.324	0.304	1.083
$^{13}C+^{32}S$	100.166	1.235	0.507	8.489	1.234	0.889	1.250
$^{14}N+^{31}P$	100.166	1.235	0.507	8.489	1.234	0.889	1.250
$^{15}N+^{30}P$	100.000	1.140	0.680	20.000	1.200	0.600	1.350
$^{15}N+^{31}P$	100.000	1.140	0.680	20.000	1.200	0.600	1.350
$^{16}O+^{29}Si$	100.000	1.140	0.680	20.000	1.200	0.600	1.350
$^{16}O+^{30}Si$	100.000	1.140	0.680	20.000	1.200	0.600	1.350
$^{17}O+^{29}Si$	60.000	1.200	0.529	15.500	1.140	0.728	1.250
$^{18}O+^{28}Si$	60.000	1.200	0.529	15.500	1.140	0.728	1.250
$^{31}P+^{28}Si$	100.000	1.203	0.500	48.76	1.203	0.500	1.250
$^{32}S+^{27}Al$	100.000	1.203	0.500	48.76	1.203	0.500	1.250

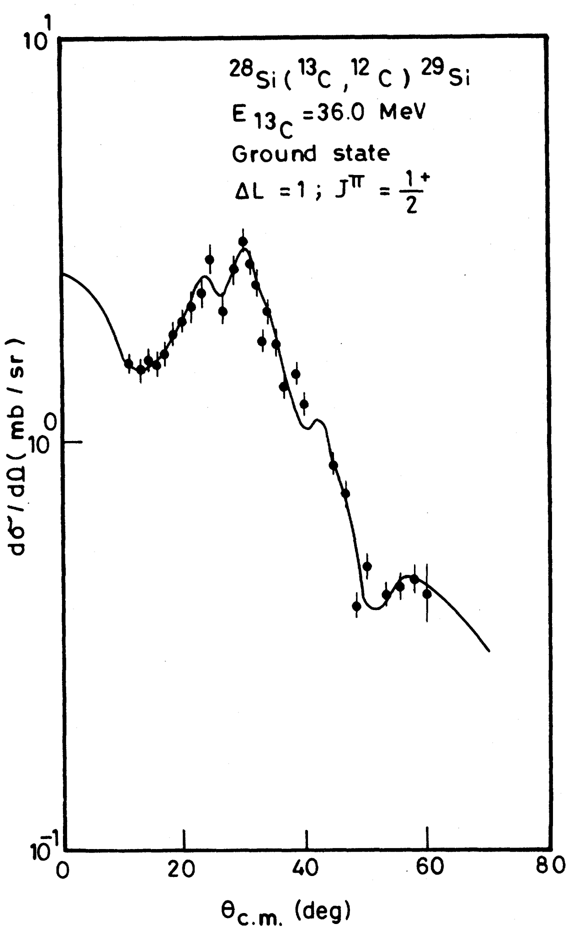

FIG. 1. The differential cross sections of the reaction $^{28}Si(^{13}C,^{12}C)^{29}Si$ at incident ^{13}C energy 36.0 MeV, leaving the residual nucleus ^{29}Si in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 12.

