# Dynamical symmetries of the U(4) model and high-lying states in the $^{20}$Ne, $^{28}$Si, and $^{30}$Si nuclei

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High-lying states of $^{20}$Ne, $^{28}$Si, and $^{30}$Si produced in $\alpha$ scattering are compared with the spectra corresponding to the two dynamical symmetries of the U(4) model. The $^{20}$Ne system prefers the O(4) limit, while in the other cases, where the experimental spectra are less complete, there are no such preferences.

[ NUCLEAR REACTIONS $^{16}$O, $^{24,26}$Mg($\alpha,\alpha$) resonances, application of the algebraic model. ]

Recently an interacting boson model was proposed by Iachello to describe the nuclear molecular states.[1] In this model $s$ ($L=0$) and $p$ ($L=1$) bosons are taken into account, indicating the dominant role of the dipole degrees of freedom.

One of the oldest and best known examples of nuclear molecules are the "core + $\alpha$-particle" states, so it is desirable to explore the applicability of the model to their case. The present paper is meant to be a contribution to the solution of this task.

The first step in the application of the interacting boson model is usually a comparison between the experimental results and the prediction of the model for the simple limiting case, in which a dynamical symmetry is valid. The model which includes $s$ and $p$ bosons has a group structure of U(4) and hence has two dynamical symmetries. These are characterized by the following group chains: (I) U(4) $\supset$ O(4) $\supset$ O(3), and (II) U(4) $\supset$ U(3) $\supset$ O(3). The O(2) group at the ends of both chains is neglected since the projection of the angular momentum has no importance in our case. The O(4) limit is discussed in detail in Ref. 1; it was applied to the case of the $^{12}$C + $^{12}$C quasimolecular states[2] and, recently, preliminary results of its application to the case of some core + $\alpha$-particle states have been reported.[3] In this paper both dynamical symmetries are considered. For this purpose it is convenient to express the Hamiltonian in terms of Casimir operators. The general form is

$$H=\alpha C_{2O4}+\beta C_{2O3}+\gamma C_{1U3}+\delta C_{2U3}+\epsilon \ , \qquad (1)$$

where $C_{2O4}$ is the quadratic Casimir operator of the O(4) group, $C_{1U3}$ is the linear Casimir operator of the U(3) group, etc., while $\epsilon$ is an additive constant showing the location of the molecular bands. The dynamical symmetries I and II mean the special cases when $\gamma=\delta=0$ and $\alpha=0$, respectively. The expectation values of the Casimir operators can be determined just as for the U(6) model,[4] and the same applies to the group decomposition.[4,5] So, the two energy formulas used here are

$$E_{\rm I}=\alpha\omega(\omega+2)+\beta L(L+1)+\epsilon \ , \qquad (2)$$

$$E_{\rm II}=\beta L(L+1)+\gamma n_p+\delta n_p(n_p+2)+\epsilon \ . \qquad (3)$$

Here $\omega$ is related to the vibrational quantum number $v$ by $v=\frac{1}{2}(N-\omega)$, where $N$ is the total number of bosons,[1] and $n_p$ is the number of bosons with $L=1$ angular momentum. The possible values of these quantities are as follows:

In case I,

$$\begin{aligned}&\omega=N,N-2,N-4,\ldots,1,\text{ or }0 \ ,\\ &L=\omega,\omega-1,\omega-2,\ldots,0 \ ;\end{aligned} \qquad (4)$$

In case II,

$$\begin{aligned}&n_p=N,N-1,\ldots,0 \ ,\\ &L=n_p,n_p-2,\ldots,1,\text{ or }0 \ .\end{aligned} \qquad (5)$$

In order to see whether the special cases I and II of the U(4) model can describe the nuclear states of core + $\alpha$-particle types, some selected high-lying levels of $^{20}$Ne, $^{28}$Si, and $^{30}$Si are considered. They are classified according to (4) or (5) and fitted by (2) or (3). The experimental data are taken from $\alpha$-scattering works; it is this reaction from among those measured that is the most selective from the point of view of the $\alpha$ reduced widths ($\alpha$ spectroscopic factors). The $^{20}$Ne, $^{28}$Si, and $^{30}$Si data are those published in Refs. 6–8, respectively. All experimental levels of the $^{28,30}$Si are used,[7,8] while only levels of $E_\alpha<15$ MeV and unambiguous spin-parity are selected for the $^{20}$Ne case.[6]

Keeping in mind the rules (4) or (5), the $\omega$ or $n_p$ values are assigned according to the relative energy

 

TABLE I. The experimental level energies (in c.m. system and MeV) used for Fig. 1.

