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Separable representation of the nuclear proton-proton interaction

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We propose a separable representation of the nuclear part of the proton-proton interaction in angular-momentum states $l=0$, 1, and 2. This representation yields an accurate fit to all experimental data currently accepted for elastic proton-proton scattering. The form factors of the separable potentials are represented by simple functions in momentum space (and also in configuration space). This makes it possible to treat the Coulomb distortion of the nuclear interaction exactly and to derive the corresponding proton-proton transition matrix in closed analytic form. Our model thus provides an adequate and convenient starting point for rigorous calculations on few-body systems with charges.

[NUCLEAR REACTIONS Separable potential proposed for p-p system; Coulomb distortion of nuclear interaction treated exactly; closed analytic formulas for transition matrix given.]

I. INTRODUCTION

Experimentally the proton-proton (p-p) system has been studied extensively. This has led to very accurate phenomenological data. The analysis of these data has produced reliable phase shifts, which have undergone only minor changes during the last decade or so. Indeed, predictions of the latest phase-shift analyses do not very much differ from what was obtained already in 1969 by MacGregor *et al.*[1] or Seamon *et al.*,[2] despite the fact that the data base has been augmented considerably.[3] Only the methods employed in the newer phase-shift analyses became more refined in one way or another. Still the results by different groups, such as Arndt *et al.*,[4] Bystricky *et al.*,[5] Bugg *et al.*,[6] and Arndt and VerWest,[7] are very similar for all p-p partial waves. Therefore, we can now safely rely on well-determined phenomenological p-p phase parameters.

A principal difficulty in the theoretical treatment of p-p scattering is the long range of the Coulomb interaction. This, in particular, causes the off-shell p-p T-matrix elements (as a function of the off-shell momentum variables) to have branch-point singularities at the on-shell point. But the *physical* on-shell amplitude can be expressed by matrix elements of the T operator between suitably defined Coulombian asymptotic states.[8]

If the short-range (purely nuclear) part of the p-p interaction is described by separable potentials with rational form factors, the T-matrix elements can be found in closed form.[9–13] The problem was previously also solved by numerical treatment[14] or by invoking approximations.[15,16] Most of the early separable models give only poor fits to experimental data. An exception is the Graz separable potential,[16] which reproduces p-p observables more or less satisfactorily.[17] In the original work by the Graz group[16] the Coulomb distortion was treated approximately (only to the first order in the fine-structure constant). Subsequently the physical on-shell amplitude was calculated exactly by numerical means.[14] Recently analytical expressions of the various Coulomb-modified transition-matrix elements for the Graz separable potential were also derived.[18]

The original Graz potential falls short in reproducing the $^1S_0$ effective-range parameters and in fitting the P- and D-wave data. The purpose of the present work is to provide a new separable potential which meets the demand of a rigorous and precise description of the p-p interaction in all partial waves with $l \leq 2$. The resulting model is an appropriate

starting point for calculations on few-body systems, which rely on an integral-equation approach.

In Sec. II we recall some formulas necessary to calculate Coulomb-modified nuclear phase shifts and low-energy parameters. In Sec. III we present our new separable potential. There we also give analytical expressions needed for the on-shell T matrix and we compute phase shifts as well as effective-range parameters. Results are compared to the latest p-p phase-shift predictions by Arndt and VerWest.[7] Under the assumption of charge symmetry we give an outlook on the neutron-neutron (n-n) system. Section IV concludes the paper with a short discussion.

II. FORMALISM

A. Coulomb-modified nuclear scattering amplitude and phase shifts

The total potential $V=V_c+V_s$ is the sum of the Coulomb potential V_c and the short-range nuclear potential V_s. According to the Gell-Mann–Goldberger two-potential formalism,[19] the total T operator can be written as $T=T_c+T_{cs}$, where T_c is the Coulomb T operator and T_{cs} is then defined by

$$T_{cs}=(1+T_cG_0)t_{cs}(1+G_0T_c)\ . \tag{2.1}$$

The operator t_{cs} satisfies a Lippmann-Schwinger equation with short-range driving terms,

$$t_{cs}=V_s+V_sG_ct_{cs}\ . \tag{2.2}$$

The resolvents G_0 and G_c correspond to the free and the pure Coulomb Hamiltonians, respectively. The structure of the above equations remains the same after partial-wave (pw) decomposition.

