Hybrid quark model of Λ nonmesonic decay in nuclei

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It is shown in the hybrid quark-two-baryon model of a ΛN cluster in nuclei that the short distance, six-quark part dominates the nonmesonic decay. For a six-quark probability of 15% for a nuclear cluster the nonmesonic lifetime is $\tau_\Lambda^{\rm NM} \approx \frac{1}{3}\tau^{\rm free}$, which is consistent with present experiment.

[NUCLEAR STRUCTURE Hypernuclear structure. Calculation of lifetime of Λ in hypernuclei. Hybrid pion-six quark model of $\Delta s = 1$ weak nuclear interactions. Quark structure of nuclei and hypernuclei.]

I. INTRODUCTION

In the study of Λ and Σ hypernuclei, atomic nuclei in which one or more Λ's or Σ's have replaced nucleons, a most basic property is the lifetime of the system. This is a fundamental aspect of the nature of these hyperons and of their interactions. For Σ's it is also of the greatest practical importance, since a rich program of study of Σ hypernuclei, comparable to Λ hypernuclei, can only be carried out if there exist Σ hypernuclear states with widths narrow compared to their separations.

The free decays of both the Λ and Σ hyperons are mainly weak mesonic decays. However, their nonmesonic decays are predominant when they are bound in complex nuclei. Knowledge of these nonmesonic processes is not only necessary for the understanding of Λ- and Σ-hypernuclear lifetimes, but provides important information about Λ-N and Σ-N interactions. Although the Σ nonmesonic decay is a strong interaction, while the Λ decays only via weak interactions, the physics of their nonmesonic decays is similar, and it is worthwhile to try to treat these processes together. Because of the large relative momentum of the final-state baryons, both of these decays involve short-distance nuclear behavior. It has been shown that short-range correlations play an important role in determining both the Λ (Ref. 1) and Σ (Ref. 2) lifetimes. Recently, other calculations of Σ widths have been carried out[3,4] without including the effects of such correlations, with quite different results.

Both of these calculations[1,2] of correlation effects were based on one pion exchange (OPE) forces, weak for the Λ and strong for the Σ. It is quite obvious that OPE is inadequate for the treatment of short-range interactions. One purpose of the present paper is to explore the effects of OPE in comparison with other (short-range) mechanisms. The main purpose is to show that the hybrid model for baryon-baryon systems,[5] with a quark shell model representing short-distance effects, can be used to treat these processes, and that they in turn give important new information about the quark structure of nuclear systems. Only the Λ will be treated in detail here.

In conventional meson-exchange models of baryonic interactions, pion exchange gives rise to the long-range potentials, with ranges $\lesssim m_\pi{}^{-1} = 1.4$ fm, while to the exchange of vector mesons are attributed short-range forces of ranges $\lesssim m_\rho{}^{-1} = 0.25$ fm. However, nucleons are complex, extended quark structures with radii ≈ 1 fm. Although one does use phenomenological Yukawa forms for short-range interactions based on such heavy meson exchange, the picture has not been justified theoretically, and could be incorrect in important aspects.

The hybrid model for two nucleons makes use of projection operators in coordinate space to define the NN wave functions as conventional nuclear wave functions for $r \gtrsim r_0$ and six-quark relativistic shell model wave functions for $r \lesssim r_0$, with $r_0 \approx 0.8$ fm. It has been shown[5] that it can give a description of electromagnetic properties of the deuteron as an alternate to the conventional treatment, with quark electromagnetic processes replacing heavy-meson currents. The model also can account for the asymmetry in pp elastic scattering which has been observed at low energies, using[6] a treatment of $\Delta S=0$ weak interactions similar to the weak interaction theory for $\Delta S=1$ used here. In the present work the hybrid model is extended to hyperon-nuclear systems. In Sec. II we review the hybrid model and give a qualitative discussion of the short-range nature of nonmesonic Λ decay. In Sec. III the model is developed for the $\Lambda N\to NN$ weak nonmesonic decay, and the results for this model are discussed in Sec. IV.

