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Short range part of the *NN* interaction: Equivalent local potentials from quark exchange kernels

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To focus on the nature of the short range part of the *NN* interaction, the intrinsically nonlocal interaction among the quark constituents of colorless nucleons is converted to an equivalent local potential using resonating group kernels which can be evaluated in analytic form. The WKB approximation based on the Wigner transform of the nonlocal kernels has been used to construct the equivalent potentials without recourse to the long range part of the *NN* interaction. The relative importance of the various components of the exchange kernels can be examined: The results indicate the importance of the color magnetic part of the exchange kernel for the repulsive part in the $(ST)=(10)$, (01) channels, in particular since the energy dependence of the effective local potentials seems to be set by this term. Large cancellations of color Coulombic and quark confining contributions, together with the kinetic energy and norm exchange terms, indicate that the exact nature of the equivalent local potential may be sensitive to the details of the parametrization of the underlying quark-quark interaction. The equivalent local potentials show some of the characteristics of the phenomenological short range terms of the Paris potential.

[NUCLEAR REACTIONS Short range part of *NN* interaction, six-quark model, RGM quark exchange kernels, equivalent local potentials via WKB and Wigner transforms.]

I. INTRODUCTION

Since the quark structure of hadrons is now well established, nuclear physics is entering a period where serious attempts are being made to understand the nucleon-nucleon interaction through the fundamental interaction among quarks in terms of the underlying quantum chromodynamics. The long-range (small momentum transfer) domain of the quark-quark interaction is far from a perturbative solution, and there are no satisfactory treatments. Most recent investigations of the *NN* interaction in terms of models of the six-quark system[1–7,22,23] have therefore concentrated on the short range part of the *NN* interaction, since this may be dominated by the one gluon exchange terms of the quark-quark interaction and least influenced by the phenomenological quark confining potentials which have to be put into the theory by hand. The short range part may also be of greatest interest since the long and medium range parts of the *NN* interaction are described successfully in terms of the Paris potential[8] based on a field theoretic treatment of one- and two-pion exchange terms and including the effects of the mesonic and nucleonic resonances. In the Paris potential the short (0—0.8 fm) range part of the *NN* interaction is constructed phenomenologically through a soft-core repulsive, energy dependent potential. It is therefore of special interest to attempt to gain a more fundamental picture of this part of the interaction. Although a fully relativistic treatment of the *NN* interaction has been attempted in terms of the two center MIT bag model,[9] the most successful detailed treatments have been based on nonrelativistic quark models (with u, d quark rest masses of ~ 350 MeV, rather than the zero rest mass of the MIT bag model).

Calculations in the framework of the resonating group method (RGM) seem to be most fruitful. One of the advantages of the RGM calculations arises from the fact that the needed exchange kernels can be evaluated in analytic form. The nonlocal *NN* interaction arising from the interaction among the quark constituents can thus be given explicitly and an attempt can be made to convert this to an equivalent local potential by techniques which have been successfully used in the scattering of complex nuclei from nuclei. The RGM is also well suited for isolating the short range part of the *NN* interaction. The nucleons are described by colorless internal wave functions. In the long range limit of the *NN* distance, therefore, both colored Fermi-Breit terms and phenomenological confining color potentials automatically go to zero. As a result, van der Waals forces between the colorless nucleons are automatically excluded. Since all simple power-law confining potentials would lead to van der Waals terms of both the wrong strength and the wrong radial dependence[10–12] for the long range part of the NN interaction, this is not a disadvantage. At short NN separation, on the other hand, the color polarizability of the nucleons can be incorporated into RGM calculations by standard coupled channels techniques by including coupling to the so-called hidden color channels.[6,7]

Recent detailed calculations have been interpreted in terms of adiabatic potentials, based on a Born-Oppenheimer type of approximation, or have focused directly on the S-wave phase shifts in the various S, T channels. In both approaches the interplay between the short range (quark-quark) and the long range (mesonic) part of the interaction plays some role. Harvey[6] has studied this effect by varying the strength of a simplified pionic color-independent term added by hand to the quark-quark Hamiltonian. Faessler, Fernandez, Lübeck, and Shimizu[7] have avoided this problem by converting their S-wave phase shifts to an equivalent hard-core radius parameter. It is the advantage of the RGM approach that the nonlocal kernel arising from the color interaction among the quark constituents of the nucleons can be converted to an equivalent local NN potential without recourse to the long range parts of the interaction. If the latter are negligible in the short range domain compared with the short range terms generated by quark exchanges, the quark exchange kernels can be converted directly to an equivalent local potential. Toki[4] has shown the repulsive character of the short range part of the NN interaction by focusing on the momentum-independent part of an expansion of the Wigner transform of the exchange kernel derived from a simplified quark-quark interaction. It may, however, be important to include the momentum dependence of the Wigner transform to all orders. Oka and Yazaki[5] have used their RGM solution to construct the so-called trivially equivalent potential for the NN interaction, but this potential has infinities at the nodal positions of the relative motion functions and is therefore not subject to easy interpretation.