If V_{sl} is an n-term separable potential acting in the pw space l,

$$V_{sl}=\sum_{i=1}^{n}|g_{li}\rangle\lambda_{li}\langle g_{li}|\ , \tag{2.3}$$

Eq. (2.2) can be solved algebraically to yield

$$T_{csl}=\sum_{i,j=1}^{n}|g_{li}^c\rangle\tau_{lij}^c\langle g_{lj}^c|\ . \tag{2.4}$$

The $n\times n$ matrix τ_l^c is defined via its inverse,

$$(\tau_l^{c\,-1})_{ij}=(\Lambda_l^{-1})_{ij}-\langle g_{li}|G_{cl}|g_{lj}\rangle\ , \tag{2.5}$$

where Λ_l is the diagonal matrix with elements $\lambda_{li}\delta_{ij}$. The Coulomb-modified form factors $|g_{li}^c\rangle$ are given by

$$|g_{li}^c\rangle=(1+T_{cl}G_{0l})|g_{li}\rangle\ . \tag{2.6}$$

In order to calculate Coulomb-modified nuclear phase shifts δ_{csl} one has to know the physical on-shell matrix elements of T_{csl}. They can be obtained from Eq. (2.4) by sandwiching between Coulombian asymptotic states $|kl\infty\pm\rangle$, which are related to the Coulomb scattering states $|kl\pm\rangle_c$ via[8]

$$|kl\pm\rangle_c=|kl\infty\pm\rangle+G_{0l}(k^2\pm i0)T_{cl}(k^2\pm i0)|kl\infty\pm\rangle=|kl\infty\pm\rangle+G_{0l}(k^2\pm i0)V_{cl}|kl\pm\rangle_c\ . \tag{2.7}$$

Then one has

$$\langle kl\infty-|T_{csl}(k^2+i0)|kl\infty+\rangle=-\frac{\hbar^2}{\pi\mu}f_{csl}(k)\ , \tag{2.8}$$

where $f_{csl}(k)$ is the Coulomb-modified nuclear scattering amplitude. The relation to the phase shifts δ_{csl} reads

$$\cot\delta_{csl}(k)-i=-\exp[2i\sigma_l(k)]\frac{\hbar^2}{\pi\mu k}\Big/\langle kl\infty-|T_{csl}(k^2+i0)|kl\infty+\rangle \tag{2.9}$$

with σ_l representing the pure Coulomb phase shift.

In Eqs. (2.1)–(2.6) we suppressed the dependence on the (generally complex) energy variable E in the T operators, the resolvents, the propagators τ^c, and the Coulomb-modified form factors $|g^c\rangle$. In Eqs. (2.7)–(2.9) we used the shorthand notation $k^2\pm i0$ for the energy $\hbar^2k^2/2\mu\pm i0$, where μ is the reduced mass. Further details about the formalism can be found in Refs. 8, 11–13, and 20. We remark that the sign conventions for λ and τ may differ in different papers.

B. Effective-range expansion

For low-energy scattering by a short-range potential V_s the effective-range (ER) function

$$K_{sl}(k^2)=k^{2l+1}\cot\delta_{sl}(k) \tag{2.10}$$

plays an important role. Under certain conditions on the potential V_s this ER function is real analytic at $k^2=0$,

$$K_{sl}(k^2)=-1/a_{sl}+\tfrac{1}{2}r_{sl}k^2+\cdots\ , \tag{2.11}$$

where the scattering length a_{sl} and the effective range r_{sl} are real.[21]

If the potential is $V=V_c+V_s$, the ER function has to be modified and may be taken as

$$K_{csl}(k^2)=k^{2l+1}{C_{l\gamma}}^{-1} \times\{2\gamma H(\gamma)+{C_0}^2[\cot\delta_{csl}(k)-i]\}, \tag{2.12}$$

where $H(\gamma)$ is related to the digamma function ψ,

$$H(\gamma)=\psi(i\gamma)+(2i\gamma)^{-1} -\ln[-i\gamma\,\mathrm{sgn}(s)]\ . \tag{2.13}$$

In our notation the Coulomb potential is $V_c(r)=-2s/r$, so that $\gamma=-2\mu s/(\hbar^2 k)$ (note that s is negative for Coulomb repulsion). The coefficients C are defined by

$${C_0}^2=\frac{2\pi\gamma}{e^{2\pi\gamma}-1}\ , \tag{2.14a}$$

$${C_{l\gamma}}^{-1}=\prod_{n=1}^{l}\left[1+\frac{\gamma^2}{n^2}\right]\ . \tag{2.14b}$$

The modified ER function $K_{csl}(k^2)$ has been proved to be real analytic in the neighborhood of $k^2=0$ for certain local[22,23] and nonlocal[13] potentials. The Taylor-expansion coefficients are related to the Coulomb-modified effective-range parameters a_{csl} and r_{csl}:

$$K_{csl}(k^2)=-1/a_{csl}+\tfrac{1}{2}r_{csl}k^2+\cdots\ . \tag{2.15}$$

From Eqs. (2.4), (2.5), (2.9), and (2.12) we observe that a_{csl} and r_{csl} can be found by investigation of the low-energy behavior of the objects $\langle g_{li}\,|\,G_{cl}\,|\,g_{lj}\rangle$, $\langle kl\,\infty-\,|\,g_{li}^c\rangle$, and $H(\gamma)$. The expansion of $H(\gamma)$ and—for simple rational form factors—also the one of $\langle kl\,\infty-\,|\,g_{li}^c\rangle$ are easily at hand.[13] Expansion of the inproducts $\langle g_{li}\,|\,G_{cl}\,|\,g_{lj}\rangle$ needs more effort. In the next section we investigate their low-energy behavior for a particular choice of form factors.

