Missing longitudinal strength cannot reappear at very high energy

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The longitudinal sum rule, computed from recent measurements of quasielastic longitudinal structure functions at high momentum transfers, is lower than the theoretical prediction by about a factor 2. Here it is shown, by comparison between the theoretical measured values of the ratio of the f sum rule to the longitudinal sum rule, that any substantial fraction of missing longitudinal strength must lie at energies low enough to have been detected. This analysis is independent of any systematic experimental errors of overall scale.

[NUCLEAR REACTIONS Quasielastic electron scattering; longitudinal response function; longitudinal sum rule and "f" sum rule.]

The longitudinal inelastic structure function of a nucleus is defined as

$$S_L(|\vec{q}|,w)=\sum_{n>0}|\langle n|J^0(\vec{q})|0\rangle|^2 \times\delta(E_n-E_0-w) \quad . \qquad (1)$$

It is measured for spacelike ($\vec{q}^{\,2}>w^2$) four-momentum transfer in electron scattering by means of Rosenbluth plots (that is, the ratio of the cross section to the Mott cross section is measured for fixed three-momentum transfer $\vec{q}$ and energy loss w, but varying scattering angles, and then plotted against $[\frac{1}{2}+\tan^2(\theta/2)]$, which is the coefficient of the transverse contribution). The measurement of S_L is a matter of some difficulty, and it is not surprising that this has only been done relatively recently,[1,2] especially since the Fermi gas model of quasielastic electron scattering in the impulse approximation seemed to describe the unseparated data at $\theta=60^\circ$ quite well.[3] Thus it has proved extremely exciting that this model, which fits the unseparated inelastic structure functions quite well, fails by a factor of ~ 2 to fit the longitudinal structure function by itself. There are two aspects of this discrepancy which must be explained: First, the theoretical curve, predicated on one-body quasielastic knockout with reasonable nuclear wave functions, closely resembles the experimental curve in shape, but is a factor of 2 higher than the data. Second, the area under the quasielastic peak is supposed to satisfy a sum rule which was first proposed by Heisenberg in 1931,[4] namely

$$\lim_{|\vec{q}|\to\infty}\int_0^\infty dw\, S_L(|\vec{q}|,w)=Z \quad . \qquad (2)$$

Strictly speaking, the large three-momentum limit in (2) applies only for pointlike charged particles—for protons we should multiply the right-hand side by the proton form factor. This gives the corrected form, Eq. (2′).

$$\lim_{|\vec{q}|\to\infty}C(|\vec{q}|)\equiv\int_0^\infty dw\, S_L(|\vec{q}|,w) =Z[F^*(|\vec{q}|)]^2 \quad . \qquad (2')$$

The area under the experimental quasielastic peak, up to the highest energy loss so far measured, and for the largest values of $|\vec{q}|$, is also low by a factor of ~ 2, compared with the prediction of the Fermi gas model in impulse approximation. The sum-rule discrepancy is much more serious than any disagreement with the detailed shape of a predicted theoretical spectrum, because the sum rule is nearly model independent. That is, changing the parameters of the nuclear wave functions can shift the strength around to some extent, but cannot alter the total area under the curve.

Only three explanations of the missing sum-rule strength seem tenable: First, the experiments might be wrong. The experiment is a difficult one *ab initio*, and the analysis is complicated by the effects of Coulomb distortion of the electron wave functions, by radiative processes, and by a negative pion background.[1,2] Since the longitudinal structure function is extracted from the data as a small difference between large numbers, extreme precision is necessary in controlling all the potential sources of error to obtain only moderate precision in measuring S_L. (At the time of this writing it is not clear whether the quoted measurements of S_L are sufficiently in error to account for the factor of 2 discrepancy.) Second, the Fermi gas model is predicated on single-nucleon quasielastic knockout; however, the hadronic interactions are expected to transfer some of the inelastic strength from one-body processes to more complex processes such as two-nucleon ejection or meson production, and thereby to push the inelastic strength to energies substantially higher than the quasielastic spectrum.[5] And third, as I have suggested previously,[6] in some models of nuclear structure the interactions modify the nucleon form factors sufficiently that it becomes significantly more difficult to eject a nucleon at a given three-momentum transfer than it would be if the nucleon's form factor were unmodified.

