Experimental evidence for dinuclear structure of $^{16}O+^{12}C$ resonances in the Coulomb barrier region

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The total reaction cross section σ_R for $^{16}O+^{12}C$ has been determined between 8.5 and 14 MeV (c.m.) from elastic data using the optical theorem. It exhibits the well-known $^{16}O+^{12}C$ resonances as pronounced structures. Elastic partial widths have been deduced from σ_R for all resonances. Some of them represent a large fraction of the single particle width. This is a strong hint that the corresponding resonances have the structure of a dinuclear molecule.

[NUCLEAR REACTIONS $^{12}C(^{16}O,^{16}O)$, $^{12}C(^{16}O,^{12}C)$, $E=8.5-15$ MeV (c.m.), $\Delta E=85$ keV; measured $\sigma(\theta,E)$, deduced total reaction cross section and quasimolecular resonances and their elastic widths.]

$^{16}O + ^{12}C$ is one of the heavy ion systems where quasimolecular resonances show up most clearly. Nevertheless there are still controversies about whether certain structures observed experimentally are due to true resonance states in the composite $A=28$ system or merely statistical fluctuations. Besides this, very little experimental information exists concerning the structural configuration of the true resonances. In this work we have tried to unambiguously determine $^{16}O + ^{12}C$ resonances in the vicinity of the Coulomb barrier. Moreover, we have investigated whether or not these resonances have the structure of a dinuclear molecule, as has been supposed for a long time.

An unambiguous determination of resonances from measured excitation functions as well as experimental investigations concerning the structure of resonance states suffer from the fact that the total width Γ of the true $^{16}O + ^{12}C$ resonances (see Table I) is comparable to the average width Γ_{cn} ($\Gamma_{cn}\approx 150-200$ keV; see Refs. 1 and 2) of normal compound nuclear states. The latter are populated by the $^{16}O + ^{12}C$ reaction at an excitation energy in ^{28}Si where many of these states (with the same angular momentum) overlap strongly. As a consequence, one has to choose an experimental energy resolution $\delta E < \Gamma_{cn}, \Gamma$ in order to reveal true $^{16}O + ^{12}C$ resonances. This results in the well-known problem that one has to disentangle true resonances from compound nuclear states in excitation functions measured for individual exit channels.

Moreover, it is rather difficult—if not

TABLE I. $^{16}O + ^{12}C$ quasimolecular resonances observed in the total reaction cross section together with resonance parameters. The J^π assignments are from Refs. 18–21. The values for σ_R^{res} and Γ_{el} are estimated to be correct within 20% and 20–30 %, respectively. The fourth column gives the partial reaction cross section $\sigma_u^J = \pi\lambda^2\cdot(2J+1)$ calculated in the unitary limit.

E_{res} (MeV)	J^π	σ_R^{res} (mb)	σ_u^J (mb)	Γ (keV)	Γ_{el} (keV)	$\frac{\gamma_{el}^2}{\gamma_w^2}$ (%)
8.86	$5^-(3^-)$	73	119	130	25	15
9.26	6^+	91	134	240	52	33
10.71	8^+	72	152	220	30	18
11.11	9^-	89	164	320	52	38
11.85	8^+	66	137	290	40	16
12.79	7^-	77	112	170	37	4.0
13.07	7^-	82	110	95	24	2.3
13.70	8^+	116	119	260	110	11

impossible—to determine partial widths Γ_i from excitation functions of individual exit channels in order to get structural information for the true resonances. This is due to the existence of interference effects between true resonances and compound nuclear states.

It is worthwhile to mention that one cannot separate the two resonance species experimentally just by using energy averaged cross sections, a procedure one would use if Γ_{cn} were significantly smaller than Γ, nor can one describe the experimental data using an energy averaged S matrix. If one analyzes, however, the excitation function of the total reaction cross section σ_R one can—in certain cases—overcome the problems mentioned above. Structures in this excitation function are most likely of a nonstatistical origin. Moreover, σ_R does not contain interference effects between true resonances and compound nuclear states if both species of states mix weakly or not at all, as will be shown in the following. Thus, the total reaction cross section offers a unique possibility to pin down true resonances and to deduce elastic partial widths Γ_{el} which are needed in order to decide whether or not the true resonances have the structure of a dinuclear molecule.