III. SEPARABLE REPRESENTATION OF THE NUCLEAR *p-p* INTERACTION IN *S*, *P*, AND *D* PARTIAL WAVES

In Sec. I we already indicated that the Graz potential[16] is a separable model which provides a reasonable description of the *p-p* interaction. The fit to experimental data, however, suffers from two main deficiencies:

(i) it fails in reproducing ${}^1S_0$ low-energy parameters accurately;

(ii) the medium- and high-energy behavior of its phase shifts is not satisfactory, especially in the ${}^3P_1$ and ${}^1D_2$ partial waves.

At present a separable potential which meets the following requirements is in demand:

(1) It allows for an accurate reproduction of all currently well-established *p-p* data;

(2) it is of low rank;

(3) its form factors are relatively simple functions amenable to rigorous analytical treatment.

Consequently such a potential should be a proper tool for application in charged few-body problems. According to these intentions the best choice turned out to be

$$V_l=|\,g_{l1}\rangle\lambda_{l1}\langle g_{l1}\,|+|\,g_{l2}\rangle\lambda_{l2}\langle g_{l2}\,| \tag{3.1a}$$

with

$$\langle p\,|\,g_{l1}\rangle=\frac{p^l}{(p^2+{\beta_{l11}}^2)^{l+1}} +\gamma_{l1}\frac{p^{l+2}}{(p^2+{\beta_{l12}}^2)^{l+2}}\ , \tag{3.1b}$$

$$\langle p\,|\,g_{l2}\rangle=\frac{p^{l+2}}{(p^2+{\beta_{l21}}^2)^{l+2}} +\gamma_{l2}\frac{p^{l+4}}{(p^2+{\beta_{l22}}^2)^{l+3}}\ , \tag{3.1c}$$

For this type of form factor it is still possible to derive analytical formulas for the on- and off-shell elements of the Coulomb-modified nuclear T operator T_{csl} of Eq. (2.4). As a consequence the phase shifts $\delta_{csl}(k)$ as well as the effective-range parameters a_{csl} and r_{csl} can also be calculated including the

Coulomb-distortion effect to all orders in the fine-structure constant. For this purpose one only needs to know explicit expressions for

$$\langle g_{li} \mid G_{cl}(k^2+i0) \mid g_{lj} \rangle$$

and

$$\langle kl\infty - \mid g^c_{li}(k^2+i0) \rangle$$

for $i,j=1,2$. For the first term in the form factor of Eq. (3.1b),

$$\langle p \mid g_{\beta l} \rangle := \frac{p^l}{(p^2+\beta^2)^{l+1}} , \tag{3.2}$$

the pertinent analytical result is known to be[13,16]

$$\langle kl\infty - \mid g^c_{\beta l}(k^2+i0) \rangle = {}_c\langle kl - \mid g_{\beta l} \rangle = e^{i\sigma_l}(C_{l\gamma})^{-1/2} C_0 B^{-i\gamma} \langle k \mid g_{\beta l} \rangle , \tag{3.3}$$

where $B=(\beta+ik)/(\beta-ik)$. The corresponding inproducts can also be found in the literature[24,25]

$$\langle g_{\alpha l} \mid G_{cl}(k^2) \mid g_{\beta l} \rangle = -\frac{\pi\mu}{\hbar^2} \frac{2l+1}{2^{2l}} \begin{bmatrix} 2l \\ l \end{bmatrix} \frac{(l+1+i\gamma)^{-1}}{(\alpha-ik)(\beta-ik)(\alpha+\beta)^{2l+1}} \, {}_2F_1(1,i\gamma-l;i\gamma+l+2;AB) , \tag{3.4}$$

where $A=(\alpha+ik)/(\alpha-ik)$. The remaining form factors in Eqs. (3.1) can also be treated analytically. Indeed, $\langle p \mid g_{li} \rangle$ $(i=1,2)$ can be written as a linear combination of $\langle p \mid g_{\beta l} \rangle$ and its derivatives with respect to β. Hence $\langle g_{li} \mid G_{cl} \mid g_{lj} \rangle$ and $\langle kl\infty - \mid g^c_{li} \rangle$ $(i,j=1,2)$ can be written as linear combinations of $\langle g_{\alpha l} \mid G_{cl} \mid g_{\beta l} \rangle$, $\langle kl\infty - \mid g^c_{\beta l} \rangle$, and their derivatives with respect to α and β.

