Elastic and inelastic scattering of protons from ^{24}Mg with coupled channels analysis for the energy range 17–185 MeV

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Elastic and inelastic differential cross sections of protons scattered from ^{24}Mg were measured for incident proton energies of 20.0, 25.0, 30.4, 34.9, 39.9, and 44.9 MeV. Angular distributions from 10° to 170° in the laboratory were obtained for seven states of ^{24}Mg, namely: 0^+ (0.00 MeV), 2^+ (1.37 MeV), the sum of 4^+ (4.12 MeV) and 2^+ (4.24 MeV), 3^+ (5.24 MeV), 4^+ (6.01 MeV), and 0^+ (6.43 MeV). These data, along with data from the literature in the energy range 17–185 MeV, were analyzed using optical model and coupled channels calculations. Using a rotational model, with triaxial deformation limited to terms in $Y_{2,2}$ and $Y_{2,-2}$, reasonable agreement with the data was obtained for all states except for the 3^+ (5.24 MeV) and 4^+ (6.01 MeV) states. The inclusion of terms of $Y_{4,2}$ and $Y_{4,-2}$ in the model improved the agreement with these two states considerably; but the 3^+ (5.24 MeV) experimental data are still not accurately reproduced.

[NUCLEAR REACTIONS ^{24}Mg$(p,p')^{24}$Mg, $T=20.0$, 25.0, 30.4, 34.9, 39.9, 44.9 MeV; measured $\sigma(\theta)$, $\theta=10°-170°$ lab; $E_x=0.00$, 1.37, 4.12+4.24, 5.24, 6.01, and 6.43 MeV; enriched target; optical model and coupled channels analyses.]

I. INTRODUCTION

The nucleus ^{24}Mg is highly deformed.[1–4] As such, the excited states are strongly populated by inelastic proton scattering and play an important part in any description of scattering processes from this nucleus. A considerable amount of data is available[5–25] in the energy range 15–185 MeV for proton elastic and inelastic scattering from the lower excited states. This wealth of data provides a testing ground for various descriptions of proton scattering from ^{24}Mg. More complex models (Refs. 17, 18, 20, 22, 24, and 25) and calculations can possibly better satisfy the data, but their validity is not as readily verified.

The existing optical model analyses[11–25] of the elastic scattering data available provide a complete (if not always systematic) phenomenological, energy dependent parametrized description. Because of the strength and structure of the inelastic processes in proton scattering from ^{24}Mg, the complex optical potential cannot realistically average the inelastic contributions. As well, the presence of strong giant multipole resonances further inhibits a systematic description. More elaborate models are needed to fully describe proton scattering from ^{24}Mg.

Coupled channel calculations have been performed by various authors[17–25] with varying success depending on the extent to which the angular distributions of the inelastic differential cross sections and analyzing powers were examined. A simple rotational model for ^{24}Mg can easily explain scattering to the ground state ($K^\pi=0^+$) rotational band [comprising the 0^+ (0.00 MeV), 2^+ (1.37 MeV), 4^+ (4.12 MeV), and 6^+ (8.12 MeV) excited states]. Asymmetric rotational models are needed, however, to include the γ ($K^\pi=2^+$) vibrational band [comprising the 2^+ (2.24 MeV), 3^+ (5.24 MeV), 4^+ (6.01 MeV), and 5^+ (7.81 MeV) excited states] in the calculations. Early asymmetric rotational model calculations obtained good agreement with the experimental results for the first four states ($0^+,2^+,4^+,2^+$) but were low by orders of magnitude for the next few states (3^+ and 4^+).

Primary interest in recent work has focused on the unnatural parity 3^+ (5.24 MeV) state. This state cannot be excited directly but only through some more involved reaction mechanism. Several models have been proposed and tested with varying success. Spin-flip processes utilizing valence or core polarizations coupling with other transitions can populate the 3^+ state. However, calculations[17] indicate that these mechanisms alone are not prominent enough to explain the high cross sections observed.

Two-step processes[17,20] going through (p,d,p') to higher spin states with giant multipole resonances similarly fail to reproduce the experimental .cross sections. However, similar two-step processes of the form (p,p'',p') give an adequate description for the 3^+ differential cross sections in a limited energy range. More complicated asymmetric rotational models[21] incorporating direct coupling to the 4^+ state of the γ vibrational band improve the agreement with the data over a wide energy range, but complete matching is still missing.