The total reaction cross section σ_R for transitions from an initial state a with total angular momentum J to all possible final states $c \neq a$ can be written (using standard notation) as follows:

$$\sigma_R = \pi \lambda\!\!\!^{-2} \sum_{c \neq a} \sum_{l=J,l',s'} (2l+1) \left| S_{ac}^J \right|^2 \tag{1}$$

where S_{ac}^J are the off-diagonal elements of the scattering matrix which can be written as

$$S_{ac}^J = -ie^{i(\delta_a^J + \delta_c^J)} \left\{ \sum_{\lambda \neq \rho} \frac{g_{\lambda a}^J \cdot g_{\lambda c}^J}{E - E_\lambda + \frac{1}{2} i \Gamma_\lambda^J} + \frac{g_{\rho a}^J \cdot g_{\rho c}^J}{E - E_\rho + \frac{1}{2} i \Gamma_\rho^J} \right\} . \tag{2}$$

The index ρ labels a true resonance state $|\rho\rangle$ embedded in a continuum of normal compound nuclear states $|\lambda\rangle$ labeled λ in Eq. (2).

The ansatz made in Eq. (2) uses the assumption that true resonance states $|\rho\rangle$ are eigenstates of the composite system rather than doorway states. The states $|\rho\rangle$ are then orthogonal to the normal compound nuclear states $|\lambda\rangle$ and do not mix with these, as already expected from the structural mismatch between both species of states. In fact, it will be shown as a result of this paper that indeed most of the states $|\rho\rangle$ investigated have the very simple structure of a dinuclear molecule compared to the much more complicated structures of the compound states $|\lambda\rangle$ (other simple states with widths larger than Γ_{cn} do not exist for the reaction $^{16}O + {}^{12}C$ in the energy range considered, as can be shown by averaging the measured total reaction cross section). A further assumption used in Eq. (2) is that direct contributions are negligibly small. Statistical analyses indeed show that these contributions are small for the $^{16}O + {}^{12}C$ reaction[1,3] (as well as for the $^{12}C + {}^{12}C$ reaction[4]) at energies in the vicinity of the Coulomb barrier.

With Eq. (2) one obtains an expression for σ_R which contains, besides two quadratic terms, the following interference terms:

$$\pi \lambda\!\!\!^{-2} \sum_{l=J,l',s'} (2l+1) \left\{ 2\,\mathrm{Re} \sum_{\lambda \neq \lambda'} \frac{g_{\lambda a}^J \cdot g_{\lambda' a}^{J*} \sum_c g_{\lambda c}^J \cdot g_{\lambda' c}^{J*}}{(E - E_\lambda + \frac{1}{2} i \Gamma_\lambda^J)(E - E_{\lambda'} - \frac{1}{2} i \Gamma_{\lambda'}^J)} + 2\,\mathrm{Re} \sum_\lambda \frac{g_{\rho a}^J \cdot g_{\lambda a}^{J*} \sum_c g_{\rho c}^J \cdot g_{\lambda c}^{J*}}{(E - E_\rho + \frac{1}{2} i \Gamma_\rho^J)(E - E_\lambda - \frac{1}{2} i \Gamma_\lambda^J)} \right\} .$$

The quantities $g_{\lambda c}^J$ and $g_{\lambda' c}^J$ are normally distributed (with respect to λ) with mean value zero. They are, however, not *a priori* statistically independent in the case of overlapping states, due to the unitarity of the scattering matrix. The mutual dependence decreases, however, with an increasing number N_c of channels c into which the states $|\lambda\rangle$, $|\lambda'\rangle$ decay. If this number is large, as in the present case, $g_{\lambda c}^J$ and $g_{\lambda' c}^J$ can be considered to be independent with respect to c. This means that products like $g_{\lambda c}^J \cdot g_{\lambda' c}^J$ are also independent variables with mean value 0. Then $\sum_c g_{\lambda c}^J \cdot g_{\lambda' c}^{J*}$ is normally distributed with mean value zero if N_c is a large number. The same holds for $\sum_c g_{\rho c}^J \cdot g_{\lambda c}^{J*}$. It should be noted that the quantities $g_{\rho c}^J$ do not have a statistical distribution since they belong to the special states $|\rho\rangle$ which are

assumed not to mix with the states $|\lambda\rangle$.