The object of this paper is to point out that the reported measurements of longitudinal inelastic strength are consistent with the electric dipole sum rule and with the energy-weighted inelastic sum rule. Hence we must conclude that there is no significant missing strength at energies where the structure function has not been measured (e.g., for timelike four-momentum transfer). If the experimental results continue to stand up, we will then be able to conclude by elimination that the only viable explanation is the failure of the impulse approximation.

Several years ago I showed[7] that for a system of interacting pointlike nucleons and pions one could establish the energy-weighted progenitor sum rule

$$\langle 0|[J^0(-\vec{q}\,'),[H,J^0(\vec{q})]]|0\rangle = M^{-1}\vec{q}\,'\cdot\vec{q}[\tilde{\rho}_p(\vec{q}-\vec{q}\,') - A^{-1}\tilde{\rho}_p(-\vec{q}\,')\tilde{\rho}_p(\vec{q})] + 2\vec{q}\,'\cdot\vec{q}\int d^3x\, e^{-i(\vec{q}\,'-\vec{q})\cdot\vec{x}}\langle 0|\phi^\dagger(\vec{x})\phi(\vec{x})|0\rangle \quad , \tag{3}$$

where $\phi(\vec{x})$ is the charged pion field. Strictly speaking, the first term of Eq. (3) holds for nonrelativistic nucleons, but the modification is straightforward. Also, strictly speaking, the term involving the pion fields is infinite because it includes (divergent) nucleon self-loop diagrams which should be included with the renormalized nucleon form factors. In the following we shall assume that these identifications have been made and that, over the range of momentum transfers we shall be interested in, Eq. (3) holds in the form

$$\langle 0|[J^0(-\vec{q}\,'),[H,J^0(\vec{q})]]|0\rangle \simeq (M^*)^{-1}\vec{q}\,'\cdot\vec{q}F_N^*(q')F_N^*(q)[\tilde{\rho}_p(\vec{q}-\vec{q}\,') - A^{-1}\tilde{\rho}_p(-\vec{q}\,')\tilde{\rho}_p(\vec{q})] + \kappa_\pi(N/A)M^{-1}\vec{q}\,'\cdot\vec{q}F_\pi^*(q')F_\pi^*(q)\tilde{\rho}_p(\vec{q}-\vec{q}\,')[1+(\vec{q}-\vec{q}\,')^2/m_\pi^{*2}]^{-1} \quad , \tag{4}$$

where κ_π is the part of the electric dipole sum-rule enhancement which is attributable to pion exchange. Typical calculations[7] give 0.2 to 0.3 for this number, in medium-mass nuclei. (The possible modification of masses and form factors in nuclei is implied by superscribed asterisks.) We shall be interested in two extremes of Eq. (4), namely very small (but different) $\vec{q}$ and $\vec{q}\,'$ (suitable for evaluating the electric dipole sum rule), and moderately large ($p_F < q < 3p_F$) values of the momentum transfer, with $\vec{q}$ and $\vec{q}\,'$ set equal. This latter limit is the nuclear equivalent of the famous f sum rule of condensed matter physics.[8] In these two limits we obtain the electric dipole sum rule

$$\sum_{n>0}|\langle n|D|0\rangle|^2(E_n-E_0) = (NZ/2AM)[(M/M^*)(\rho_s/\rho)+\kappa_\pi] \tag{5}$$

and the f sum rule

$$f(|\vec{q}|) = \sum_{n>0}|\langle n|J^0(\vec{q})|0\rangle|^2(E_n-E_0) = \left(\frac{q^2}{2M^*}\right)ZF_N^{*2}(q) + \left(\frac{q^2}{2M}\right)\left(\frac{NZ}{A}\right)\kappa_\pi F_\pi^{*2}(q) \tag{6}$$

which may be expressed in terms of the longitudinal strength function as

$$f(|\vec{q}|) = \int dw\, wS_L(|\vec{q}|,w) \quad . \tag{7}$$

Now, according to Viollier and Walecka,[9] the effects of two-body short-range correlations (as distinguished from Pauli correlations) on the longitudinal sum rule (LSR), Eq. (2′), are in the range 1–10% when q is $\gtrsim 1.5p_F$. However, their calculation does not tell us where the strength lies. If we suppose that there is some missing strength which has not been measured in electron scattering, then it will affect the ratio of the f sum rule to the LSR. Let us take the ratio of the integral of missing strength to that of the known strength to be χ. We can define an effective energy where this missing strength is to be found, in terms of the assumed missing integrated LSR strength, χ:

$$w_{\rm eff} = [(1+\chi)(f_{\rm theor}/C_{\rm theor}) - (f_{\rm expt}/C_{\rm expt})]/\chi \quad . \tag{8}$$