II. Λ AND Σ NONMESONIC DECAY AND SHORT-DISTANCE NUCLEAR PROPERTIES

A. Cluster model

In this subsection a qualitative description of Λ and Σ nonmesonic decay is given, with emphasis on the short-range nature of these processes. Consider the reactions

$$\Lambda+N\to N+N \;, \tag{1a}$$

$$\Sigma_0+N\to\Lambda+N \;, \tag{1b}$$

which can follow a (K^-,π^-) reaction in a nucleus. We consider these reactions as occurring in the hypernuclear ground state, with the initial ΛN or $\Sigma_0 N$ system being a two-baryon cluster in the bound state. In this subsection we shall review the calculation of the reaction widths of such clusters using typical cluster wave functions.

An essential part of the physics of both processes is that large momentum transfer is involved. Let us consider the kinematics. Taking the initial cluster at rest and neglecting binding effects, which only slightly modify the kinematics, one finds that for process 1(a) the final momenta of the two nucleons is $k_1=k_2=417$ MeV/c, while for process 1(b) the momentum of the nucleon or Λ is $k_1=k_2=283$ MeV/c. Thus the final relative momentum

$$q=|k_1-k_2|/2 \tag{2}$$

is 2.11 fm^{-1} for the $\Lambda N\to NN$ and 1.43 fm^{-1} for the $\Sigma_0 N\to\Lambda N$ decays. At such momentum transfers short-distance properties of nuclei enter, and indeed the quark structure of nuclei could be deeply involved. In order to illustrate this let us use the simple model of Ref. 2. Neglecting final state distortions, the widths for reactions (1a) and (1b) satisfy

$$\Gamma\propto\left|\int d^3r\, e^{i\vec{q}\cdot\vec{r}}\, V\Psi_i(\vec{r})\right|^2 \;, \tag{3}$$

where Ψ_i is the initial cluster wave function and V is the interaction leading to the width. Since $q>1$ fm^{-1} for both reactions, it is clear that the behavior of Ψ_i at small r is important. In Ref. 2 it was shown for $\Sigma_0 N\to\Lambda N$ with V given by one pion exchange that there is indeed strong sensitivity. In that work a cluster wave function with a radial form

$$R_i(r)\alpha(e^{-\alpha r}-e^{-\beta r})$$

was used, and it was shown that Σ_0 widths $\Gamma_{\Sigma_0}\approx 1-2$ MeV are predicted. This small width, to be compared to $\Gamma\approx 50$ MeV for an impulse approximation treatment, is due to the short-distance nuclear repulsion, which greatly reduces the matrix element.

In the present work we treat process (1a), which involves considerably higher relative momentum for the final state projectiles. Moreover, these final particles have momenta considerably larger than the nuclear Fermi momentum, so that the treatment given in the preceding paragraph is even more relevant than for the Σ_0 nuclear decay. Thus for the understanding of the lifetime of Λ's in nuclei we can expect very short-distance behavior to be the dominant nuclear physics. Therefore, this will involve the quark structure of hypernuclei in a crucial way.

B. Review of the hybrid model

A complete quark model description for Λ or Σ hypernuclei is not available at the present time, since nuclei are low-density systems and quark/QCD models have not been developed for accurate descriptions of nuclear processes at distances $\gtrsim 1$ fm. Here we use the hybrid model of Ref. 5, which combines two successful models for hadrons: the Schrödinger model with an NN potential for large distances and a quark bag model to describe the system at short distances. The objective is to see how much one can learn about the quark structure of nuclei without introducing specific dynamic models. For example, the effects of quantum chromodynamics (QCD) are *represented,* rather than calculated.

The wave functions are defined in terms of projections. For a two-baryon system, the wave function is

$$\Psi_{BB}=\begin{Bmatrix}\Psi_{BB}(r), & r>r_0\\ \Psi_{6q}, & r<r_0\end{Bmatrix} , \tag{4}$$

where r is the distance between the centers of mass of the two baryons, $\Psi_{BB}(r)$ is a conventional wave function, and Ψ_{6q} is a six-quark wave function given in a quark shell model by

$$\Psi_{6q}=\sum_i C_i\Psi_{6q}^{(i)} \;, \tag{5}$$

where C_i are spectroscopic amplitudes and $\Psi_{6q}^{(i)}$ are wave functions for various six-quark configurations.