The color interaction among the quark constituents of colorless nucleons is intrinsically nonlocal. To interpret the nature of the short range part of the NN interaction it is therefore very useful to attempt to construct equivalent effective local potentials. It is the aim of this investigation to study the nature of such potentials in terms of the underlying quark exchange kernels. The method of equivalent local potentials has been widely used in the analysis of the scattering of complex nuclei from nuclei (see, e.g., discussions of the Perey-Buck effect[13]), usually in the form of a local momentum approximation.[14] Recently the Wentzel-Kramers-Brillouin (WKB) approximation using the Wigner transform of the nonlocal exchange kernels was studied in detail by Horiuchi,[15] and the equivalent local potentials derived by this technique have been used with considerable success in a number of nuclear problems.[16] The simplicity of this method makes it particularly attractive for a study of the short range behavior of the NN interaction in terms of the underlying quark-quark interaction, and this method has therefore been chosen. Owing to the approximations inherent in the use of nonrelativistic RGM kernels and the use of the WKB method in converting these to equivalent local potentials, no attempt will be made to give a detailed comparison of the many quark-quark interactions which have been used in recent studies of the NN interaction. Instead, we focus on a few characteristic examples chosen from among those cases which give reasonable fits to the nucleon mass, the $\Delta-N$ mass difference, and the size of the nucleon. The quark-quark interactions which have been singled out are discussed briefly in Sec. II, together with the parts of the RGM formalism needed in this investigation. The equivalent local potentials for the short range part of the NN interaction are exhibited and discussed in Sec. III. Since few of the recent RGM treatments exhibit the detailed analytic expressions for the needed exchange kernels, these are collected in an appendix together with their Wigner transforms, which are central to the present method of calculation. A few concluding remarks are intended to emphasize that we are only at the beginning stage of a deeper understanding of the NN interaction.

II. THE QUARK-QUARK INTERACTION AND THE RGM FORMALISM

The one gluon exchange potential leads to a color fine structure interaction of Fermi-Breit form. In all recent treatments these Fermi-Breit terms have been combined with a phenomenological quark confining potential of two-body form. Tensor and spin orbit terms in the Fermi-Breit interaction are omitted since the emphasis is on S-wave scattering. In setting the details of the quark-quark interaction to be used in a nonrelativistic RGM calculation, the most important considerations include (1) the quark mass, (2) the binding energy of the nucleon and the delta, and (3) the size parameter of the quark single particle wave function. In a nonrelativistic treatment the mass of the u, d quarks must be chosen to lie in the 300—350 MeV range. In the nonrelativistic kinematics the center of mass motion can be separated rigorously, but the reduced mass of the six-quark *N-N* system is $\frac{3}{2}m$ (m = quark mass), whereas the reduced mass in the *N-N* RGM equation should be $\frac{1}{2}M_N$ (M_N = nucleon mass). To avoid the problem of an impossible mass renormalization, the two reduced masses should be nearly equal. The importance of fitting the $\Delta - N$ mass difference has been emphasized by many authors. This parameter essentially fixes the color fine structure constant. The size parameter of the quark single particle wave function is related to the size of the nucleon. In the harmonic oscillator approximation, the root mean square radius of the nucleon, 0.8 fm, is equal to the oscillator constant $[b=(\hbar/m\omega)^{1/2}]$. Using the philosophy of the little bag,[17] size parameters considerably less than 0.8 fm have also been chosen. In this investigation we focus on a few characteristic examples of recent calculations chosen from among those potentials which give good fits to both the nucleon mass and the $\Delta - N$ mass difference and use u, d quark masses of ~ 350 MeV. These include (1) one of the potentials of Oka and Yazaki[5], (2) the potentials of Faessler, Fernandez, Lübeck, and Shimizu (FFLS) with the quadratic confining term,[7] and (3) the potential of Harvey[6] which has been parametrized to reproduce the baryon spectrum in the 1.2 to 2 GeV range as well as the N and Δ masses. In all cases the quark-quark interaction has the form (with $V=\Sigma_{i<j}v_{ij}$)