Thus, the above interference terms vanish and one is left with the following expression for σ_R:

$$\sigma_R=\pi\lambda\!\!\!^{-2}\left\{\sum_{l=J,l',s'}(2l+1)\sum_{\lambda\neq\rho}\frac{\Gamma^J_{\lambda a}\cdot\Gamma^J_{\lambda\,\mathrm{reac}}}{(E-E_\lambda)^2+(\frac{1}{2}\Gamma^J_\lambda)^2}\right.$$
$$\left.+(2J+1)\frac{\Gamma^J_{\rho a}\cdot\Gamma^J_{\rho\,\mathrm{reac}}}{(E-E_\rho)^2+(\frac{1}{2}\Gamma^J_\rho)^2}\right\}, \quad (3)$$

where

$$\Gamma^J_{\lambda\,\mathrm{reac}}=\sum_{c\neq a}\Gamma^J_{\lambda c}$$

and

$$\Gamma^J_{\rho\,\mathrm{reac}}=\sum_{c\neq a}\Gamma^J_{\rho c}\ .$$

It can be assumed that the first term in Eq. (3) has a smooth energy dependence if the number of states $|\lambda\rangle$ within δE is large. This is the case for the $^{16}O+{}^{12}C$ reaction in the energy range studied.[5,6] The important consequence of these considerations is that the total reaction cross section σ_R decomposes into a background term and a resonance term σ_R^{res} and that therefore the determination of $\Gamma^J_{\rho a}=\Gamma_{\mathrm{el}}$ can be readily achieved using Eq. (4):

$$\sigma_R^{\mathrm{res}}=4\pi\lambda\!\!\!^{-2}(2J+1)\frac{\Gamma_{\mathrm{el}}(\Gamma-\Gamma_{\mathrm{el}})}{\Gamma^2}\ , \quad (4)$$

where $\Gamma=\Gamma^J_\rho$.

We have deduced the $^{16}O+{}^{12}C$ total reaction cross section σ_R in the vicinity of the Coulomb barrier with the aim of determining elastic partial widths for $^{16}O+{}^{12}C$ resonances. The method used to get σ_R is that proposed by Holdeman and Thaler[7] (the sum-of-differences method), who have shown that σ_R can be obtained from carefully measured elastic data. Starting from the optical theorem the authors of Ref. 7 show that σ_R can be expressed in terms of the measured elastic cross section $\sigma_{\mathrm{el}}(\theta)$ in the following way:

$$\sigma_R=A+2\pi\int_{\theta_0}^{\pi}(\sigma_C(\theta)-\sigma_{\mathrm{el}}(\theta))\sin\theta\,d\theta\ , \quad (5)$$

where $\sigma_C(\theta)$ is the Coulomb cross section and θ_0 is an angle which should be equal to or smaller than the angle θ_C for which $\sigma_{\mathrm{el}}(\theta)$ starts to deviate from $\sigma_C(\theta)$. The first term on the right hand side of Eq. (5) depends on the nuclear scattering amplitude $f_N(0)$ at zero degree. It can be dropped in the case of heavy ion reactions with moderate absorption and large values of the Sommerfeld parameter due to the fact that the absolute value of $f_N(0)$ is small in this case.[8–10] It is obvious that the application of Eq. (5) for the determination of σ_R is limited to small energies due to the condition $\theta_0\leq\theta_C$.

We have measured angular distributions consisting of 45 angles equally spaced between $\theta_{\mathrm{lab}}=6°$ and 50° every 85 keV in the energy range 8.5–15 MeV. Each angular distribution was measured simultaneously using a multidetector array in which thin Si detectors were positioned on both sides of the beam axis both below and above the horizontal plane. The most forward angles were used to correct the data for the effect of small changes in the beam spot position and to get an absolute normalization of the data.