As an example we give corresponding results obtained by first-order differentiation. Defining

$$\langle p \mid g_{\beta\beta l} \rangle := -\frac{1}{2\beta} \frac{1}{l+1} \frac{\partial}{\partial\beta} \langle p \mid g_{\beta l} \rangle = \langle p \mid g_{\beta l} \rangle / (p^2+\beta^2) \tag{3.5}$$

we find (suppressing the dependence on k^2+i0)

$$\langle kl\infty - \mid g^c_{\beta\beta l} \rangle = \frac{1+\dfrac{k\gamma}{(l+1)\beta}}{k^2+\beta^2} \langle kl\infty - \mid g^c_{\beta l} \rangle , \tag{3.6}$$

$$\langle g_{\alpha l} \mid G_{cl} \mid g_{\beta\beta l} \rangle = \frac{1}{k^2+\beta^2} \left[\frac{\pi\mu}{\hbar^2} \frac{2l+1}{2^{2l}} \begin{bmatrix} 2l \\ l \end{bmatrix} \frac{1}{2(l+1)\beta(\alpha+\beta)^{2l+2}} + \left[1+\frac{k\gamma}{(l+1)\beta} \right] \langle g_{\alpha l} \mid G_{cl} \mid g_{\beta l} \rangle \right] , \tag{3.7a}$$

$$\langle g_{\alpha\alpha l} \mid G_{cl} \mid g_{\beta\beta l} \rangle = \frac{1}{(k^2+\alpha^2)(k^2+\beta^2)} \left[\frac{\pi\mu}{\hbar^2} \frac{2l+1}{2^{2l-1}} \begin{bmatrix} 2l \\ l \end{bmatrix} \frac{(l+1)(\alpha^2+\alpha\beta+\beta^2+k^2)+k\gamma(\alpha+\beta)}{(2l+2)^2\alpha\beta(\alpha+\beta)^{2l+3}} + \left[1+\frac{k\gamma}{(l+1)\alpha} \right] \left[1+\frac{k\gamma}{(l+1)\beta} \right] \langle g_{\alpha l} \mid G_{cl} \mid g_{\beta l} \rangle \right] . \tag{3.7b}$$

For the derivative of the hypergeometric function we used

$$\frac{d}{dz} {}_2F_1(1,i\gamma-l;i\gamma+l+2;z) = \frac{1}{z(1-z)} \{ (i\gamma+l+1)+[(i\gamma-l)z-(i\gamma+l+1)] \, {}_2F_1(1,i\gamma-l;i\gamma+l+2;z) \} . \tag{3.8}$$

Let us now consider the low-energy behavior according to Eq. (2.12). We already mentioned that the low-energy expansion of $H(\gamma)$ and $\langle kl\infty - \mid g^c_{li} \rangle$ is relatively easy. Therefore, we concentrate on the inproducts

TABLE I. $^1S_0$ effective-range parameters.

	a_{cs} (fm)	r_{cs} (fm)	$a_s^{n\text{-}n}$ (fm)	$r_s^{n\text{-}n}$ (fm)
Separable potential	−7.81	2.75	−19.23	2.85
Experiment (Refs. 26 and 27)	−7.828±0.008	2.80±0.02	−18.6±0.5	2.83±0.11

$\langle g_{li} \mid G_{cl} \mid g_{lj} \rangle$. The principal role is played by the term $\langle g_{\alpha l} \mid G_{cl} \mid g_{\beta l} \rangle$ of Eq. (3.4). A tedious calculation reveals that at $k^2=0$

$$\langle g_{\alpha l} \mid G_{cl}(0) \mid g_{\beta l} \rangle = -\frac{\pi\mu}{\hbar^2} \frac{1}{2^{2l}\alpha\beta(\alpha+\beta)^{2l+1}} \begin{bmatrix} 2l \\ l \end{bmatrix} \times \left\{ \sum_{n=0}^{2l} \left[2k\gamma \left[\frac{1}{\alpha}+\frac{1}{\beta} \right] \right]^n \Big/ (-2l)_n - \frac{\left[2k\gamma \left[\frac{1}{\alpha}+\frac{1}{\beta} \right] \right]^{2l+1}}{(2l)!} e^{2k\gamma \left[\frac{1}{\alpha}+\frac{1}{\beta} \right]} \Gamma \left[0, 2k\gamma \left[\frac{1}{\alpha}+\frac{1}{\beta} \right] \right] \right\}. \tag{3.9}$$

Note that $k\gamma>0$, $\alpha>0$, $\beta>0$, and that the imaginary part of this object vanishes. For $\alpha=\beta$ and $2\mu=\hbar^2=1$ the above expression is in agreement with the corresponding one in Ref. 24.

For the evaluation of the effective range we require the term linear in k^2 of $\langle g_{\alpha l} \mid G_{cl} \mid g_{\beta l} \rangle$. It turns out that the pertinent imaginary part vanishes. Therefore, it is sufficient to study its real part. The hypergeometric function occurring in Eq. (3.4) can be reduced to ${}_2F_1(1,i\gamma;1+i\gamma;AB)$. The expansion in powers of k^2 for the real part of the latter hypergeometric function exists in the literature.[13] Collection of the appropriate results leads to analytic expressions both for the Coulomb-modified scattering lengths a_{csl} and effective-range parameters r_{csl} of our potential (3.1).