$$v_{ij}=(\lambda_i\cdot\lambda_j)[f(r_{ij})+(\vec{\sigma}_i\cdot\vec{\sigma}_j)g(r_{ij})] , \quad (1)$$

where λ_i is the color SU(3) generator for the ith quark normalized such that

$$P_{ij}(\text{color})=\tfrac{1}{2}(\lambda_i\cdot\lambda_j)+\tfrac{1}{3} .$$

For the potentials of Oka-Yazaki and FFLS

$$f(r)=-ar^2+\frac{\alpha_s\hbar c}{4r} ,$$
$$g(r)=-\frac{\pi\alpha_s\hbar^3}{6m^2c}\delta(\vec{r}) . \quad (2)$$

For the potential of Harvey

$$f(r)=A\exp[-r^2/\alpha^2]+Br^2+C+K\delta(\vec{r}) ,$$
$$g(r)=\tfrac{2}{3}K\delta(\vec{r}) . \quad (3)$$

The numerical values of the constants are shown in Table I, together with the predicted N and Δ masses. The FFLS potential has been chosen consistently; that is, the coeffcient a of the quark confining potential is related variationally to the gluon-quark coupling constant α_s and the oscillator constant b by requiring that the nucleon mass be a minimum. The need for this consistency requirement is not absolute in view of the missing long-range terms in the interaction. No such consistency was required for the parameters of the two other examples. Note also that the three examples chosen cover a wide range of size parameters. The b values range from 0.475 fm (FFLS) to 0.8 fm (Harvey).

The RGM wave function for the six-quark *NN* system can be written as

$$\psi=\mathscr{A}\{[\phi_N(\zeta_1\zeta_2\zeta_3)\times\phi_N(\zeta_4\zeta_5\zeta_6)]_{ST}\chi_N(\vec{R}_{12})\} . \quad (4)$$

The internal wave function ϕ_N of the nucleons include, among the internal degrees of freedom ζ_i, color, spin, isospin, and the internal orbital degrees of freedom such as $(\vec{r}_1-\vec{r}_2)$ and $\frac{1}{2}(\vec{r}_1+\vec{r}_2)-\vec{r}_3$ in $\phi_N(\zeta_1\zeta_2\zeta_3)$, where $\vec{r}_i$ is the position vector for the ith quark. The internal orbital functions are 0*s* functions. For simplicity these have been chosen to

TABLE I. Parameters of the quark-quark interaction.

	mc^2 (MeV)	b (fm)	a (MeV fm^{-2})	α_s	M_Nc^2 (MeV)	$M_\Delta c^2$ (MeV)
Oka-Yazaki[a]	300	0.6	62.5	1.39	1105	1397
FFLS[b]	355	0.475	34.5	0.97	1191	1485

	mc^2 (MeV)	b (fm)	A (MeV)	α (fm)	B (MeV fm^{-2})	C (MeV)	K (MeV fm^3)	M_Nc^2 (MeV)	$M_\Delta c^2$ (MeV)
Harvey[c]	355	0.8	952.5	0.8	−3.125	−119.95	−227.775	939	1240

[a]Reference 5.
[b]Reference 7.
[c]Reference 6.

be harmonic oscillator functions with oscillator constant b. Each ϕ_N is orbitally symmetric of space symmetry [3], each is spin-isospin symmetric of SU(4) symmetry [3], and each is a color singlet of color symmetry $[1^3]$. The spins and isospins of the nucleons are coupled to total S and T. [$\Delta\Delta$ and the so-called hidden color state can easily be included in a coupled channels treatment. They differ only in the color and SU(4) symmetry of the three-quark functions, and only the color and spin-isospin coefficients of the various kernels are affected; see the Appendix. Since the effects of coupling to $\Delta\Delta$ and hidden color channels are not of major importance, and since it is the aim of the present investigation to gain a simple picture of the short range part of the NN interaction, such coupled channel effects are not included.]