In order to provide the basis for new, improved coupled channels calculations, measurements were made of the differential cross section angular distributions for seven states in ^{24}Mg via ^{24}Mg$(p,p')^{24}$Mg at six energies between 20 and 50 MeV. The next section gives a short description of the experiment followed by a description of the data reduction. An outline of the theoretical analysis is followed by a discussion of the results of the current work along with some comments concerning other analyses which have been recently reported.

II. EXPERIMENT

The proton beam facility of the University of Manitoba sector focused cyclotron was used to measure the elastic and inelastic scattering of protons from ^{24}Mg at 20.0, 25,0, 30.4, 34.9, 39.9, and 44.9 MeV. Two isotopically enriched ($>99\%$) targets of ^{24}Mg were used for this experiment. Their thicknesses were 4.93 ± 0.13 mg/cm^2 (used for energies below 35 MeV) and 10.11 ± 0.25 mg/cm^2 (for energies 35 MeV and above). All handling of the targets was performed in an inert atmosphere to reduce contamination. Analysis of the experimental data indicated an approximate 0.04 mg/cm^2 thickness due to oxygen present in both targets. The experimental aparatus and procedures used have been described in Ref. 26. Data were accumulated at laboratory angles from 10° to 90° in 2.5° steps and from 90° to 170° in 5.0° steps.

III. DATA REDUCTION

The energy spectra collected were analyzed using a computer program to unfold the peaks of interest and to correct for background and contributions due to the oxygen contamination. The typical energy resolution of the NaI(T1) detectors used was 300 keV at 30 MeV. This did not permit the states at 4.12 and 4.24 MeV to be separated, so the sum of their cross sections was determined. Corrections were made for computer deadtime, proton reaction losses in the NaI(T1) detectors,[27] multiple scattering in the target,[28] the finite size and divergence of the incident proton beam, and the finite apertures of the solid angle defining collimators.[29] Differential cross sections in numerical form can be obtained from the authors upon request. A sample of the angular distributions is shown in Figs. 1 and 2.

IV. THEORETICAL ANALYSIS

Additional differential cross section data for elastic and inelastic scattering of protons from ^{24}Mg at incident proton energies of 17.5,[6] 49.5,[12] 100,[8] 155,[15] and 185 MeV (Ref. 9) are available in the literature. Polarization data are also available at 17.5,[5] 20.3,[20] 49.5,[13] 155,[15] and 185 MeV.[10] Together with the results from the present experiment, these data were analyzed using a standard optical model and various coupled channels descriptions.

The optical model codes SEEK (Ref. 30) and MAGALI (Ref. 31) were used to find acceptable fits to the elastic scattering data. The Coulomb radius parameter was fixed at 1.05 fm, as determined from electron scattering data.[1–4] No attempt was made at this point to find a common geometry or energy dependence for the optical potential. The agreement with the data and the optical potential parameters obtained were quite reasonable. Since the latter served only as starting points for the following analysis, the fit and the parameters will not be presented in this paper.

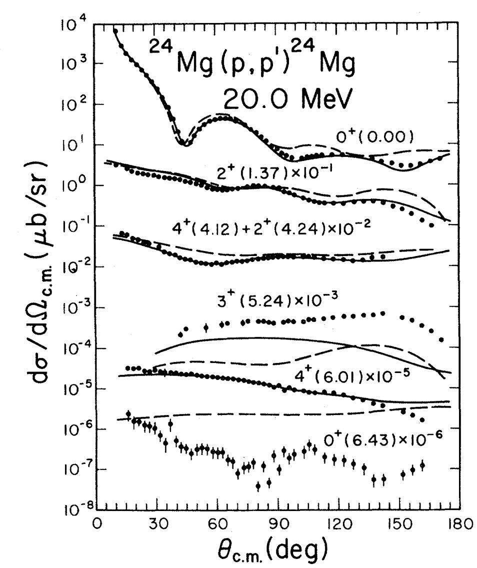

FIG. 1. Differential cross sections for 20.0 MeV protons scattering from the 0^+ (0.00 MeV), 2^+ (1.37 MeV), 4^+ (4.12 MeV) and 2^+ (4.24 MeV), 3^+ (5.24 MeV), 4^+ (6.01 MeV), and 0^+ (6.43 MeV) states of ^{24}Mg. The solid lines show the results for the coupled channels calculation using the extended coupling scheme while the dotted line was produced using the simple asymmetric rotational model.