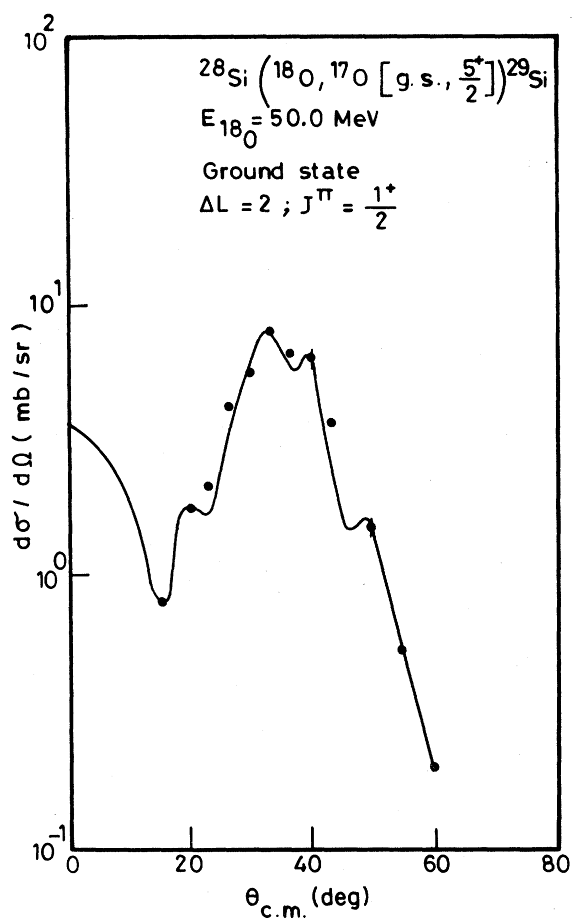

FIG. 2. The differential cross sections of the reaction $^{28}Si(^{18}O,^{17}O)^{29}Si$ at incident ^{18}O energy 50.0 MeV, leaving the residual nucleus ^{29}Si in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 13.

composed of the outgoing nucleus plus one nucleon. For this purpose, the single nucleon transfer process is considered in the framework of the distorted wave Born approximation theory. The interaction potential responsible for the transition is calculated by folding in an effective nucleon-nucleon force of a Brink-Boeker or Skyrme type with the density distribution of the outgoing nucleus. A theoretical expression is obtained for the differential cross section. The interaction processes in the initial and final channels are described by optical model potentials. The nuclear interactions of the nucleon-nucleus potentials are taken to be of the Skyrme-type potentials. The obtained expressions are applied in studying the different heavy ion single nucleon transfer reactions $^{28}Si(^{13}C,^{12}C)^{29}Si$, $^{28}Si(^{18}O,^{17}O)^{29}Si$, $^{28}Si(^{10}B,^{9}Be)^{29}P$, $^{29}Si(^{16}O,^{15}N)^{30}P$, $^{30}Si(^{16}O,^{15}N)^{31}P$, $^{27}Al(^{32}S,^{31}P)^{28}Si$, and $^{32}S(^{13}C,^{14}N)^{31}P$. The heavy ion projectile incident energies have different values between 36.0 and 100.0 MeV. Numerical calculations for the differential cross sections of these reactions are carried out using the DWBA calculations. The theoretically calculated angular distributions are compared with the experimental measurements.[12–16] From the fitting between the theoretical calculations and experimental data, spectroscopic factors are extracted.

In Sec. II, the theoretical expressions for the differential cross section are introduced. In Sec. III,