Since each ϕ_N is internally totally antisymmetric, the antisymmetrizer $\mathscr{A}$ in Eq. (4) which makes ψ totally antisymmetric under exchange of quarks among the nucleons can be reduced in terms of double coset generators[18] to the simple form

$$\mathscr{A}=(1-9P_{36})(1-\mathscr{P}) , \tag{5}$$

where P_{36} exchanges quarks 3 and 6 and $\mathscr{P}=P_{14}P_{25}P_{36}$ induces nucleon exchange. The P's act on the full space, color, spin, and isospin degrees of freedom. Because of the symmetry of the wave function ψ, $\mathscr{P}$ can be replaced by $(-1)^{S+T}\mathscr{P}(\text{space})$, where $\mathscr{P}(\text{space})$ acts on the space parts only. The evaluation of the spin-isospin and color parts of all matrix elements is carried out with the use of three-particle coefficients of fractional parentage (cfp's) and simple recoupling transformations (for details, see, e.g., Ref. 19).

The relative motion function in ψ is specified in terms of the baryon position vectors through $\vec{R}_{12}=\vec{R}_1-\vec{R}_2$ where, e.g.,

$$\vec{R}_1=\tfrac{1}{3}(\vec{r}_1+\vec{r}_2+\vec{r}_3) .$$

The RGM kernels for the operators $\mathscr{O}$ (with $\mathscr{O}=1$ (norm kernel), T (kinetic energy), V [interaction, see Eq. (1)] are evaluated in terms of the parameter coordinates $\vec{R}$, $\vec{R}\,'$ by

$$K_{\mathscr{O}}(\vec{R},\vec{R}\,')=\tfrac{1}{2}\langle[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R})\psi(\vec{R}_{\text{c.m.}})\,|\,\mathscr{O}\mathscr{A}\,|\,[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R}\,')\psi(\vec{R}_{\text{c.m.}})\rangle . \tag{6}$$

The factor $\frac{1}{2}$ takes account of the identity of the two nucleons. The center of mass motion function $\psi(\vec{R}_{\text{c.m.}})$ is included so that orbital integrations can be carried out over the full set of six $\vec{r}_i$'s. The kernels split into direct and exchange parts

$$K_{\mathscr{O}}(\vec{R},\vec{R}\,')=K_{\mathscr{O}}^{(D)}(\vec{R},\vec{R}\,')+K_{\mathscr{O}}^{(E)}(\vec{R},\vec{R}\,') , \tag{7}$$

through

$$K_{\mathscr{O}}^{(D)}(\vec{R},\vec{R}\,')=\tfrac{1}{2}\langle[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R})\psi(\vec{R}_{\text{c.m.}})\,|\,\mathscr{O}(1-\mathscr{P})\,|\,[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R}\,')\psi(\vec{R}_{\text{c.m.}})\rangle , \tag{8a}$$

$$K_{\mathscr{O}}^{(E)}(\vec{R},\vec{R}\,')=\tfrac{1}{2}(-9)\langle[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R})\psi(\vec{R}_{\text{c.m.}})\,|\,\mathscr{O}P_{36}(1-\mathscr{P})\,|\,[\phi_N\times\phi_N]_{ST}\delta(\vec{R}_{12}-\vec{R}\,')\psi(\vec{R}_{\text{c.m.}})\rangle . \tag{8b}$$

With

$$\chi(\vec{R})=\tfrac{1}{2}[\chi_N(\vec{R})-(-1)^{S+T}\chi_N(-\vec{R})] , \tag{9}$$

the RGM equation for the relative motion function becomes

$$-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}}{}^2\chi(\vec{R})+\int K^{(E)}(\vec{R},\vec{R}\,')\chi(\vec{R}\,')d\vec{R}\,' =E\chi(\vec{R}) , \tag{10}$$

where

$$K^{(E)}(\vec{R},\vec{R}\,')=K_V^{(E)}(\vec{R},\vec{R}\,')+K_T^{(E)}(\vec{R},\vec{R}\,') -[E+2\langle\phi_N\,|\,H_B\,|\,\phi_N\rangle]\times K_1^{(E)}(\vec{R},\vec{R}\,') \tag{11}$$

is given in terms of the exchange kernels, and where E is the energy of the relative motion of the two nucleons. The nucleon binding energy is given by the baryon internal Hamiltonian

$$H_B=\sum_{i=1}^{3}\left[-\frac{\hbar^2}{2m}\nabla_i{}^2\right]-T_{\text{c.m.}}^{(B)}+\sum_{i<j}^{3}v_{ij} . \tag{12}$$