Figure 1 shows angular distributions measured. The arrows mark the cutoff angle θ_0 used to evaluate σ_R according to Eq. (5). The choice of θ_0 is a

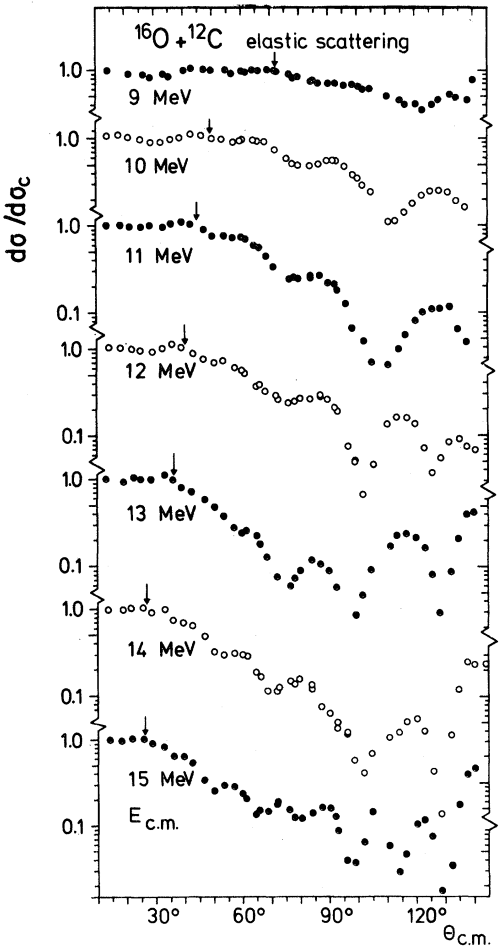

FIG. 1. Angular distributions of the elastic scattering of ^{16}O on ^{12}C. The arrows mark the cutoff angle θ_0 [see Eq. (5)].

delicate problem if the scattering data oscillate about the Coulomb cross section at forward angles as in the $^{16}O + ^{12}C$ case. It is then almost impossible to choose θ_0 according to the prescription given above, since the oscillations start at extremely small angles. It has been shown, however, that only a relatively small error is made if θ_0 is chosen within or at the limit of the oscillations.[15] We have studied this point for the $^{16}O + ^{12}C$ case, i.e., we have investigated the influence of different θ_0 values on the absolute value of σ_R. For this purpose σ_R was calculated according to Eq. (5) with different θ_0 values from the experimental data and from optical model angular distributions obtained as a best fit to the experimental data. θ_0 was varied between θ_C (the angle for which the final breakoff from the Coulomb cross section occurs) and the smallest angle possible (0° for optical model angular distributions). Analysis of the optical model angular distributions showed that one obtains

$$\sigma_R^{\mathrm{OM}} = \pi\lambda^2 \sum_l (2l+1)T_l$$

if $\theta_0 \approx 0°$ and σ_R values which are consistently 10% to 20% higher than σ_R^{OM} if $\theta_0 = \theta_C$. θ_0 values in between give cross sections between the above limits. Similar results were obtained for the analysis of the experimental data: $\theta_0 = \theta_C$ gives cross sections which are roughly 10% higher than those obtained for $\theta_0 < \theta_C$ in the energy range up to 11 MeV and roughly 20% higher for higher energies.

The cross sections shown in Fig. 2 are calculated with $\theta_0 = \theta_C$. This choice is the best possibility to define θ_0 in a clearcut way consistently over the whole energy range. It follows, however, that the actual values of σ_R might be consistently up to 10% smaller for energies below 11 MeV and up to 20% smaller for higher energies.