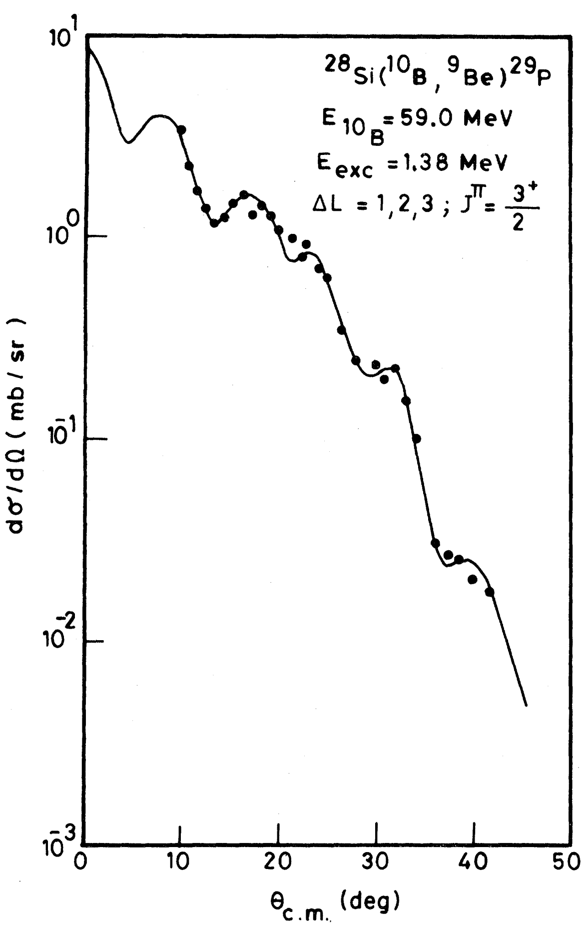

FIG. 3. The differential cross sections of the reaction $^{28}Si(^{10}B,^{9}Be)^{29}P$ at incident ^{10}B energy 59.0 MeV, leaving the ^{29}P residual nucleus with excitation energy 1.38 MeV. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 14.

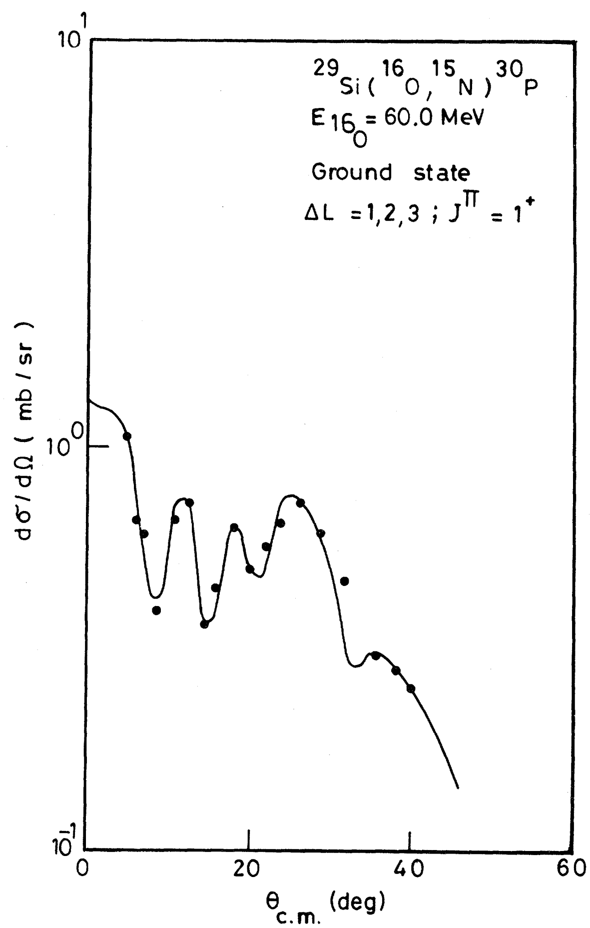

FIG. 4. The angular distributions of the reaction $^{29}Si(^{16}O,^{15}N)^{30}P$ at incident ^{16}O energy 60.0 MeV, leaving the residual nucleus ^{30}P in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 15.

numerical calculations and results are given. A discussion and conclusions are presented in Sec. IV.

II. THE DWBA AMPLITUDE WITH SKYRME-TYPE POTENTIAL

The heavy ion reactions with single nucleon transfer will be considered. These reactions are treated theoretically as direct nuclear reactions. The initial channel is the interaction between the heavy projectile A incident on the target T. The projectile A is taken to be the bound state of a nucleon N and a core C. This direct nucleon transfer reaction leads to the final channel of an interaction between the outgoing particle C and the residual heavy nucleus R. The residual nucleus R is taken to consist of a nucleon N bound to the target T. This reaction is represented as

$$A(N+C)+T\rightarrow C+R(N+T)\,, \qquad (1)$$

where N is bound to the core C in the projectile A with relative angular momentum l_i, while N is bound to T in the residual nucleus R with relative angular momentum l_f. The distorted wave Born approximation (DWBA) amplitude for this one nucleon transfer reaction in the no-recoil approximation is given as

$$T_{fi}\propto\int\chi_f^{(-)*}(\vec{x})A_{l_il_f}(\vec{x})\chi_i^{(+)}(\vec{x})d\vec{x}\,, \qquad (2)$$