The baryon mass is thus given by

$$M_Bc^2=3mc^2+\langle\phi_B\,|\,H_B\,|\,\phi_B\rangle .$$

Harvey has chosen the parameters of his quark-quark interaction to fit both M_N and M_Δ so that $\langle\phi_B\,|\,H_B\,|\,\phi_B\rangle$ gives the true binding energy of the baryons. The potentials of Oka-Yazaki and FFLS fit the $\Delta-N$ mass difference but give only an approximate fit to the nucleon mass (see Table I). In the actual calculations of this investigation, experimental binding energies were used in Eq. (11), but the final results are not qualitatively sensitive to the relatively small differences between the experimental and theoretial values.

It is to be noted that there is no direct potential term in Eq. (10), a feature characteristic of a color interaction of the type of Eq. (1) and the wave equation of a color singlet two-body system. [The direct (delta function) terms in the interaction kernel $K_V^{(D)}(\vec{R},\vec{R}\,')$ give the contribution of the internal potential energies to the nucleon binding energy, and they make up part of $\langle \phi_N | H_B | \phi_N \rangle$.] The RGM equation for the *NN* relative motion thus involves no local potential term if the underlying quark-quark interaction is made up of color Fermi-Breit and color confining terms. The full solution of the *NN* scattering problem would clearly require additional interactions of a local long-range character (of one-pion exchange form, for example). Since our focus is on the short range part of the *NN* interaction, no such terms will be added. If such additional terms are negligible in the short range (0—0.8 fm) domain, compared with the short range terms generated by quark exchanges, they will not influence the equivalent *NN* potentials constructed by the present method. Only the nonlocal kernels generated by quark pair exchanges will be considered.

The RGM kernels needed can be given in analytic form. Despite the fact that a number of RGM calculations of the six-quark system have been made, previous investigations give the needed kernels only for special radial forms[4,22] or give the kernels in GCM (generator coordinate) form.[5,7] Explicit expressions for the needed kernels are therefore given in the Appendix. The quark-quark interaction includes a term of Gaussian radial dependence [see Eq. (3)]. This is particularly useful since many other radial forms can easily be expanded in terms of Gaussians.

III. EQUIVALENT LOCAL POTENTIALS

To interpret the nonlocal kernels generated by the quark pair exchanges, an attempt can be made to convert these to equivalent local potentials. This will be particularly useful if we want to compare the effects of the nonlocal kernel with the phenomenological short range terms which have heretofore been added by hand to the *NN* interaction (cf. the Paris potential[8]). Equivalent local potentials have been widely used in the analysis of the scattering of nuclei from complex nuclei. If the RGM equation, Eq. (10), for $\chi(\vec{R})$ is replaced by

$$-\frac{\hbar^2}{2\mu}\nabla_{\vec{R}}{}^2\chi^{(\mathrm{loc})}(\vec{R})+U(R)\chi^{(\mathrm{loc})}(\vec{R})=E\chi^{(\mathrm{loc})}(\vec{R})\ , \tag{13}$$

with the condition

$$\chi^{(\mathrm{loc})}(\vec{R}) \underset{R\to\infty}{\longrightarrow} \chi(\vec{R})\ ,$$

this defines an equivalent local potential, $U(R)$. One possible choice of $U(R)$ is the so-called trivially equivalent local potential. This is obtained by solving the RGM equation and substituting the RGM $\chi(\vec{R})$ into Eq. (13). This has been examined by Oka and Yazaki for the six-quark model of the short range *NN* interaction. It suffers from the serious drawback that it is undefined at points where $\chi(\vec{R})=0$. Usually some form of local momentum approximation is used in defining the equivalent local potential. Recently the WKB approximation based on the Wigner transform of the nonlocal kernels was used successfully by Horiuchi[15] to show that it yields equivalent local potentials which simulate the effects of the nonlocal RGM kernels of the nuclear cluster model with good accuracy by making detailed phase shift comparisons.[16] This method has been chosen because of its simplicity and the fact that it permits us to construct an equivalent local potential for the short range part of the *NN* interaction without recourse to the long range part of the interaction. The validity of the method is subject to two basic conditions. First, the usual WKB approximation is $\hbar \ll$ characteristic action.[15,16] Second, the nonlocality range parameter, given approximately by b in our case, must be such that $(bP/\pi)<\hbar$. This is satisfied for relative motion energies up to ~ 1 GeV in our case. We would therefore expect the WKB approximation to be a reasonable approximation in the ~ 300 MeV range.

