Analytic structure of the $\bar{p}$-nucleus optical potential

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(Received 16 August 1982)

The positions of zeros and poles of the optical model S matrix are tracked for the case of antiproton scattering from ^{16}O. It is found that moderately rapid energy variations can arise from zeros near the real axis and that this is a likely occurrence for expected strengths.

[NUCLEAR REACTIONS Scattering theory, antiproton-nucleus optical model, S-matrix analytic structure.]

In a recent paper[1] Auerbach, Dover, and Kahana point out that for the $\bar{p}$-nucleus optical potential there may exist resonancelike structures in the elastic scattering cross section. They consider the 180° differential cross section and, by comparing (for ^{16}O) three different sets of Woods-Saxon potentials, they show that the cross section can be much enhanced at backward angles for certain energies. The present paper seeks to look at the same cases but to study the S-matrix behavior in the complex energy plane so that the role of isolated structures can be made clear.

Note first that the amplitude at 180° is given by

$$f=\frac{1}{2ik}\sum(-l)^l(1-S_l)(2l+1) \; .$$

The general behavior of the magnitude of an S-matrix element as a function of (real) energy is for it to be 1 up to some energy and then to decrease rapidly. Of course, S_l may show phase changes as a function of energy due to nearby zeros and poles. For strongly absorbing potentials there is a tendency for S_l to remain real. Note that the sum for $S_l=0$ or 1 gives 1, -2, $+3$, -4, . . . , so that there will be an oscillation each time a new l comes into the picture (whenever $|S_l|$ goes to zero). In fact, the S_l's go to zero gradually so that the actual diffraction pattern is observed. If the S_l's vanish very smoothly the oscillation is held close to its central (zero) value and the cross section is small. If this smoothness is disturbed the oscillations become greater and the cross section becomes (on the average) larger. We can see this behavior by plotting

$$f_l\equiv\left|\frac{(2l+1)(1-S_l)}{2k}\right| \tag{2}$$

as a function of energy.

Figure 1(c) shows these quantities for the case (c) of Ref. 1, i.e., $V=100$ MeV, $W=200$ MeV, $r_{0R}=r_{0I}=1.2$ fm, and $a_R=a_I=0.52$ fm. As may be seen, the behavior is very smooth and the cross section is correspondingly small.

Figure 1(b) shows the same functions for the case (b) of Ref. 1, i.e., $V=300$, $W=100$ MeV, $r_{0R}=r_{0I}=1.2$ fm, and $a_R=a_I=0.52$ fm. The behavior is less smooth and the cross section is correspondingly larger.

Figure 1(a) shows the case (a) of Ref. 1 (the one

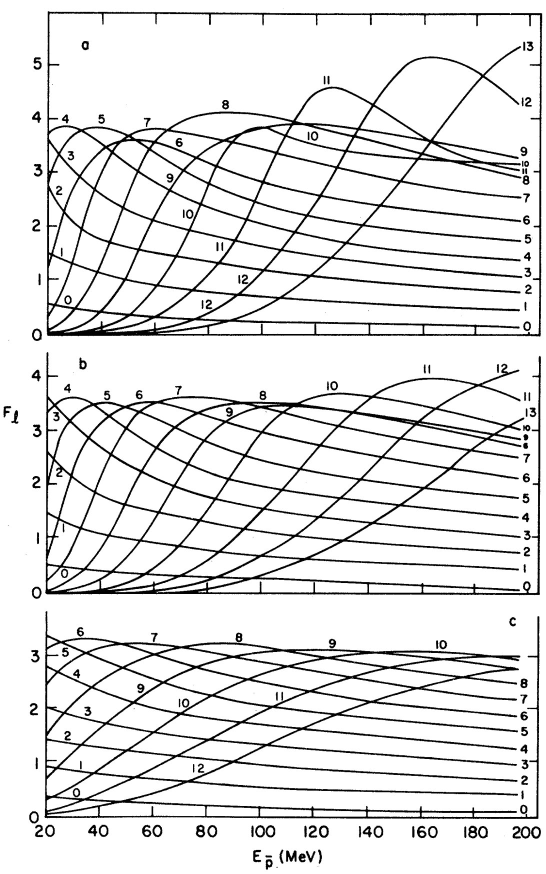

FIG. 1. (a)–(c) Comparison of the quantities F_l [defined by Eq. (2) in text] for the three cases of elastic $\bar{p}$ scattering from ^{16}O considered in Ref. 1.

hoped to be similar to the physical case), i.e., $V=300$, $W=100$, $r_{0R}=1.3$ fm, $r_{0I}=1.1$ fm, and $a_R=a_I=0.52$ fm. Here the behavior is most irregular and gives a much larger cross section. The object is to try to understand this behavior in terms of structures in the complex energy plane.