The computer code CHUCK (Ref. 32) was used to perform the calculation of the differential cross sections and analyzing powers for the first two states using a simple rotational model. This coupled channels calculation is based on the expansion

$$R = R_0(1+\beta_2 Y_{2,0}+\beta_4 Y_{4,0}) .$$

Because of the relative simplicity of these calculations it was possible to perform fairly detailed searches for the best overall parameter sets (optical potential and deformation parameters) describing the data. A common geometry for all energies was found to be satisfactory with $r=1.16$ fm, $r_I=1.40$ fm, $a=a_I=0.60$ fm, $r_{SO}=0.96$ fm, and $a_{SO}=0.67$ fm.

The computer code JUPITOR (Ref. 33) was used to calculate the differential cross sections and analyzing powers of the first four states with an asymmetric rotational model. In this model the expansion used was

$$R = R_0[1+\beta_2\cos(\gamma)Y_{2,0}+\beta_2(\sin(\gamma)/\sqrt{2})(Y_{2,2}+Y_{2,-2})+\beta_4 Y_{4,0}] .$$

The starting value for γ ($=21°$) was taken from the literature.[20] Searches were made to obtain the best parameter set to fit the angular distributions for the 0^+ (0.00 MeV), 2^+ (1.37 MeV), 4^+ (4.12 MeV), and 2^+ (4.24 MeV) states. At some energies the 4^+ (4.12 MeV) and 2^+ (4.24 MeV) states were not resolved, so the sum of the differential cross sections was used in the fits. Reasonable agreement to the differential cross section and analyzing power angular distributions was obtained for these states.

Calculations were then performed to find the agreement with the 3^+ (5.24 MeV) and 4^+ (6.01 MeV) states. These calculations always gave very poor agreement with both differential cross section angular distributions. No reasonable change in the parameters corrected the disagreement. The available analyzing power data were likewise not well reproduced. It was thought that the lack of a deformed spin orbit potential in the version of JUPITOR used provided a possible explanation for the disagreement.

Using the parameters as determined above, asymmetric rotational model searches were performed with the computer code ECIS (Ref. 34) encompassing the first six excited states of ^{24}Mg. The radius and diffuseness parameters were held fixed at the above

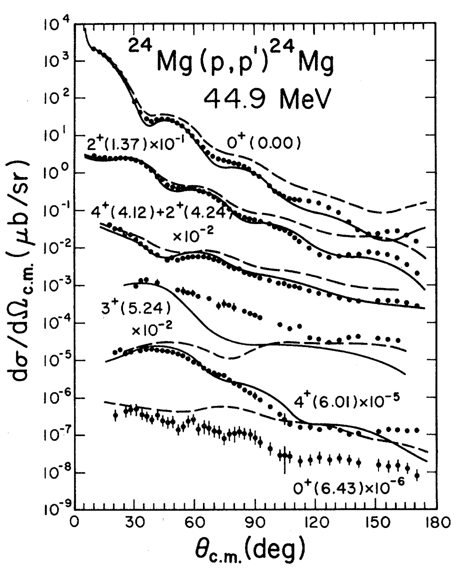

FIG. 2. Differential cross sections for 45.0 MeV protons scattering from the 0^+ (0.00 MeV), 2^+ (1.37 MeV), 4^+ (4.12 MeV) and 2^+ (4.24 MeV), 3^+ (5.24 MeV), 4^+ (6.01 MeV), and 0^+ (6.43 MeV) state of ^{24}Mg. The solid lines show the results for the coupled channels calculation using the extended coupling scheme while the dotted line was produced using the simple asymmetric rotational model.