In the WKB approximation, the kernel $K^{(E)}(\vec{R},\vec{R}\,')$ is first converted to a momentum dependent potential

$$\hat{K}^{(E)}(\vec{R},\vec{P}_{\mathrm{op}})=\int d\vec{s}\,\exp\left[\frac{i}{2\hbar}\vec{s}\cdot\vec{P}_{\mathrm{op}}\right]K^{(E)}\left[\vec{R}-\frac{\vec{s}}{2},\vec{R}+\frac{\vec{s}}{2}\right]\exp\left[\frac{i}{2\hbar}\vec{s}\cdot\vec{P}_{\mathrm{op}}\right]. \tag{14}$$

This momentum dependent operator is then approximated by the Wigner transform of the kernel

$$K_W^{(E)}(R^2,P^2,(\vec{R}\cdot\vec{P})^2)=\int d\vec{s}\,\exp\left[\frac{i}{\hbar}\vec{s}\cdot\vec{P}\right]K^{(E)}\left[\vec{R}-\frac{\vec{s}}{2},\vec{R}+\frac{\vec{s}}{2}\right], \tag{15}$$

where $\vec{P}$ is treated as a c number. Because of the rotational invariance and the symmetry of the exchange kernel, K_W is a function only of R^2, P^2, and $(\vec{R}\cdot\vec{P})^2$. The Wigner transforms for the relevant kernels are collected in the Appendix. In terms of these the equivalent local potential $U(R)$ for relative motion orbital angular momentum, L, is given by the transcendental equation

$$U(R)=K_W^{(E)}\left[R^2,2\mu[E-U(R)],2\mu R^2\left[E-U(R)-\frac{\hbar^2(L+\frac{1}{2})^2}{2\mu R^2}\right]\right], \tag{16}$$

where the WKB replacement

$$L(L+1)\rightarrow(L+1/2)^2$$

has been made in the last variable (although final results are not very sensitive to this change).

The transcendental equation has been solved numerically for the $U(R)$ for the quark-quark interactions of Table I, and the $U(R)$ will be interpreted as effective local potentials for the short range part of the NN interaction. In solving the transcendental equation the true nucleonic reduced mass has been used, rather than $\frac{3}{2}m$; but since our parameter choices have been limited to cases where the difference between the two values is small, the solutions are not very sensitive to which choice is made. Figures 1 and 2 show the effective local potentials for the $(ST)=(10)$ and (01) channels, for $L=0$ (S-wave relative motion functions). The short range nature of these potentials is dictated by the color nature of the quark-quark interaction among the color singlet nucleons. The differences in the ranges between the three examples are related to the choice of oscillator constants for the three cases. The shortest ranges are obtained with the Faessler, Fernandez, Lübeck, and Shimizu parameters. The longest range potentials are those using Harvey's parameters, consistent with the differences in oscillator constants (0.475 fm for FFLS, 0.8 fm for Harvey). In comparison with the trivially equivalent local potentials obtained by Oka and Yazaki, these potentials have a rather soft core. However, compared with the phenomenological short range terms of the Paris potential, these potentials show a rather hard core. For all cases the potentials at the origin $(R=0)$ for $(ST)=(10)$ are higher than those for $(ST)=(01)$. Since the ^3S phase shifts are less repulsive than the ^1S phase shifts as calculated with the short range interactions of Oka-Yazaki and FFLS, this result seems surprising. However, it is to be noted that the $(ST)=(10)$ potentials are smaller (less repulsive) than the $(ST)=(01)$ potentials for the larger R values, with a crossover in the 0.35 to 0.45 fm range for the Oka-Yazaki parameters and in the 0.25 to 0.35 fm range for those of FFLS. The potentials exhibit a significant energy dependence. This is shown explicitly in Fig. 3, which gives the energy dependence of the core height ($R=0$ values) of the effective potentials. The increase with energy is almost precisely linear with rates of increase given by $\sim 0.5E$ and $\sim 0.3E$ (E in MeV) for the potential parameters of Oka-Yazaki and FFLS, respectively. This compares with