The Ψ_{6q}^{i} are composed of products of single-quark bag wave functions, ϕ_v, which satisfy the MIT bag[6] boundary condition which gives color current confinement at the boundary $r=r_0$,

$$i\vec{\gamma}\cdot\hat{r}\phi_v\,|_{r_0}=\phi_v\,|_{r_0}\ . \tag{6}$$

These latter are simply the stationary wave functions for a Dirac particle in a uniform potential

$$\phi_v=\mathcal{N}\begin{bmatrix} j_l(\kappa_v r)[Y_l \chi]_j \\ j_{l'}(\kappa_v r)[Y_{l'}\chi]_j \end{bmatrix}, \tag{7}$$

so that the upper components are the Schrödinger wave functions for a spin $\frac{1}{2}$ particle with angular momentum j in a uniform potential. The κ_v are wave number eigenvalues determined by the boundary condition of Eq. (6). Note that the quark part of the wave function is the inside part of the complete two-baryon wave function, and thus the second MIT boundary condition concerning energy flow does not apply. For this reason all j values in Eq. (7) are possible, and there is a complete set of states in the spherical representation being used. Note also that both spherical and deformed states can be represented with this spherical basis, as in the nuclear shell model.

Since the $\Psi_{6q}^{(i)}$ form a complete set within the interior region, given an effective quark-quark Hamiltonian (e.g., derived from QCD) it is always possible to represent the interior region with a wave function of the form of Eq. (5) for the valence quarks. In previous work[5] the spectroscopic amplitudes were simply phenomenological parameters, with $\sum C_i^2=P_{6q}$ = the six-quark probability. In Ref. 5 it was shown that vector meson current contributions can be reproduced with $\sum_i C_i^2\approx 0.05$ for the deuteron and $r_0\approx 0.8$ fm. In other words, about 5% of a deuteron is best described as a six-quark system. More recently,[7] further theoretical developments have enabled us to determine a number of these parameters for the p-p system. However, for the present calculations we present our results in terms of phenomenological six-quark probabilities, since we are stressing the use of experiments on Λ decay to learn about hypernuclear quark structure. In the future we hope to be able to determine from theory the values of the most important C_i for the ΛN and ΣN coupled systems, which will enable us to predict various hypernuclear properties.

The Hamiltonian also is projected into an inside and outside part:

$$\mathcal{H}=\begin{cases}\mathcal{H}_{\pi}, & r>r_0\\ \mathcal{H}_{qq}, & r<r_0\end{cases}, \tag{8}$$

where $\mathcal{H}_\pi$ is the one pion exchange (OPE) interaction Hamiltonian, and $\mathcal{H}_{qq}$ the effective quark interaction Hamiltonian. As we gain more experience with this model, we shall add uncorrelated two-pion contributions to the outside part. Also, in principle there can be surface terms in the Hamiltonian which connect the inside and outside parts, but these can be removed and represented by the inside and outside effective Hamiltonians.

The Λ decay is an especially favorable process for this model, since the weak quark Hamiltonian is quite well known [and thus H_{qq}^{w} of Eq. (8) is known]. For the strong processes, such as those involved in $\Sigma N\rightarrow\Lambda N$, the quark Hamiltonian is not as well known. For this reason we shall not treat process (1b) in the present paper.

III. HYBRID MODEL FOR Λ DECAY IN NUCLEI

In its free decay the lifetime of the Λ

$$\Lambda^{\text{free}}\rightarrow\pi+N(\text{weak}) \tag{9}$$

is

$$\tau_\Lambda^{\text{free}}=2.63\times10^{-10}\ \text{sec}\ . \tag{10}$$

Noting that the energy of the N in the free Λ decay is $E_N\approx 5$ MeV, and that the Λ shell model potential in nuclei is -32 MeV, it is clear that the Pauli principle will strongly inhibit the mesonic decay in hypernuclei. It has long been recognized[8] that the nonmesonic process

$$\Lambda+N\rightarrow N+N\ (\text{weak}) \tag{11}$$

will probably dominate the decay for heavy nuclei, and the reactions of Eqs. (11) and (9) have been used to analyze[9] the hypernuclear Λ decay. However, due to the high momentum transfer, short-distance Λ-N