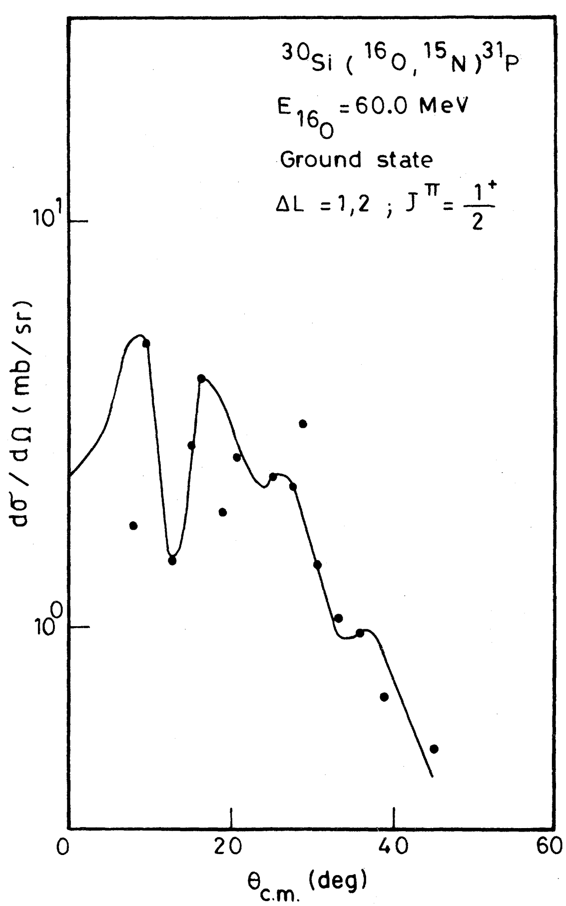

FIG. 5. The angular distributions of the reaction ^{30}Si(^{16}O,^{15}N)^{31}P at incident ^{16}O energy 60.0 MeV, leaving the residual nucleus ^{31}P in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 15.

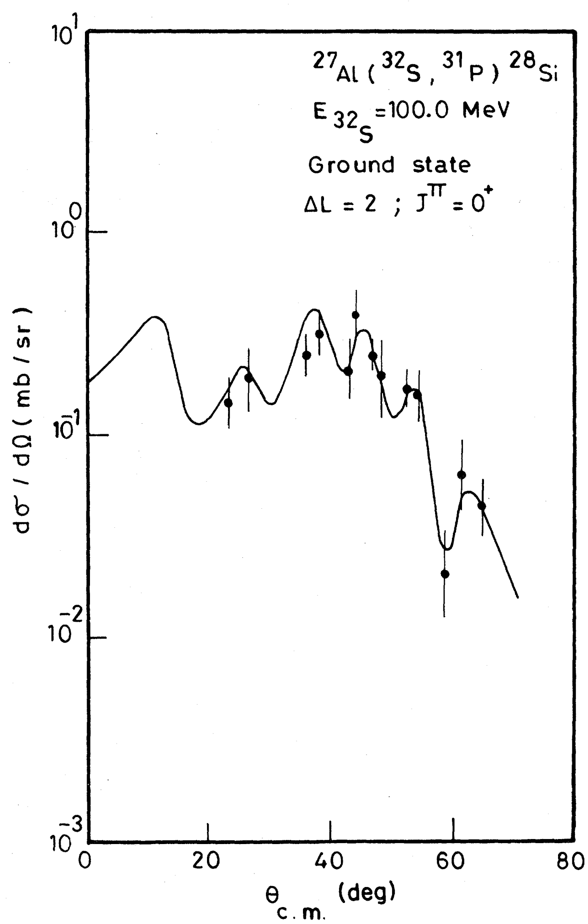

FIG. 6. The differential cross sections of the reaction ^{27}Al(^{32}S,^{31}P)^{28}Si at incident ^{32}S energy 100.0 MeV, leaving the residual nucleus ^{31}P in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 16.