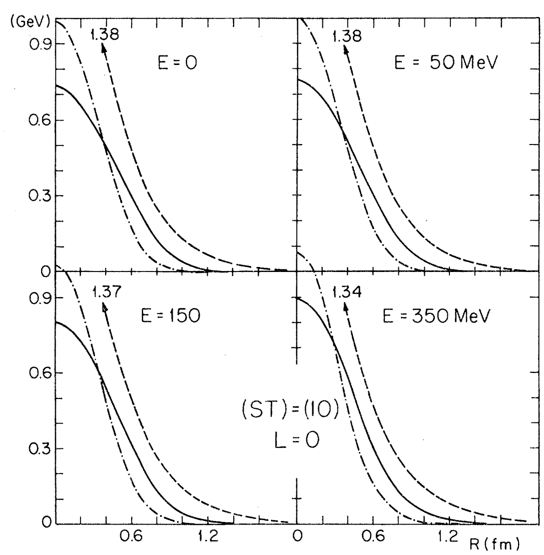

FIG. 1. The equivalent local potentials for the $(ST)=(10)$ S-wave channels. Solid line, Oka-Yazaki. Dashed-dotted line, Faessler, Fernandez, Lübeck, and Shimizu. Dashed line, Harvey.

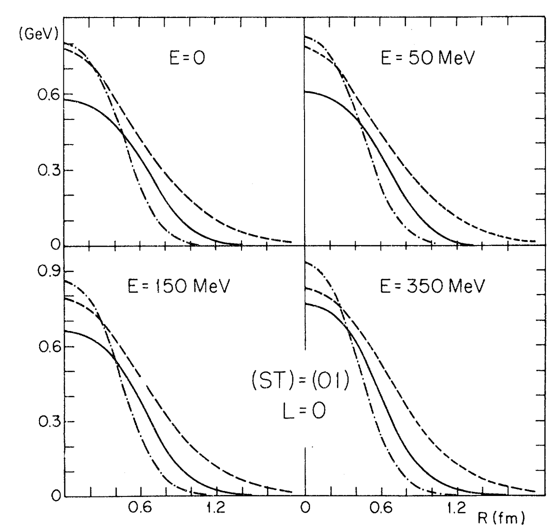

FIG. 2. The equivalent local potentials for $(ST)=(01)$ S-wave channels (as in Fig. 1).

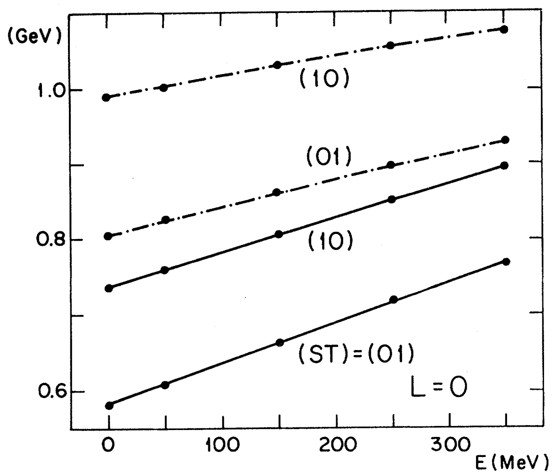

FIG. 3. The energy dependence of the core height ($R=0$ value) of the equivalent local potentials. Solid line, Oka-Yazaki. Dashed-dotted line, FFLS.

the linear energy-dependent terms of $0.998E$ and $0.952E$ (for $T=0$ and $T=1$ channels, respectively) of the Paris potential.

It is interesting to examine the relative importance of the various components of the exchange kernels in their contribution to the effective local potentials. The transcendental equation, Eq. (16), can be put in the form

$$U(R)=\Sigma_i K_{W_i}^{(E)}(R,U(R)) ,$$

where the terms with $i=C$, M, Q, K, and N gives the contributions of the various exchange kernels, color Coulombic, color magnetic, quadratic confining potential, kinetic energy, and norm exchange

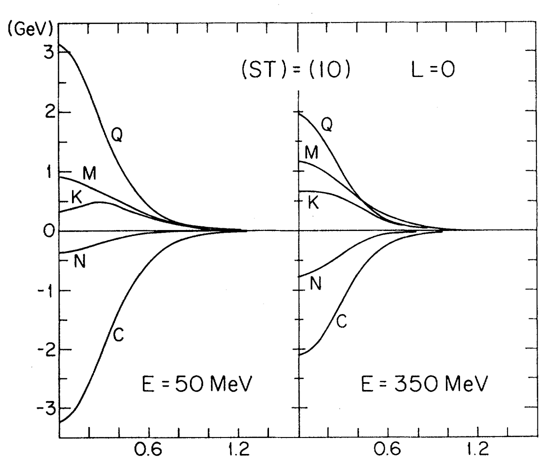

FIG. 4. The contributions of the various exchange kernels to the equivalent local potentials (Oka-Yazaki). Q—quadratic confining exchange kernel, M—color magnetic, K—kinetic energy, N—norm, C—color Coulombic.