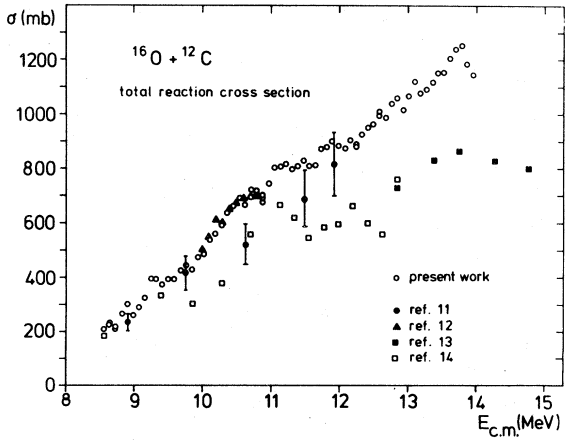

FIG. 2. Total reaction cross section for the reaction $^{16}O + ^{12}C$ obtained from elastic data together with fusion cross sections (Refs. 13 and 14) and cross sections obtained from γ-yield measurements (Refs. 11 and 12).

The limited angular range investigated ($\theta_{\max} = 140°$ instead of 180°) is an additional source of error. In general one obtains σ_R values which are smaller than or equal to the actual σ_R value [equality holds for the case that $\sigma_{\mathrm{el}}(\theta) = \sigma_C(\theta)$ for all $\theta > \theta_{\max}$]. The maximum missing cross section $\Delta\sigma_R^{\max}$ is given by

$$\Delta\sigma_R^{\max} = 2\pi \int_{\theta_{\max}}^{\pi} \sigma_C(\theta)\sin\theta\, d\theta \, .$$

It drops from 60 mb at 9 MeV to 20 mb at 15 MeV. The actual missing cross section was estimated to be roughly one third of $\Delta\sigma_R^{\max}$ using angular distributions which were measured up to very backward angles.[16,17] This error could be approximately accounted for using $\theta_{\max} = 180°$ and assuming $\sigma_{\mathrm{el}}(\theta) = \sigma_{\mathrm{el}}(140°)$ for $140° < \theta \leq 160°$ and $\sigma_{\mathrm{el}}(\theta) = \sigma_c(\theta)$ for $\theta > 160°$ in our analysis.

Figure 2 also contains fusion cross sections and reaction cross sections obtained from γ-yield measurements. Agreement with the latter data is rather satisfactory as far as absolute values for σ_R are concerned. Unfortunately this comparison is not possible for higher energies since total reaction cross section data are missing for these energies.

It should be mentioned that the absolute values for σ_R are ill defined for energies larger than 14 MeV. This is due to the fact that the corresponding angular distributions do not contain enough angles smaller than θ_0 (see Fig. 1) in order to normalize these distributions as mentioned above (one should have at least four angles less than θ_0; up and down from and left and right of the beam direction). Therefore these σ_R values have been eliminated in the following.

Figure 3 also shows the total reaction cross section for the reaction $^{16}O + ^{12}C$. It is obvious from this figure that σ_R exhibits pronounced structures superimposed on a smooth background. It should be noted that these structures are to a large extent independent of the choice of the cutoff angle θ_0 as long as $\theta_0 \leq \theta_C$. We observe deviations of the resonance structures from the average cross section $\langle \sigma_R \rangle$ (averaged over 1 MeV) of the order of 10% to 40%. The probabilities for these deviations being statistical fluctuations have been calculated using N_{eff} values (the number of effective channels) obtained from Hauser-Feshbach calculations (parameters of Ref. 1). The calculated probabilities are smaller than 10^{-4} in all cases. We therefore conclude that these structures are of a nonstatistical origin. They are true resonances of the $A = 28$ system. In fact, all resonances found in the total reaction

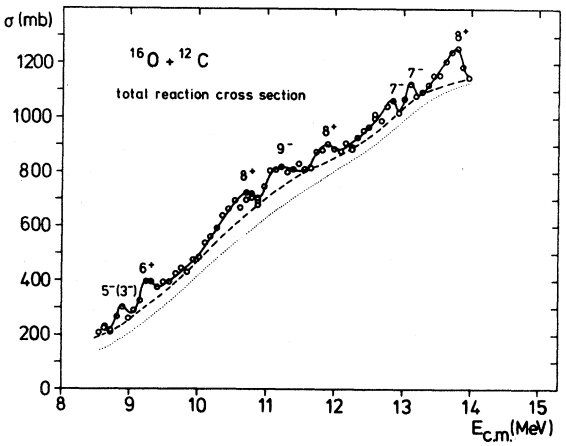

FIG. 3. $^{16}O + ^{12}C$ total reaction cross section. The J^{π} values are from Refs. 18–21. The smooth lines (dashed, dotted) represent assumed background cross sections (see text). The solid line is drawn to guide the eye.