where χ_i and χ_f are the distorted wave functions in the initial and final channels, respectively. The factor $A_{l_i l_f}(\vec{x})$ is given by

$$A_{l_i l_f}(\vec{x}) = \int U^{*}_{l_f m_f}(\vec{x}+\vec{x}_1) V_{NC}(x_1) \times U_{l_i m_i}(\vec{x}_1) d\vec{x}_1 , \quad (3)$$

where $U_{l_i m_i}$ and $U_{l_f m_f}$ are the bound state wave functions of the transferred nucleon in the initial and final channels, respectively. $V_{NC}(x_1)$ is the nuclear interaction potential between the transferred nucleon N and the outgoing nucleus C.

In the folding model representation we have:

$$V_{NC}(x_1) = \int d\vec{\xi} \phi_0^{*}(\vec{\xi}) \sum_{i=1}^{A_0} V_{Ni}(\vec{\xi},\vec{x}_1) \phi_0(\vec{\xi}) , \quad (4)$$

where

$$\phi_0(\vec{\xi}) = \phi_0(\xi_1, \xi_2, \ldots, \xi_{A_0}) \quad (5)$$

is the internal wave function of the outgoing nucleus C, and A_0 is its mass number. In Eq. (4), we have V_{Ni} standing for the interaction potential between the transferred nucleon (N) and the nucleon (i) of the outgoing nucleus C. Then, Eq. (4) can be written as

$$V_{NC}(x_1) = \int \rho_0(\vec{r}) V_{Ni}(\vec{r}-\vec{x}_1) d\vec{r} , \quad (6)$$

where $\rho_0(\vec{r})$ is the outgoing particle density distribution at the point $\vec{r}$.

Then, we get for the differential cross section of the heavy ion reactions with single nucleon transfer an expression

$$\frac{d\sigma}{d\Omega} = \frac{m^{*}_{AT} m^{*}_{CR}}{(2\pi\hbar^2)^2} \frac{k_f}{k_i(2I_A+1)(2I_T+1)} \sum_{\substack{\mu_A \mu_T \\ \mu_C \mu_R}} |T_{fi}|^2 . \quad (7)$$

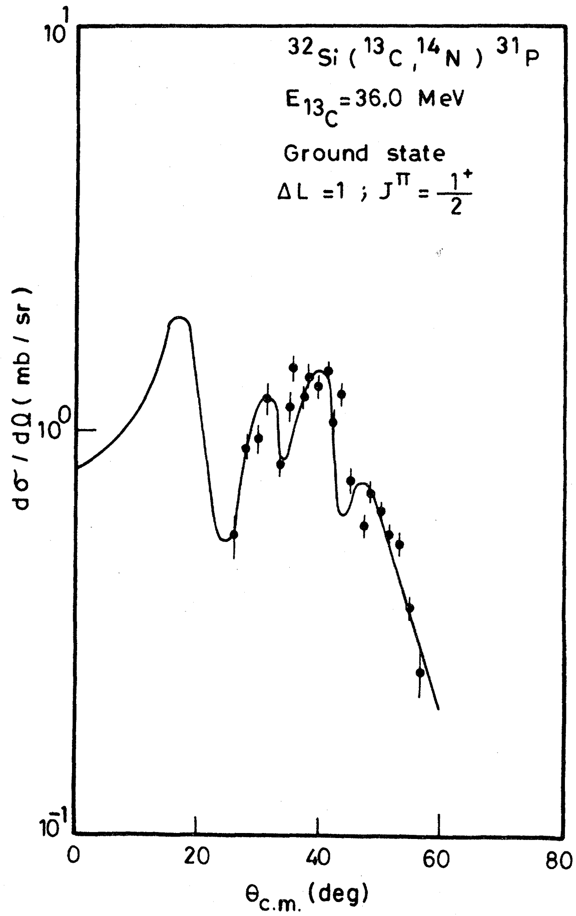

FIG. 7. The differential cross sections of the reaction ^{32}Si(^{13}C,^{14}N)^{31}P at incident ^{13}C energy 36.0 MeV, leaving the residual nucleus ^{31}P in its ground state. The solid curve is our present calculations. The optical model parameters are listed in Table I. The points are the experimental data and are taken from Ref. 12.