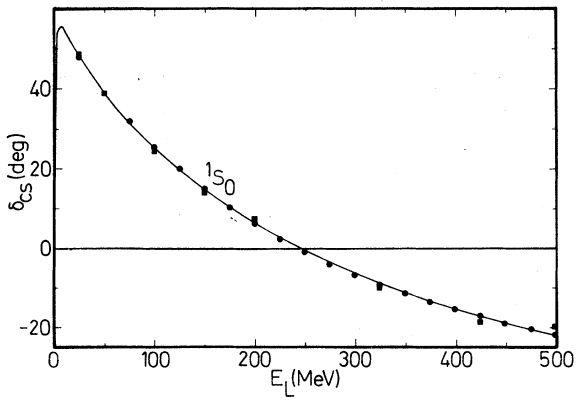

FIG. 1. Separable potential results for $^1S_0$ Coulomb-distorted nuclear phase shifts compared to predictions of the phase-shift analysis by Arndt and VerWest (Ref. 7).

Using the closed formulas for the on-shell matrix elements of T_{csl} and the low-energy parameters $a_{csl=0}$ and $r_{csl=0}$ we determined the open parameters of the separable potential. For this purpose we employed a least-squares minimization technique to fit the latest phenomenological p-p phase shifts by Arndt and VerWest[7] (energy-dependent solution) up to $E_{\text{lab}}=500$ MeV. In the $^1S_0$ partial wave we additionally imposed the constraint that the effective-range parameters a_{cs} and r_{cs} were reproduced in agreement with modern experimental data.[26]

For the most important partial wave requiring an exact treatment of the Coulomb distortion, namely, the $^1S_0$ state, we provided the potential (3.1) to be of rank 2. This led to a perfect fit of the phenomenological phase shifts over the whole energy range up to $E_{\text{lab}}=500$ MeV and to an accurate reproduction of the low-energy parameters a_{cs} and r_{cs} (see Fig. 1 and Table I).

Among the P waves only the $^3P_0$ state demanded a rank-2 potential (the corresponding phase shift changes sign at $E_{\text{lab}} \approx 200$ MeV). The $^3P_1$ and $^3P_2$ states could already be described by an ansatz of rank 1 ($\lambda_2=0$). Also for the P waves the fit to phenomenological data is observed to be quite satisfactory (see Figs. 2–4). The partial wave $^3P_2$ was treated as if it were uncoupled. This assumption seems to be well justified in view of the smallness of the mixing parameter ϵ_2.

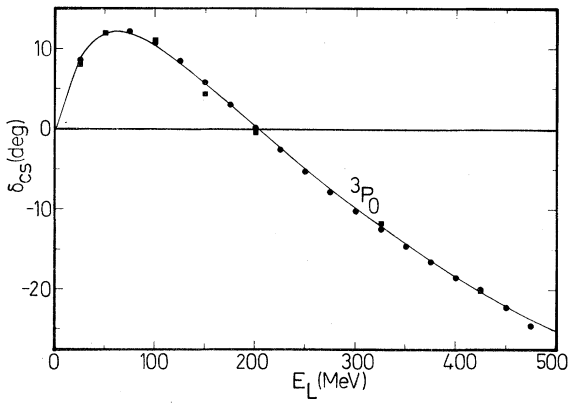

FIG. 2. Same as Fig. 1 for $^3P_0$.

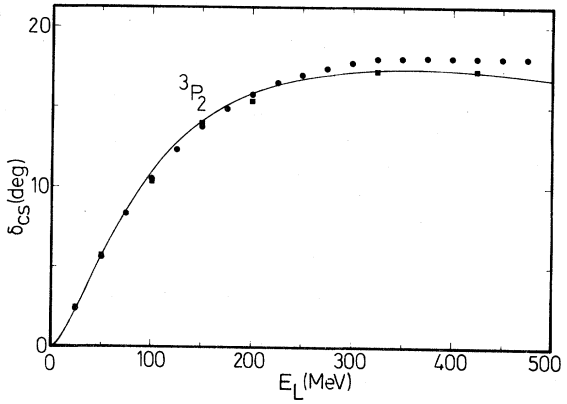

FIG. 4. Same as Fig. 1 for $^3P_2$.

For $l=2$ only the $^1D_2$ partial wave was to be considered. Also here a rank-1 potential was sufficient. The quality of the fit may be seen from Fig. 5.

It is interesting to study the influence of the Coulomb distortion on phase shifts δ_{csl} and effective-range parameters a_{csl} and r_{csl}; the Coulomb-distortion mechanism is known to be the most important electromagnetic effect modifying the short-range nuclear interaction[28] also on the level of polarization observables.[17] Our formalism allows us to reveal this influence by simply switching off the Coulomb field. Then all quantities labeled "cs" become purely nuclear ones. For scattering phase shifts the Coulomb-distortion effect is observed to be quite important only in the lowest partial wave $^1S_0$ and there predominantly at low and moderate energies, $E_{\rm lab}\lesssim 20$ MeV (see Fig. 6). Because of the occurrence of the centrifugal barrier, the Coulomb distortion becomes less significant or even negligible for higher partial waves[16] (Fig. 7).