(a)
$^{16}O+\alpha$ system (Ref. 6)

| $L=0$ | $L=1$ | $L=2$ | $L=3$ | $L=4$ | $L=5$ | $L=6$ | $L=7$ | $L=8$ |
|---|---|---|---|---|---|---|---|---|
| 1.99 | 4.00 | 2.70 | 2.44 | 4.32 | 3.72 | 4.05 | 8.61 | 7.22 |
| 2.47 | 4.12 | 3.11 | 4.39 | 5.26 | 5.53 | 7.83 | 10.70 | |
| 3.92 | 6.50 | 4.08 | 5.67 | 5.82 | 7.95 | 9.58 | | |
| 6.24 | 7.23 | 4.77 | 6.11 | 6.06 | | | | |
| 7.69 | | 5.85 | 7.66 | 6.29 | | | | |
| | | 6.11 | | 7.20 | | | | |
| | | 6.59 | | 7.51 | | | | |
| | | 7.14 | | 8.04 | | | | |
| | | 7.62 | | | | | | |

(b)
$^{24}Mg+\alpha$ system (Ref. 7)

| $L=0$ | $L=1$ | $L=2$ | $L=3$ | $L=4$ |
|---|---|---|---|---|
| 2.99 | 2.83 | 2.74 | 3.26 | 3.99 |
| 3.06 | 2.99 | 2.91 | 3.85 | |
| 3.25 | 3.83 | 3.65 | 3.89 | |
| | 3.91 | 3.69 | 4.11 | |
| | | 3.96 | | |
| | | 4.00 | | |
| | | 4.08 | | |

(c)
$^{26}Mg+\alpha$ system (Ref. 8)

| $L=0$ | $L=1$ | $L=2$ | $L=3$ | $L=4$ |
|---|---|---|---|---|
| 3.13 | 2.85 | 3.61 | 4.07 | 4.42 |
| 3.36 | 3.21 | 3.98 | 4.27 | |
| 3.55 | 3.25 | 4.32 | | |
| 3.67 | 3.45 | | | |
| 3.73 | 3.67 | | | |
| 3.86 | 4.03 | | | |
| 3.95 | | | | |
| 4.25 | | | | |
| 4.36 | | | | |

differences. At this status of the model, in which the description involves only level energies, there is no other way for the assignment.[2] This procedure contains some ambiguity, but it becomes less and less when the experimental spectrum becomes more and more complete in the sense that the number of the measured levels comes near the number of the levels predicted by the model. From this point of view our most favorable system is that of the $^{16}O+\alpha$. The choice of $N$ is a delicate question. If the model had a microscopic foundation, $N$ would be defined by the microscopic structure, as it is in the U(6) model. At the present phenomenological level, however, $N$ is treated as a quantity to fit. The parameters of (2) and (3) include $N$, as it can be seen, e.g., from the comparison of (2) with Eq. (4) in Ref. 1.

Table I shows the experimental data concerning the $^{20}Ne$ nucleus, while the comparison with the model states is shown in Fig. 1. The small crow's feet in the experimental spectra indicate the fragmentation of the "boson states" due to some degrees of freedom neglected in this description. In such a case the energy displayed is a simple average of the energies of the actual levels. The parameters of the fits,

TABLE II. The parameters (in MeV) of the energy formulas for the fits shown in Fig. 1.

| | | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ | $\epsilon$ |
|---|---|---|---|---|---|---|
| $^{16}O+\alpha$, | O(4) | −0.048 | 0.119 | | | 7.37 |
| $^{24}Mg+\alpha$, | O(4) | −0.018 | 0.069 | | | 3.97 |
| $^{26}Mg+\alpha$, | O(4) | −0.016 | 0.066 | | | 4.15 |
| $^{26}Mg+\alpha$, | U(3) | | 0.060 | 0.19 | −0.021 | 2.62 |