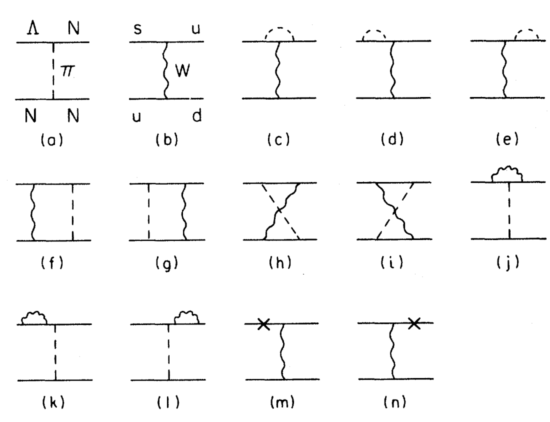

FIG. 1. (a) and (b) Lowest order long-range and short-range $\Delta S=1$ interactions in a hybrid model. (c)—(k) One loop and renormalization diagrams used in renormalization group procedure. See Ref. 11.

properties must be carefully considered, as discussed in the previous section. For example, in the calculation by Adams[1] it was shown that with a weak interaction arising from pion exchange a hard core nucleon-nucleon repulsion reduces the nonmesonic ratio by more than an order of magnitude. This is an excellent situation for application of the hybrid model.

A. Weak interaction Hamiltonian

We shall calculate the nonmesonic process using the Hamiltonian symbolically given in Eq. (8). For the pion contribution we use the usual model, with a weak potential given for $r > r_0$ corresponding to Fig. 1(a). The interaction Hamiltonian is given by [see, e.g., Ref. 1 and Fig. 1(a)]

$$H_\pi^w(r>r_0)=\frac{g_w g_\pi}{4\pi}[V_1(r)\vec{\sigma}_2\cdot\hat{r}+V_2(r)\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r}+V_3(r)\vec{\sigma}_1\cdot\vec{\sigma}_2]\tau_1\cdot\tau_2\ , \tag{12}$$

with

$$V_1(r)=\frac{i}{2m}\frac{d}{dr}u(r)\ , \tag{13a}$$

$$V_2(r)=\frac{\lambda}{2M(M+M_\Lambda)}\frac{d^2}{dr^2}u(r)\ , \tag{13b}$$

$$V_3(r)=\frac{\lambda}{2M(M+M_\Lambda)}\frac{1}{r}\frac{d}{dr}u(r)\ , \tag{13c}$$

where $u(r)=e^{-mr}/r$. The coupling constants have been determined empirically (see, e.g., Ref. 1). We use $G_w g_\pi/4\pi=9.6\times10^{-8}$ and $\lambda=-6.7$. It is understood that H_π^w of Eq. (12) is to be used only in the outside region, $r>r_0$. For the inside region $r<r_0$, the weak Hamiltonian is given by the quark model. In the absence of strong interactions, the intermotion Hamiltonian is the standard Cabbibo model for $\Delta S=1$, corresponding to Fig. 1(b),

$$H_{6q}^{w(0)}(r<r_0)=\frac{G}{\sqrt{2}}\sin\theta_c\cos\theta_c\bar{u}\gamma_\mu \times(1-\gamma_5)s\,\bar{d}\gamma^\mu(1-\gamma_5)u+\text{c.c.}, \tag{14}$$

which corresponds to the W exchange of Fig. 1(b) for energy $\ll M_w$. We restrict ourselves to the 1s quarks with single-quark wave functions given by[10]

$$\phi_{s_{12}}=\frac{N}{\sqrt{4\pi}}\begin{bmatrix} j_0(\kappa_0 r)\ \chi \\ -j_1(\kappa_0 r)\ \ \vec{\sigma}\cdot\hat{r}\chi\end{bmatrix}\ . \tag{15}$$