TABLE II. Extracted spectroscopic factors.

Reaction	Incident energy (MeV)	Excitation energy (MeV)	ΔL	J^{π}	Spectroscopic factors
^{28}Si(^{10}B,^{9}Be)^{29}P	59.0	1.38	1,2,3	$\frac{3}{2}^{+}$	0.85
^{28}Si(^{13}C,^{12}C)^{29}Si	36.0	0.00	1	$\frac{1}{2}^{+}$	0.83
^{32}S(^{13}C,^{14}N)^{31}P	36.0	0.00	1	$\frac{1}{2}^{+}$	0.88
^{29}Si(^{16}O,^{15}N)^{30}P	60.0	0.00	1,2,3	1^{+}	0.80
^{30}Si(^{16}O,^{15}N)^{31}P	60.0	0.00	1,2	$\frac{1}{2}^{+}$	0.87
^{28}Si(^{18}O,^{17}O)^{29}Si	50.0	0.00	2	$\frac{1}{2}^{+}$	0.84
^{27}Al(^{32}S,^{31}P)^{28}Si	100.0	0.00	2	0^{+}	0.80

III. NUMERICAL CALCULATIONS AND RESULTS

In the present work, heavy ion reactions with single nucleon transfer have been studied. Theoretical investigations of these reactions have been introduced. The distorted wave Born approximation is used in developing a theoretical expression for the differential cross sections. In the distorted wave Born approximation (DWBA) calculations, optical model potentials have been used to describe the heavy ion interactions in the initial and final channels. A Woods-Saxon form has been used for the optical model potentials. The Woods-Saxon potential form used is

$$V_{\rm opt}(r)=V_0(1+e^{(r-r_0/a_0)})^{-1}+iW_0(1+e^{(r-r_W/a_W)})^{-1}+V_c\ . \quad (8)$$

The different parameters of the optical model potential given by Eq. (8) are changed and adjusted such that the theoretically calculated elastic scattering cross section of the corresponding heavy ions fit the corresponding experimental measurements. These obtained values of the different parameters are then fixed and used in the calculations of the differential cross sections of the transfer reactions. Also, for the calculations using the present form of the Skyrme-type potential for the nucleon-nucleus interactions, we need the outgoing particle density distribution $\rho_0(r)$. In the present work, $V_{NC}(x_1)$ is calculated using Eq. (6) with $\rho_0(r)$ given by

$$\rho_0(r)=\rho_0'\left[1+\frac{wr^2}{\alpha^2}\right]e^{-(r/\alpha)^2}\ , \quad (9)$$

where

$$\rho_0'=A_0\Big/\left[\pi^{3/2}\alpha^3\left[1+\frac{3w}{2}\right]\right]\ . \quad (10)$$

The parameters w and α are chosen such that they fit the density distribution deduced from electron scattering. The wave functions U_{l_i} and U_{l_f} appearing in Eq. (3) are assumed to be oscillator model wave functions.