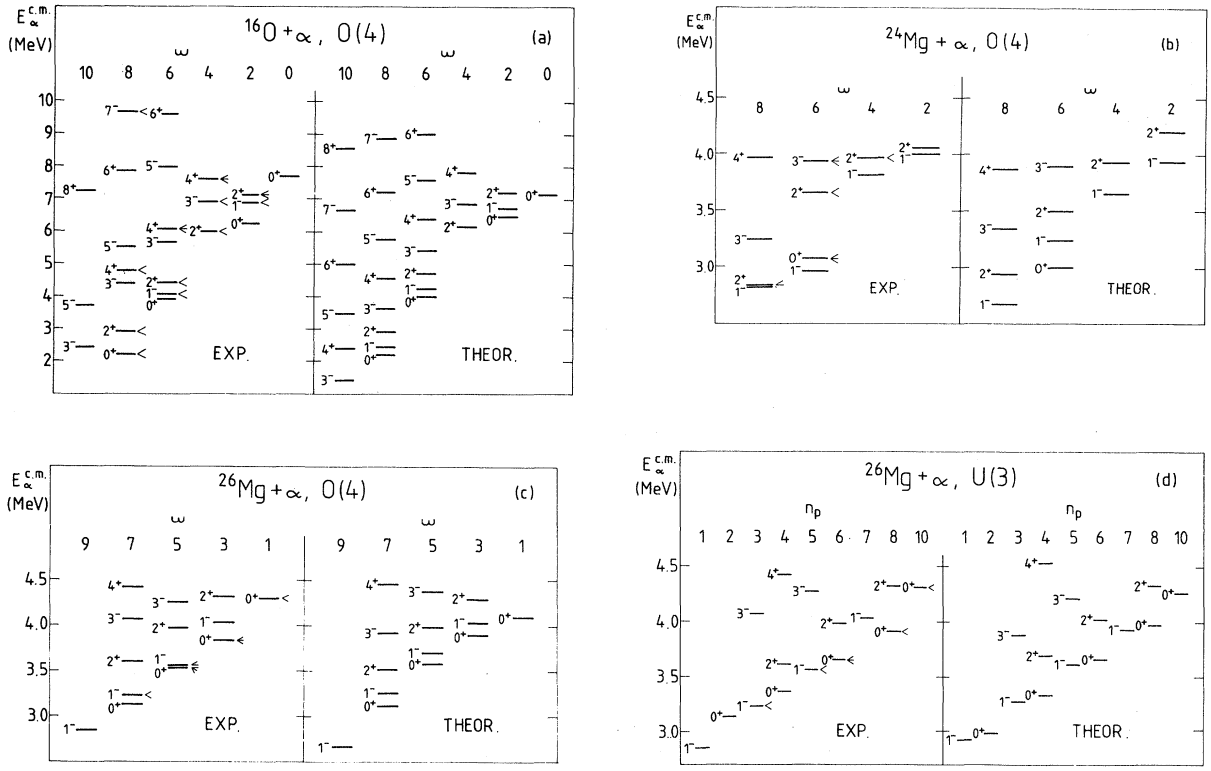


FIG. 1. (a)–(d) Comparison between the experimental and theoretical spectra.

shown in Fig. 1, are given in Table II.

As for the "missing states" in the experimental spectra, especially at the ends of the multiplets, one should be aware of the difficulties of this kind of investigation. These states are usually found as resonances, corresponding to high-lying states of the compound system. However, the resonance region is strongly limited both at the low and high energy sides by the small penetration and by the high level density, respectively.

In the $^{30}$Si and $^{28}$Si cases more or less similar agreement can be attained between the experimental spectra and the two different model spectra. This may be a result of the incompleteness of the experimental level schemes. On the other hand, $^{20}$Ne definitely prefers the O(4) limit, which indicates the presence of this dynamical symmetry at least in some approximation. It was impossible to achieve any acceptable agreement with the U(3) limit. So, in spite of the possible ambiguity in the assignment of $\omega$, which we think to be small, an unambiguous choice could be made between the dynamical symmetries, when the experimental spectrum was rich enough.

According to the correspondence between the limits of the U(4) model and the geometrical picture, the presence of the O(4) symmetry indicates the usefulness of the Morse potential in the description of the core $+\alpha$ states.[9]

The $N$ values calculated from $\alpha$ and $\epsilon$ are: 11.4, 13.9, and 15.1 for the $^{20}$Ne, $^{28}$Si, and $^{30}$Si, respectively. In fact, keeping the $N$ values fixed at 10, 14, and 15, and having only two parameters to fit in the O(4) limit, a description almost as good as the ones shown in the figures could be reached. This fact is in agreement with the interpretation of bosons in terms of nucleon pairs and does not allow the interpretation in terms of $\alpha$ particles[10] in these cases.

To achieve a more accurate description of the experimental spectra, not only more complete experiments seem to be needed, but more elaborate versions of the model as well. Some straightforward steps would be to: (i) use the general case of the U(4) model; (ii) make a distinction between proton and neutron bosons, which is obviously important when speaking about $\alpha$ clusters; (iii) involve bosons with $L=2$ angular momentum in the same way as in the case of heavy nuclei[11]; and (iv) take into account the influence of other channels.[1] Nevertheless, we think that the simple approximation used here as a first step is not bad, and shows the usefulness of the IBM in the field of the nuclear molecular states.

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