Taking the quarks in a singlet state, one finds for the basic two-quark weak matrix element

$$\langle ud;J=0\,|\,H_{6q}^{w(0)}\,|\,su;J=0\rangle =\frac{G}{\sqrt{2}\pi}\sin\theta_c\cos\theta_c N^4 R, \tag{16}$$

where

$$R=\int_0^{r_0}dr\,r^2(j_0{}^2(\kappa_0 r)+j_1{}^2(\kappa_0 r))^2\ . \tag{17}$$

Assuming that the Λ-N pair is $\frac{1}{2}\Lambda p+\frac{1}{2}\Lambda n$, and taking $r_0=0.8$ fm,[5] and the quark probability[11] of 5%, one finds

$$\langle NN\,|\,H_{6q}^{w(0)}\,|\,\Lambda N\rangle=3.38\text{ eV}\ . \tag{18}$$

Recall that this result [Eq. (18)] holds only for "bare" quarks.

One of the most important recent developments in the treatment of hadronic weak interactions is the derivation of the effective weak Hamiltonian for systems of quarks with the inclusion of strong interaction effects. This has been carried out by using renormalization group techniques based on one-loop diagrams of W's and gluons, such as those of Figs. 1(c) and (d), plus renormalization terms. The form of the weak Hamiltonian becomes

$$H^w\approx\frac{G}{2\sqrt{2}}\sin\theta_c\cos\theta_c[(K^{\gamma_1}+K^{\gamma_2})\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)u-(K^{\gamma_1}-K^{\gamma_2})\bar{d}\gamma_\mu(1-\gamma_5)s\bar{u}\gamma^\mu(1-\gamma_5)u]\ , \tag{19}$$

where γ_1,γ_2 are anomalous dimensions and K is determined by the QCD coupling constant α_s. Using the values of Gilman and Wise,[12] one obtains

$$H^w\approx\frac{G}{\sqrt{2}}\sin\theta_c\cos\theta_c[1.51\bar{u}\gamma_\mu(1-\gamma_5)s\bar{d}\gamma^\mu(1-\gamma_5)u-0.856\bar{d}\gamma_\mu(1-\gamma_5)s\bar{u}\gamma^\mu(1-\gamma_5)u]\ . \tag{20}$$

Evaluating this interaction in the states of Eq. (18), one finds

$$\langle NN\,|\,H_{6q}^{w}\,|\,\Lambda N\rangle=0.65\langle NN\,|\,H_{6q}^{w(0)}\,|\,\Lambda N\rangle =2.2\text{ eV}\ . \tag{21}$$

That is, there is a 35% reduction in the matrix element. Note that the treatment leading to Eq. (20) greatly improves the $\Delta I=\frac{3}{2}$ to $\Delta I=\frac{1}{2}$ ratio, but still does not give the $\Delta I=\frac{1}{2}$ rule. If one uses the form of Eq. (19) and arbitrarily chooses α_s to fit the

$\Delta I = \frac{1}{2}$ rule, then the reduction of the matrix element is 64%. This indicates that there is still some uncertainty in the result of Eq. (20) and thus of Eq. (21). Note however, that although Eqs. (19) and (20) are approximate, the operators which are used are the ones best determined by the renormalization group method,[12] and H^w is quite accurate for our purposes.

B. The many-body model

In an energy-independent potential description, the Λ-N potential, like the N-N potential, consists of a short-range repulsive core and a longer range attractive region. Thus the Brueckner many-body methods, which were developed for ordinary nuclei, should be satisfactory for the treatment of hypernuclei. This suggests a bound-state reaction matrix approach such as has been used by Dabrowski and Rozynek.[4,13] Since we are only calculating a weak decay width here, this is equivalent to calculating a weak transition matrix element using the solutions to the Bethe-Goldstone equation for the hypernuclear wave functions. Thus the width is given by

$$\Gamma = (2\pi)^{-2} \int_{<k_f} d^3k \int Q_N(\vec{k}, \vec{k}_f) \rho_f \times \sum_{i,f} |T_{f,i}|^2 , \tag{22}$$

where ρ_f is the density of final states and $Q_N(\vec{k}, \vec{k}_f)$ is the Pauli exclusion operator. Since the free $\Lambda N \rightarrow NN$ decay leads to nucleons with a momentum of 417 MeV/$c \gg k_f$, $Q_N \approx 1$ for this process. The transition matrix element is given by

$$T_{\rm fi} = T_{6q} + \langle \phi_f(NN) | H_\pi^w | \phi_i(\Lambda N) \rangle . \tag{23}$$