The present expressions are applied and numerical calculations are carried out for different heavy ion reactions with single nucleon transfer. The neutron transfer reaction ^{28}Si(^{13}C,^{12}C)^{29}Si is studied for ^{13}C incident energy 36.0 MeV leaving the ^{29}Si residual nucleus in its ground state. The differential cross sections of the single neutron transfer heavy ion reaction ^{28}Si(^{18}O,^{17}O)^{29}Si are calculated for ^{18}O incident energy 50.0 MeV leaving ^{29}Si in its ground state. Using ^{10}B projectiles with incident energy 59.0 MeV, the angular distributions of the reaction ^{28}Si(^{10}B,^{9}Be)^{29}P leaving the ^{29}P residual nucleus with excitation energy 1.38 MeV are calculated. The ^{16}O projectiles are used with incident energy 60.0 MeV in studying the single proton transfer reactions ^{29}Si(^{16}O,^{15}N)^{30}P and ^{30}Si(^{16}O,^{15}N)^{31}P leaving the residual nuclei ^{30}P and ^{31}P in their ground states. The 100.0 MeV ^{32}S projectiles are used in the reaction ^{27}Al(^{32}S,^{31}P)^{28}Si leaving the ^{28}Si nucleus in the ground state. Also, the proton pickup reaction ^{32}S(^{13}C,^{14}N)^{31}P is studied with ^{13}C projectile of energy 36.0 MeV and for the ground state of the residual nucleus ^{31}P. The different parameters of the optical model potential for the different reactions are given in Table I. The calculated differential cross sections are shown in Figs. 1–7. The present theoretical calculations are shown by the solid curves. The points are the experimental data. The present theoretically calculated angular distributions are compared with the experimental measurements. From these comparisons, the spectroscopic factors are extracted. The obtained values of the spectroscopic factors are given in Table II.

IV. DISCUSSION AND CONCLUSIONS

In the present work, we studied different heavy ion reactions with single nucleon transfer. The present study is introduced by using the distorted wave Born approximation. The DWBA calculations are carried out by using the optical model potentials. For the nuclear interactions of the nucleon-nucleus potentials, Skyrme-type potentials are used. From Figs. 1–7, we see that the present calculations produce the shapes and the oscillatory pattern of the angular distributions. Also, the theoretical calculations fit the right positions of the peaks, the maxima and the minima of the differential cross sections. Also, the obtained values of the spectroscopic factors are reasonable, as is clear from Table II.

ACKNOWLEDGMENTS

One of us (A.O.) is very thankful and grateful to Professor Abdus Salam and Professor Paolo Budinich as well as to the International Atomic Energy Agency and UNESCO for hospitality at the International Centre For Theoretical Physics, Trieste, Italy, where the numerical calculations were done. He also would like to thank the Centro di Calcolo dell'Università di Trieste, Italy, for the use of the facilities.

[1]N. K. Glendenning, Rev. Mod. Phys. 47, 659 (1975).
[2]A. Osman, Nuovo Cimento 25A, 571 (1975).
[3]J. W. Negele, Phys. Rev. C 1, 1260 (1970).
[4]D. Vautherin and D. M. Brink, Phys. Rev. C 5, 626 (1972).
[5]J. W. Negele and D. Vautherin, Phys. Rev. C 5, 1472 (1972).
[6]J. W. Ehlers and S. A. Moszkowski, Phys. Rev. C 6, 217 (1972).
[7]C. B. Dover and N. Van Giai, Nucl. Phys. A190, 373 (1972).
[8]R. W. Manweiller, Nucl. Phys. A240, 373 (1975).
[9]A. Osman and M. Y. Ismail, Atomkernenergie 37, 217 (1981).
[10]A. Osman and M. Y. Ismail, Ann. Phys. (Leipzig) 37, 340 (1980).
[11]A. Osman, M. Y. Ismail, and M. M. Osman, J. Phys. G 7, 347 (1981).
[12]G. D. Westfall and S. A. A. Zaidi, Phys. Rev. C 14, 610 (1976).
[13]J. F. Petersen, D. Dehnhard, and B. F. Bayman, Phys. Rev. C 15, 1719 (1977).
[14]W. Treu, C. W. Towsley, K. G. Nair, and H. Vogt, Phys. Rev. C 19, 96 (1979).
[15]J. V. Maher, J. C. Peng, D. A. Sink, C. M. Cheng, H. S. Song, and T. J. Lewis, Phys. Rev. C 14, 2174 (1976).
[16]J. D. Garrett, H. E. Wegner, T. M. Cormier, E. R. Cosman, O. Hansen, and A. J. Lazzarini, Phys. Rev. C 12, 489 (1975).