Obviously the Coulomb distortion also has an essential influence on the $^1S_0$ effective-range parameters. Among the on-shell data these parameters, especially a_{cs}, are most sensitive to electromagnetic effects. Therefore, they constitute a decisive criterion for the validity of charge symmetry. In Table I we have quoted purely nuclear effective-range parameters a_s and r_s predicted by our new separable potential. We calculated them by switching off the Coulomb interaction and correcting for the different reduced mass of the n-n system. Therefore, the corresponding values can be regarded as n-n effective-range parameters (under the assumption of charge symmetry and neglecting all further electromagnetic effects). It is seen that n-n low-energy parameters thus obtained from the separable p-p potential are also in good accordance with the latest experimental data. Therefore, the purely nuclear part of the separable potential, i.e., the ansatz (3.1) together with the parameters of Table II, can safely be employed for describing the n-n system. The corresponding n-n scattering phase shifts result by subtracting the Coulomb-distortion effect Δ_l (Figs. 6 and 7) from the p-p phases δ_{csl} (Figs. 1–5):

$$\delta_{sl}^{n\text{-}n}=\delta_{csl}-\Delta_l\ . \tag{3.10}$$

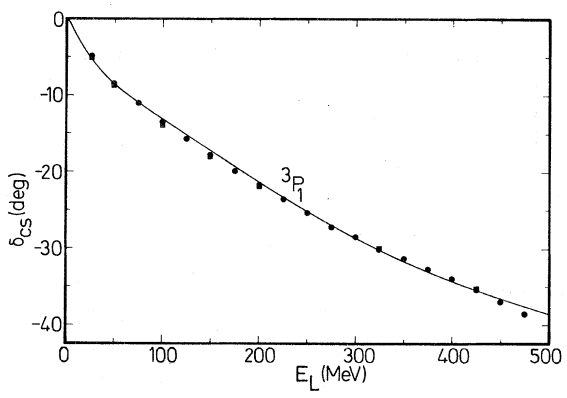

FIG. 3. Same as Fig. 1 for $^3P_1$.

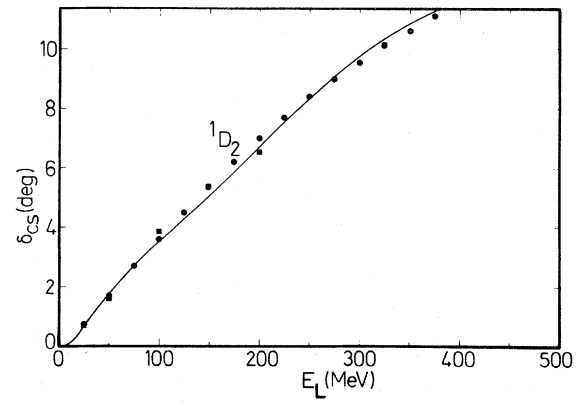

FIG. 5. Same as Fig. 1 for $^1D_2$.

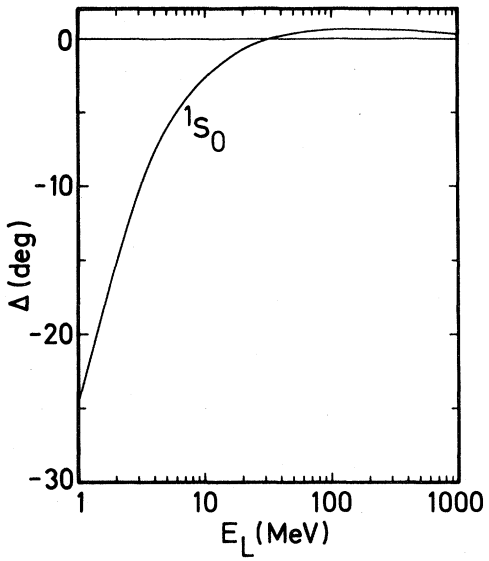

FIG. 6. Coulomb-distortion effect $\Delta=\delta_{cs}-\delta_s$ for $^1S_0$.

Here the correction for the different reduced mass turns out to be negligible. Because of the lack of experimental data, no comparison to *n-n* phenomenological phase shifts is possible at the present time.