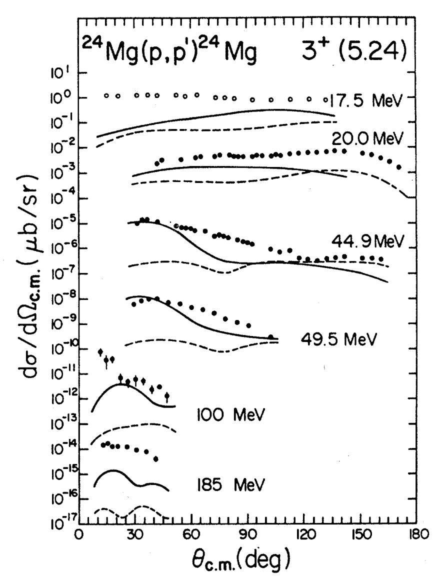

FIG. 3. Differential cross sections for protons scattering from the 3^+ (5.24 MeV) state of ^{24}Mg. The dotted lines show the results of the calculations using the simple asymmetric rotational model and the solid lines show the results of the calculations using the extended coupling scheme. The top angular distribution is plotted with the correct scale. All other angular distributions are separated by two decades for clarity.

quoted values. No significant improvement was made by including the 3^+ (5.24 MeV) and 4^+ (6.01 MeV) states explicitly in the searches. The results of the calculations for a representative sample of the 11 energies included in the analysis are shown in Figs. 1–6 by the dotted lines. It was concluded that the form of the nuclear deformation used in the expansion was not adequate to describe the 3^+ (5.24 MeV) and 4^+ (6.01 MeV) states.

The next stage in the analysis used the asymmetric rotational model including terms in $Y_{4,2}$ and $Y_{4,-2}$, as suggested by Ray *et al.*[21]:

$$R = R_0[1+\beta_2\cos(\gamma)Y_{2,0}+\beta_2(\sin(\gamma)/\sqrt{2})(Y_{2,2}+Y_{2,-2})+\beta_4\cos(\gamma_1)Y_{4,0}+\beta_4(\sin(\gamma_1)/\sqrt{2})(Y_{4,2}+Y_{4,-2})] .$$

Calculations were made for the differential cross sections and analyzing powers of the first six states. The starting value for the additional term in the expansion was chosen to match the result of Ref. 21 ($\gamma_1 = 94°$). The searches were performed only at the lower eight energies. The final parameter sets are given in Table I. No obvious energy dependence of the parameters is discernible. Literature values for β_2 range from 0.47 to 0.56. The corresponding value for this analysis is $\beta_2\cos(\gamma)=0.51$. Similarly, literature values for β_4 range from -0.05 to -0.06, which are comparable to $\beta_4\cos(\gamma_1)=-0.07$. The value for γ, 13.4°, obtained in the present analysis is somewhat lower than the literature values of 20°–23°. The value for γ_1 obtained in the present analysis is 107° compared to 94° obtained by Ray *et al.*[21]

V. DISCUSSION

From the results of the analysis it is evident that both asymmetric rotational models reproduce the differential cross sections for the first four excited states very well. Only at far backward angles are there slight discrepancies in the magnitudes and shapes of the calculated angular distributions. The available analyzing power data are equally well reproduced by both coupling schemes though there are large deviations past 90° center of mass (c.m.) for the 0^+ (0.00 MeV) and 2^+ (1.37 MeV) states.

However, the simple asymmetric rotational model and the extended model, incorporating additional terms in $Y_{4,2}$ and $Y_{4,-2}$, differ significantly in the calculations for the differential cross sections for the 3^+ (5.24 MeV) and 4^+ (6.01 MeV) states (see Figs. 3 and 4). Firstly, for the 4^+ (6.01 MeV) state, the best

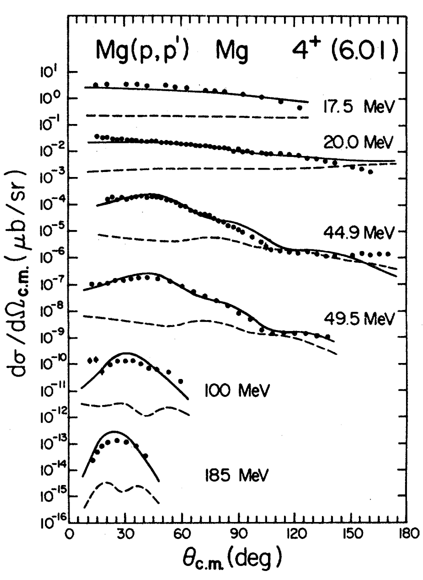

FIG. 4. Differential cross sections for protons scatterings from the 4^+ (6.01 MeV) state of ^{24}Mg. The dotted lines show the results of the calculations using the simple asymmetric rotational model and the solid lines show the results of the calculations using the extended coupling scheme. The top angular distribution is plotted with the correct scale. All other angular distributions are separated by two decades for clarity.