Here $f_{\rm theor}$ and $f_{\rm expt}$ are the theoretical and experimental values of the f sum, the $C_{\rm theor}$ and $C_{\rm expt}$ the corresponding values of the LSR. In Table I below are plotted the derived values of this effective energy, for various values of M^*, the effective nucleon mass in the relativistic Fermi gas model. The values of κ_π appearing in the table were derived under the assumption that the net enhancement of the electric dipole sum rule for medium-A nuclei is 1.6, which may be expressed as

$$1+\kappa \simeq (940/M^*)(\rho_s/\rho)+\kappa_\pi = 1.6 \quad , \tag{9}$$

where ρ and ρ_s are, respectively, the ordinary and Lorentz-scalar densities of the nucleus. In calculating the theoretical ratio $f_{\rm theor}/C_{\rm theor}$, relativistic corrections have been inserted according to the relativistic

TABLE I. Values of the effective energy where missing strength may be found, for two assumed values of the fractional missing strength, $\hat{\chi}=\chi/(1+\chi)$, and for two different sets of measurements, for fixed three-momentum transfer $q=410$ MeV/c, as functions of the effective nucleon mass M^* in the relativistic Fermi gas. Also tabulated are the pionic contribution to the dipole sum rule and the relativistically corrected version $\langle\epsilon\rangle$ of the average quasielastic kinetic energy (given nonrelativistically by $q^2/2M^*$).

M^* (MeV)	κ_π	$\langle\epsilon\rangle$ (MeV)	$w_{\rm eff}$ (MeV) ^{40}Ca ($f_{\rm expt}/C_{\rm expt}=98.7$[a]) $\hat{\chi}=20\%$	$\hat{\chi}=50\%$	^{56}Fe ($f_{\rm expt}/C_{\rm expt}=107$[b]) $\hat{\chi}=20\%$	$\hat{\chi}=50\%$
940	0.616	83	89	94	56	86
900	0.573	86	105	101	73	93
850	0.515	91	127	110	94	102
800	0.451	95	151	119	119	111
750	0.378	101	177	130	145	122
700	0.295	107	207	142	174	133
650	0.200	113	239	155	207	146
600	0.093	120	275	169	243	161
550	0.0	128	315	185	283	177

[a]M. Deady, private communication. [b]R. Altemus, private communication and Ref. 1.

Fermi gas model.

We see from Table I that, for reasonable values of M^* (the value which best fits the quasielastic spectrum is ~700 MeV), the assumed missing strength cannot lie in uncharted realms of the energy-loss spectrum, but would have to lie precisely where the measurements have been made. This result is sensitive neither to the assumed value of the Fermi momentum p_F nor to the estimate of the pionic enhancement of the electric dipole sum rule. The values of $w_{\rm eff}$ derived from the two sets of experiments, in which the targets were ^{40}Ca and ^{56}Fe, are mutually consistent within the quoted experimental errors. (It is perhaps worth noting that, barring systematic errors, the relative uncertainties in integrated quantities such as $f_{\rm expt}$ and $C_{\rm expt}$ are smaller by roughly $1/\sqrt{N}$ than those in the individual experimental points, where N is the number of points in the integral. Thus in this case the errors are $\lesssim 10\%$. Moreover, since only the ratio $f_{\rm theor}/C_{\rm expt}$ enters here, errors in overall scale will not affect Table I.) We may therefore conclude that the requirement of consistency between the f-sum rule and the LSR precludes the possibility that any significiant longitudinal strength remains at energies too high to be measured by the present experiments. Although the results are not presented here, I have performed the corresponding calculations for the experiments at lower momentum transfers, and found similar consistency between the f sum and the LSR. This tends to confirm the findings of Viollier and Walecka[9] that deviations from the asymptotic form of the LSR should be smaller than 10% or so for $q/p_F > 1.5$.

Thus we may reiterate that only two explanations for the present experiments on longitudinal strength functions now survive: First, that the experiments themselves contain some hitherto-overlooked systematic errors, and second, that the impulse approximation fails in the manner suggested earlier.

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