IV. CONCLUSION

We presented a separable potential for the nuclear part of the *p-p* interaction in the partial waves $^1S_0$, $^3P_{0,1,2}$, and $^1D_2$. Our model is capable of precisely fitting recent phenomenological phase shifts up to $E_{\rm lab}\approx 500$ MeV, while it also reproduces accurate effective-range parameters in the $^1S_0$ partial wave. The problems associated with the r^{-1} tail of the potential were treated rigorously. Thereby it was possible to derive closed analytic expressions for all relevant quantities, and in particular for the Coulomb-modified nuclear transition matrix. Hence the Coulomb-distortion effect was treated exactly in phase shifts δ_{csl} as well as in low-energy parameters a_{csl} and r_{csl}. In agreement with former findings, it turns out that this effect is quite important and must not be neglected in the $^1S_0$ state (at low and moderate energies); only for higher partial waves can it be considered negligible. By subtracting the Coulomb distortion from *p-p* quantities a reasonable description of the *n-n* system could also be provided.

Since our ansatz for the separable potential is of low rank and amenable to analytic treatment, it should be well suited for use in few-body calculations of charged-hadron systems. Under the assumption of charge symmetry (and neglecting further, less important, electromagnetic effects) the *n-n* system can be treated at the same time via its purely nuclear part. We note that a complete separable representation of the whole nucleon-nucleon system can be constituted by supplying the neutron-proton (*n-p*) interaction by our previously published separable *n-p* potential.[29] This model uses an ansatz of the same type as in Eqs. (3.1) and is fitted to *n-p* phenomenological phases resulting from the same phase-shift analysis by Arndt and VerWest.[7]

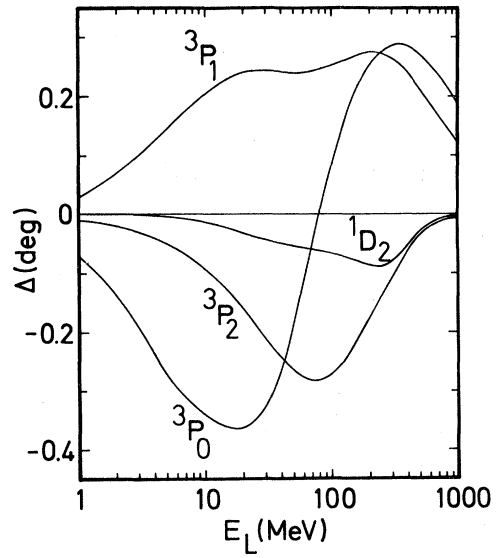

FIG. 7. Same as Fig. 6 for $^3P_{0,1,2}$ and $^1D_2$.

TABLE II. Numerical values of potential parameters. In our system of units the dimensions are $(\beta)=\text{fm}^{-1}$, $(\gamma)=\text{fm}^0$, and $(\lambda)=\text{MeV fm}^{-(2l+1)}$.

Partial wave	Parameters	
$^1S_0$	$\beta_{11}=0.813\,167\,8$	$\gamma_1=2.698\,168$
	$\beta_{12}=1.288\,463$	$\gamma_2=0.327\,066\,4$
	$\beta_{21}=7.496\,476$	$\lambda_1=-17.87\,098$
	$\beta_{22}=1.661\,389$	$\lambda_2=82\,710.93$
$^3P_0$	$\beta_{11}=0.832\,289\,4$	$\gamma_1=6.397\,468$
	$\beta_{12}=1.262\,785$	$\gamma_2=0.0$
	$\beta_{21}=2.645\,783$	$\lambda_1=-12.100\,82$
		$\lambda_2=27\,7071.8$
$^3P_1$	$\beta_{11}=0.971\,344\,1$	$\gamma_1=31.979\,81$
	$\beta_{12}=2.180\,297$	$\lambda_1=59.892\,49$
$^3P_2$	$\beta_{11}=1.977\,127$	$\gamma_1=5.242\,138$
	$\beta_{12}=3.147\,108$	$\lambda_1=-1477.246$
$^1D_2$	$\beta_{11}=2.522\,169$	$\gamma_1=-0.266\,832\,5$
	$\beta_{12}=1.651\,999$	$\lambda_1=-918\,408.7$

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[1]M. H. MacGregor, R. A. Arndt, and R. M. Wright, Phys. Rev. 182, 1714 (1969).

[2]R. E. Seamon, K. A. Friedman, G. Breit, R. D. Haracz, J. M. Holt, and A. Prakash, Phys. Rev. 165, 1579 (1969).

[3]J. Bystricky, P. Carlson, C. Lechanoine, F. Lehar, F. Mönnig, and K. R. Schubert, in *Landolt-Börnstein (New Series),* edited by H. Schopper (Springer, Berlin, 1980), Vol. I/9.

[4]R. A. Arndt, R. H. Hackman, and L. D. Roper, Phys. Rev. C 9, 555 (1974); 15, 1002 (1977).

[5]J. Bystricky, C. Lechanoine, and F. Lehar, CEN-Saclay Report DPhPE 79-01, 1979; J. Bystricky, C. Lechanoine, and F. Lehar, in *Landolt-Börnstein (New Series),* edited by H. Schopper (Springer, Berlin, 1980), Vol. I/9, p. 186.