To this end the Schrödinger equation was solved for complex energies with the corresponding generalized boundary conditions in terms of "ingoing" and "outgoing" Hankel functions of complex momentum. This brute force method works within certain limits. To understand these bounds one notes that h^+ and h^- now differ (at $r\sim 10$ fm) by many orders of magnitude. To disentangle the amount of ingoing and outgoing solutions (which is necessary to obtain the S matrix) an accurate solution is necessary. The larger the matching radius the more accurate the solution must be.

A second condition is given by the constraint that the matching must be done "outside" the potential. That is to say, the ingoing and outgoing pieces must not be mixed by the potential. This requires that the matching radius be large. The further from the real axis the calculation is attempted, the more the two limits approach each other and, at some point, the calculation is not possible.

If an exact solution to the exponential potential were available (for $l>0$) the matching could be done to these functions and the matching radius could be moved in. In fact, the matching was done to a Born approximation representation of these functions, which improved errors somewhat.

In any case, there is a limit to the amount one can go away from the real axis, and this causes a limited field of view. For the present case reasonable calculations were possible for $|\mathrm{Im}E|\lesssim 76$ MeV.

The zeros and poles of the S matrix were located by printing out the magnitude of S_l in one plot and the phase of S_l in another. The resolution of these plots was 4 MeV and, while some interpolation was done, the accuracy of the pole and zero positions cannot be considered to be much better than this. Since the purpose of this paper is to find qualitative insight into the analytic structure of S_l, this is deemed to be adequate.

Let us first consider a real potential. For the weak case we find a set of zero-pole pairs symmetrically placed about the real axis at increasing distances for increasing real energy. Note that the zeros and poles always occur in pairs. It is often useful to consider the meromorphic part of the S matrix in the form of an infinite product[2]

$$S\sim\prod_\alpha \frac{E-E_\alpha-i\Gamma_\alpha/2}{E-E_\alpha+i\Gamma_\alpha/2}\ . \tag{3}$$

The absolute value of each factor is 1, while the numerator (zero) provides a factor $e^{i\delta}$ and the denominator (pole) provides another factor $e^{i\delta}$. Thus the zero and pole combine to each give "half" of the resonance. It is these standard "single particle" poles which give the smooth behavior seen in Fig. 1(c).

If the strength of the real potential is increased, pockets of confining potential can form in the surface region giving "orbiting" states. The behavior of the poles due to this effect is shown in Fig. 2 for the case of $l=8$. When the potential strength reaches about 70 MeV, a pole-zero pair comes into the field of view (curve A). Only the zeros are plotted since the poles are known to be symmetrically placed for real potentials. The position of the zero-pole pair approaches the real axis and decreases in energy. The pair shown at $V=300$ MeV would cause a very narrow resonance (if there were no imaginary part to the potential). At a strength of about 200 MeV, a second zero-pole pair enters the field of view. Thus for the case to be considered with $V=300$ MeV, there are two orbital zero-pole pairs to be dealt with. The variation of the position of the zero-pole pair with "a" is also shown. Very narrow resonances could also be obtained in this way (for unrealistic values of a).

Also in Fig. 2 is illustrated the "rotation" of a zero-pole pair as an imaginary potential is turned on. The zero moves toward the real axis and the pole away. This causes the magnitude of S_l to decrease, in some cases very rapidly, with an attendent structure in the cross section. Of course, the "standard" poles are moving away and the corresponding zeros approaching the real axis as well (not shown).

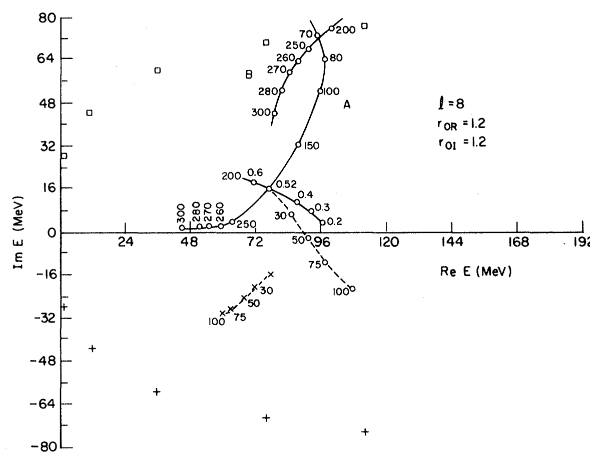

FIG. 2. Behavior of zeros and poles of S_l for $l=8$. The solid lines correspond to a purely real potential and the dotted curves show the variation with W. The symbol □ shows the location of (nearly) fixed zeros and + shows the location of corresponding poles. For the purely real cases, mirror poles exist but are not shown. The symbol o denotes zeros corresponding to orbiting states and × denotes their poles.