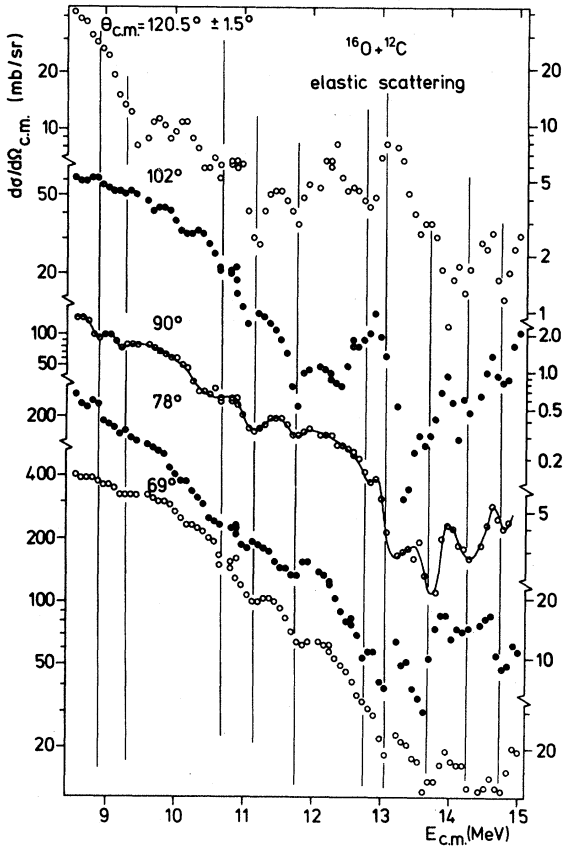

FIG. 4. Excitation functions of the elastic scattering of ^{16}O on ^{12}C measured for different angles. The 90° data points are connected by a solid line. The vertical lines mark the position of the $^{16}O + ^{12}C$ resonances.

cross section have already been observed as anomalies in other investigations. The fact that they show up in the total reaction cross section is an ultimate proof that they are true resonances. Table I lists the resonances observed in this investigation. It is interesting to note that all resonances can already be found as pronounced minima in the 90° elastic excitation function (see Fig. 4).

From the measured total reaction cross section we finally deduced elastic partial widths Γ_{el} according to Eq. (4) for the $^{16}O + ^{12}C$ resonances below 14 MeV. To do that one has to know the background cross section. Instead of using an optical model prediction we preferred to choose the smooth background contribution shown in Fig. 3 (dashed line). It represents a hand-drawn line and gives the smallest Γ_{el} values, among other possible choices for background contributions. From the two solutions of Eq. (4) the smaller one was chosen. The values obtained for Γ_{el} are given in Table I. They are estimated to be accurate to 20–30 % as a result of calculations using different absolute values for σ_R (within the estimated limits) and different background contributions (for instance, the dotted line shown in Fig. 3).

The elastic widths have been converted to reduced widths γ_{el}^2 using $r_0 = 1.5$ fm as the radius parameter. They are compared in Table I with single particle widths γ_w^2 calculated in the Wigner limit. It turns out that the reduced elastic widths represent in all cases—with the exception of the two $J^{\pi} = 7^-$ resonances at 12.79 and 13.07 MeV—a large fraction of the corresponding single particle width even if one takes the lowest limit given by the 30% uncertainty of Γ_{el}. It is of course obvious that the elastic width of a resonance populated by the $^{16}O + ^{12}C$ entrance channel cannot be very small. The values obtained in this investigation are, however, particularly large. In fact, we think that such large values are direct evidence that these resonances have the structure of a dinuclear molecule.

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