The final state NN wave function $\phi_f(NN)$ is calculated using an eikonal distortion with the standard nuclear optical potential. For the initial state wave function, $\phi_i(\Lambda N)$, the solution to the Bethe-Goldstone equation $(r > r_0)$

$$(\text{Kinetic energy} + K_{\Lambda N}) \phi_i = \epsilon \phi_i , \tag{24}$$

where $K_{\Lambda N}$ is the reaction matrix, is approximated by the form[14] used by Adams,[1] with a hard core of 0.4 fm. As will be evident, the details of the bound state wave function are not important so long as the repulsive correlation is included.

IV. RESULTS AND CONCLUSIONS

The numerical value for lifetime for nonmesonic (NM) decay of the Λ given by the calculation described in the previous section is approximately three times the free Λ lifetime if one uses 0.05 for the fraction of the six-quark bag in the two baryon state, which was the result obtained for the deuteron.[5] Therefore, in terms of the $6q$ part of a two baryon cluster in a complex nucleus one obtains

$$\tau_\Lambda^{NM} \approx 9.8 \times 10^{-10} \left[\frac{0.05}{P_{6q}} \right]^2 \text{sec}$$

$$\approx 3 \times \tau_\Lambda^{\text{free}} x \left[\frac{0.05}{P_{6q}} \right]^2 , \tag{25}$$

where P_{6q} is the average of the initial and final six-quark probabilities. Another important result of our calculation is that the pion contribution is very small:

$$\tau_\Lambda^{\text{NM}(\pi)} \approx 0.16 \times 10^{-6} \text{ sec} \gg \tau_\Lambda^{\text{NM}(6q)} . \tag{26}$$

This explains the very large lifetime found by Adams.[2] In a recent calculation[15] in the conventional weak interaction model, there was found an approximately equal contribution of π and ρ mesons, with $\tau_\Lambda^{\text{NM}} \approx \frac{1}{3} \tau_\Lambda^{\text{free}}$. In our model the π contribution vanishes for $r < r_0$, which accounts for some, but not all, of the difference in the pion contribution in these approaches. Also, the weak ρ coupling, which should be considered as an essentially phenomenological weak potential rather than a process derived from rho mesons (due to the very short range), is not known. Thus it is most useful to consider the Λ nonmesonic lifetime as a means of determining the ΛN "ρ" coupling in the conventional weak model.

For the hybrid model essentially all of the $\Lambda N \rightarrow NN$ process takes place within the $6q$ regions. Note that if $P_{6q} \approx 15\%$ for complex nuclei, which is not unexpected since the internucleon distance in the deuteron $\gtrsim 1.5$ times that of complex nuclei, one obtains from Eq. (25)

$$\tau_\Lambda^{\text{NM}} \approx \frac{1}{3} \tau_\Lambda^{\text{free}} , \tag{27}$$

which is compatible with present experimental information.[16] This result is consistent with the conventional treatment[15] of weak nuclear reactions. Note that in a recent study of inclusive electron scattering[17] from ^{3}He it has been concluded that $P_{6q} \approx 15\%$. A model[18] of the He form factors including quark components is also compatible with these results.

We conclude that the nonmesonic decay of Λ's in complex nuclei is a short-distance phenomenon which is best described via the $6q$ part of the ΛN system, using the fundamental gauge theory of weak interactions. Further experiments on hyperon decays in nuclei can teach us more about the quark structure of nuclear systems.

Note added in proof: Bruce McKellar has informed us that the coupling constant of Ref. 1 is too small. Even with his suggested change, our OPE contribution is very small.

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