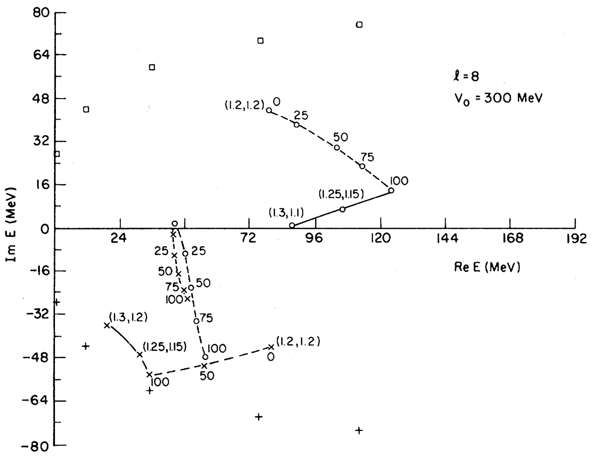

FIG. 3. Behavior of zeros and poles of S_l for $l=8$. The dotted curve shows the motion under the influence of W and the solid curve its motion under the change of the pair (r_{0R}, r_{0I}). The meaning of the symbols is the same as Fig. 2.

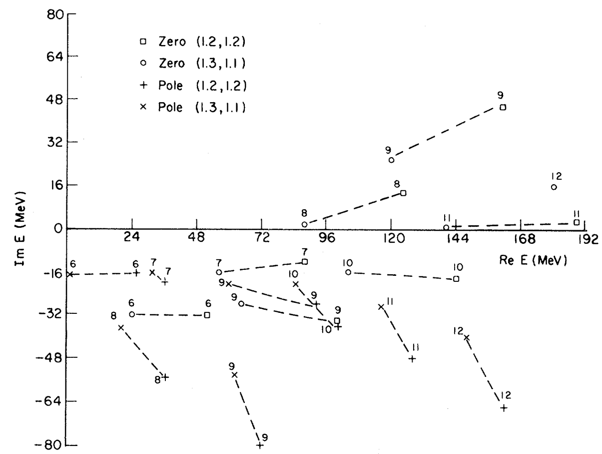

FIG. 4. Motion of the zeros and poles of various S_l. The dotted curve connecting the points is only meant to connect them logically and does not imply that the motion follows a straight line path.

In Fig. 3 the real potential is fixed at 300 MeV and the imaginary part of the potential is varied up to 100 MeV. Then the pole-zero pair is followed as we go to case (a) ($r_{0R}=1.3$, $r_{0I}=1.1$). As can be seen, the pole is now playing a very small role and the zero is dominating this partial wave around 120 MeV. In some sense one is left with a very strong "half resonance."

This is by no means always the case, however, as can be seen from Fig. 4. The behavior of the $l=10$ case can now be understood since the pole and zero are about equidistant from the real axis but on the same side. Thus Fig. 1(a) shows a rapid rise and a rapid drop in this partial wave. The strong enhancement of the $l=8$ and $l=11$ partial waves is seen to be due to the lowering of the position of the zero to near the real axis.

The importance of zeros lying near the real axis can be seen in the case of compound nucleus reactions,[2,3] but the present case shows their effect also in the "direct" part of the amplitude.

The situation is seen to be one of moderate complexity with a handful of partial waves dominating the behavior in any given energy region. The study of the zeros and poles of the S matrix experimentally would appear to give a useful characterization of the $\bar{p}$-nucleus interaction.

This work was done under the auspices of the Department of Energy.

[1]E. H. Auerbach, C. B. Dover, and S. H. Kahana, Phys. Rev. Lett. 46, 702 (1981).

[2]W. R. Gibbs, Phys. Rev. 181, 1414 (1969).

[3]W. R. Gibbs, Phys. Lett. 103B, 281 (1981).