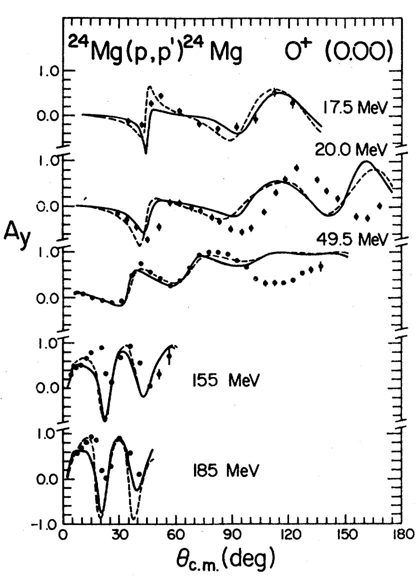

FIG. 5. Analyzing powers for protons scattering from the 0^+ (0.00 MeV) state of ^{24}Mg. The dotted lines show the results of the calculations using the simple asymmetric rotational model and the solid lines show the results of the calculations using the extended coupling scheme.

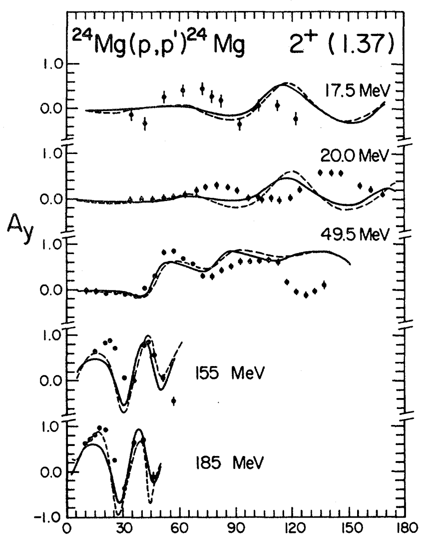

FIG. 6. Analyzing powers for protons scattering from the 2^+ (1.37 MeV) state of ^{24}Mg. The dotted line show the results of the calculations using the simple asymmetric rotational model and the solid lines show the results of the calculations using the extended coupling scheme.

fits using the simple asymmetric rotational model are low by an order of magnitude and do not resemble the shape of the experimental angular distributions. The inclusion of terms in $Y_{4,2}$ and $Y_{4,-2}$ resulted in theoretical predictions which agree with the data very well. This was to be expected since there is a direct coupling to the 4^+ (6.01 MeV) state from the 0^+ (0.00 MeV) state through an adjustable parameter. Secondly, for the 3^+ (5.24 MeV) state, the calculations using the simple asymmetric rotational model were low and of the wrong shape, particularly as the incident energy increased. Using the new coupling scheme the calculations are still low, but the discrepancy is not as pronounced, and the shapes of the angular distributions calculated are greatly improved. Also, as the incident energy increases, the agreement improves, suggesting that at the higher energies the new coupling scheme is important while at lower energies some other reaction mechanism may be involved in populating the 3^+ state. Again the analyzing powers calculated using the two coupling schemes are in reasonable agreement with the data with no preference to either coupling scheme.

A recent publication[18] shows spin-flip probabilities for proton scattering from the 2^+ (1.37 MeV) state of ^{24}Mg. In Fig. 7 these data are compared with predictions using the new coupling scheme. The agreement is quite good. Total reaction cross sections, predicted using the new coupling scheme, are compared with experiemntal values[35] in Table I. Again the agreement is quite good.

The extended coupling scheme was first proposed by Ray *et al.*[21] in order to improve the agreement with the 800 MeV differential cross section angular distribution for the 4^+ (6.01 MeV) state. These authors also obtained reasonable agreement with the 20.3 and 40.0 MeV data using the 800 MeV deformation parameters. Furthermore, to simplify the calculation Ray *et al.* did not include the spin orbit

TABLE I. Optical potential (geometry parameters and potential strengths) and deformation parameters obtained in fitting the experimental data. Total reaction cross section experimental (Ref. 35) and theoretical values are also given.