[6]D. V. Bugg, J. A. Edgington, C. Amsler, R. C. Brown, C. J. Oram, K. Shakarchi, N. M. Stewart, G. A. Ludgate, A. S. Clough, D. Axen, S. Jaccard, and J. Vávra, J. Phys. G 4, 1025 (1978).

[7]R. A. Arndt and B. J. VerWest, Texas A&M University Report DOE/ER/05223-29, 1981; also in *Polarization Phenomena in Nuclear Physics—1980 (Fifth International Symposium, Santa Fe),* Proceedings of the Fifth International Symposium on Polarization Phenomena in Nuclear Physics, AIP Conf. Proc. No. 69, edited by G. G. Ohlsen, R. E. Brown, N. Jarmie, M. W. McNaughton, and G. M. Hale (AIP, New York, 1981).

[8]H. van Haeringen, J. Math. Phys. 17, 995 (1976).

[9]W. W. Zachary, J. Math. Phys. 12, 1379 (1971); 14, 2018 (1973).

[10]Ž. Bajzer, Nuovo Cimento A 22, 300 (1974).

[11]H. van Haeringen and R. van Wageningen, J. Math. Phys. 16, 1441 (1975).

[12]H. van Haeringen, Nucl. Phys. A253, 355 (1975).

[13]H. van Haeringen, J. Math. Phys. 18, 927 (1977).

[14]W. Plessas, L. Mathelitsch, F. Pauss, and H. F. K. Zingl, *Non-Local Separable Interactions in the Two-Nucleon System* (University Graz, Graz, Austria 1977); L. Mathelitsch and W. Plessas, in *Few-Body Systems and Nuclear Forces,* edited by H. Zingl *et al.* (Springer, Berlin, 1978), Vol. II, p. 505.

[15]D. R. Harrington, Phys. Rev. 139, B691 (1965); Thien Vo-Dai and Y. Nogami, Nucl. Phys. A141, 369 (1970); S. Ali, M. Rahman, and D. Husain, Phys. Rev. D 6, 1178 (1972); G. Cattapan, G. Pisent, and V. Vanzani, Lett. Nuovo Cimento 11, 650 (1974); G. Cattapan, G. Pisent, and V. Vanzani, Nucl. Phys. A241, 204 (1974).

[16]W. Plessas, L. Streit, and H. Zingl, Acta Phys. Austriaca 40, 272 (1974); L. Črepinšek, C. B. Lang, H. Oberhummer, W. Plessas, and H. F. K. Zingl, *ibid.* 42, 139 (1975).

[17]W. Plessas, L. Mathelitsch, and F. Pauss, Phys. Rev. C 23, 1340 (1981).

[18]W. Plessas and W. Schweiger, J. Phys. G 7, 1019 (1981); H. van Haeringen, L. P. Kok, and W. Plessas, *ibid.* 7, L193 (1981).

[19]M. Gell-Mann and M. L. Goldberger, Phys. Rev. 91, 398 (1953).

[20]H. van Haeringen, Groningen University, Report 150, 1979; Delft University Report 82 05, 1982, 2nd ed.

[21]M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1967), p. 286.

[22]H.Cornille and A. Martin, Nuovo Cimento 26, 298 (1962).

[23]J. Hamilton, I. Øverbö, and B. Tromborg, Nucl. Phys. B60, 443 (1973).

[24]H. van Haeringen and L. P. Kok, Phys. Lett. 82A, 317 (1981); H. van Haeringen, *ibid.* 86A, 359 (1981); H. van Haeringen, J. Math. Phys. (to be published). In these works the form factors $|g_{\beta l}\rangle$ are $\sqrt{2/\pi}$ times ours in Eq. (3.2).

[25]Another expression for $\langle g_{\alpha l}|G_{cl}|g_{\beta l}\rangle$, which is equivalent to the one in Eq. (3.4), was obtained in a different way by N. Maleki and J. Macek, Phys. Rev. A 21, 1403 (1980).

[26]J. P. Naisse, Nucl. Phys. A278, 506 (1977).

[27]B. Gabioud, J.-C. Adler, C. Joseph, J.-F. Loude, N. Morel, A. Perrenoud, J.-P. Perroud, M. T. Tran, E. Winkelmann, W. Dahme, H. Panke, D. Renker, C. Zupančič, G. Strassner, and P. Truöl, Phys. Rev. Lett. 42, 1508 (1979); B. Gabioud, J.-C. Adler, C. Joseph, J.-F. Loude, N. Morel, A. Perrenoud, J.-P. Perroud, M. T. Tran, E. Winkelmann, W. Dahme, H. Panke, D. Renker, G. Strassner, G. F. de Téramond, and P. Truöl, Phys. Lett. 103B, 9 (1981).

[28]A. Gersten, Nucl. Phys. A290, 445 (1977); Phys. Rev. C 18, 2252 (1978).

[29]L. Mathelitsch, W. Plessas, and W. Schweiger, Phys. Rev. C 26, 65 (1982).