	Deformation parameters				$\bar{\beta}_2=0.526$ $\bar{\beta}_4=0.247$		$\bar{\gamma}=13.4°$ $\bar{\gamma}=107°$			
	Geometry parameters (fm)				$r=1.16$ $a=0.60$		$r_I=1.40$ $a_I=0.60$		$r_{SO}=0.96$ $a_{SO}=0.67$	
T_p (MeV)	V (MeV)	W (MeV)	W_D (MeV)	V_{SO} (MeV)	σ_R^{th} (mb)	σ_R^{exp} (mb)	β_2	β_4	γ (deg)	γ_1 (deg)
17.5	53.9	0.00	3.47	4.32	870	841±17	0.5734	0.3400	9.27	84.9
20.0	47.6	0.00	3.55	4.02	800		0.5375	0.3102	11.65	125.2
25.0	44.6	0.00	4.19	5.70	775	773±13	0.5658	0.2863	10.69	121.4
30.4	43.1	1.12	3.64	6.89	737	724±11	0.5337	0.2074	13.56	112.8
34.9	43.4	3.14	2.62	6.21	719	673±11	0.5243	0.2073	15.13	107.5
39.9	40.0	5.38	0.95	6.29	647	645±11	0.4790	0.1944	17.63	99.4
44.9	40.3	7.15	0.00	4.08	618	618±10	0.5158	0.2049	15.85	105.6
49.5	39.3	7.74	0.00	5.72	618	601±10	0.4798	0.2245	13.32	102.3
100	25.4	8.40	0.00	5.47	484	399±11				
155	15.5	7.00	0.00	3.75	363					
185	14.7	8.00	0.00	4.52	380	(362±15)[a]				

[a]Interpolated from values for nearby nuclei.

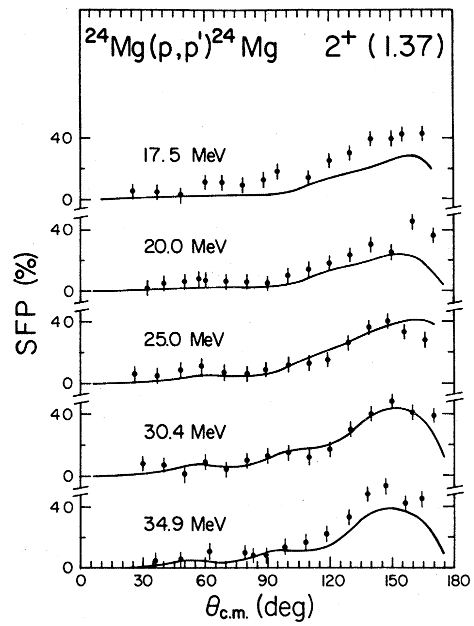

FIG. 7. Spin-flip probabilities for proton scattering from the 2^+ (1.37 MeV) state of ^{24}Mg. The solid lines show the results of the calculations using the extended coupling scheme.

interaction. In the present analysis the spin orbit potential was included in all calculations and was deformed in the calculations using ECIS.

In two other recent works,[17,22] other reaction mechanisms have been proposed to explain the large differential cross sections for the 3^+ (5.24 MeV) state. These analyses make use of the existence of strong multipole resonances in proton scattering from ^{24}Mg at the lower energies (usually thought to be maximum about 24 MeV). Both show significant improvement in the agreement with the 3^+ (5.24 MeV) state experimental angular distributions. Both analyses are restricted, however, to the lower energies, 15—35 MeV. Also, no mention is made of results for differential cross sections for the 4^+ (6.01 MeV) state.

Lovas *et al.*[22] calculate contributions arising from two-step processes in the framework of an excited-core model, first forming a higher excited state which then decays into the inelastic channel. Similarly, De Leo *et al.*[17,18] calculate the effect of two-step processes of the form (p,p'',p') and (p,d,p') coupled to higher collective excited states. Only the former process was found to provide good agreement with the 3^+ (5.24 MeV) data. Spin-flip mechanisms via valence or core polarization were also considered and found to be insufficient to give agreement with the data. In addition to the restricted energy range it should be remarked that both calculations follow a phenomenological approach in treating the two-step processes considered without corroborating data to indicate the strength of these processes.

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