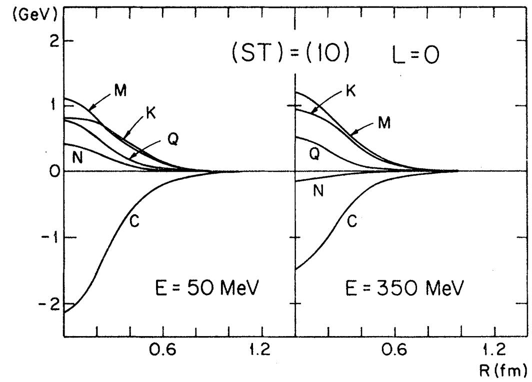

FIG. 5. The contributions of the various exchange kernels to the equivalent local potentials (FFLS) (as in Fig. 4).

kernels to the effective local potential. The results are shown for the $(ST)=(10)$ channel for the parameters of Oka-Yazaki (Fig. 4) and FFLS (Fig. 5). Quite significant differences are observed. For the Oka-Yazaki parameters the Coulombic (C) and confining potential terms (Q) almost cancel each other so that the net effective local potential appears to be set by the color magnetic term (M). For the FFLS parameters, however, the somewhat smaller color Coulombic contribution is not canceled by the confining potential terms (Q) alone. The energy dependence of the various exchange contributions is not fully understood. However, it is clear that it does not arise from explicit E dependence of the coefficient of the norm exchange kernel. The cancellation of the large attractive contribution of the color Coulombic exchange kernel by the effects of the other kernels appears to indicate that the precise parametrization of the quark-quark interaction may be important. Owing to the phenomenological character of the quark confining potentials, it may therefore be premature to draw conclusions about the underlying character of the short range part of the NN interaction, and it would be difficult to conclude that the color magnetic term is the key to the short range effective repulsion. However, Figs. 4 and 5 seem to indicate that the contribution of the color magnetic exchange kernel, with magnitude fixed by the $\Delta-N$ mass difference, may largely determine the magnitude of the effective repulsive core potential, because the linear energy dependence of the effective potentials is matched quantitatively by the energy dependence of the color magnetic term.

Figure 6 shows the angular momentum dependence of the effective local potential. The figure shows that the L dependence is much more important than in the corresponding case of nuclear scattering.

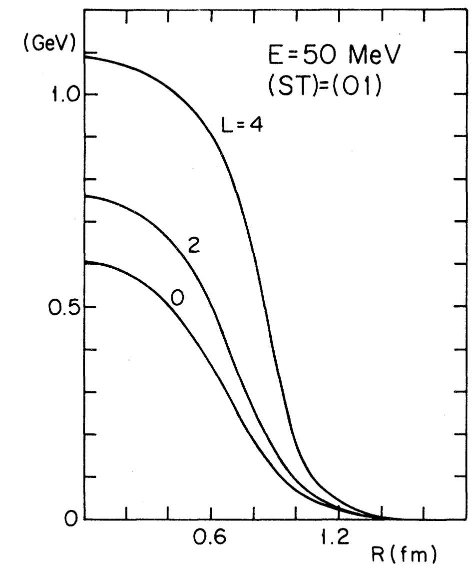

FIG. 6. The L dependence of the equivalent local potential (Oka-Yazaki).

Finally, Fig. 7 shows some of the difficulties encountered in attempts to gain an effective local potential with the WKB approximation. For the $(ST)=(00)$, $L=1$ channel and $E=0$, the transcendental Eq. (16) gives two solutions for small values of R, one attractive and one repulsive. There are no solutions for $1.0 \lesssim R \lesssim 1.4$ fm, and again two solutions for large values of R. For $E=150$ MeV there are two solutions for all values of R, and one would expect to choose the solution which goes to zero as $R \to \infty$ as the physically relevant one. This would lead to the conclusion that the effective $(ST)=(00)$ potential is attractive. Although it would be a surprising result, the experimental evidence against it may not be overwhelming.[20] The parametrizations of the quark-quark interactions which have been used are based on experimental input which is

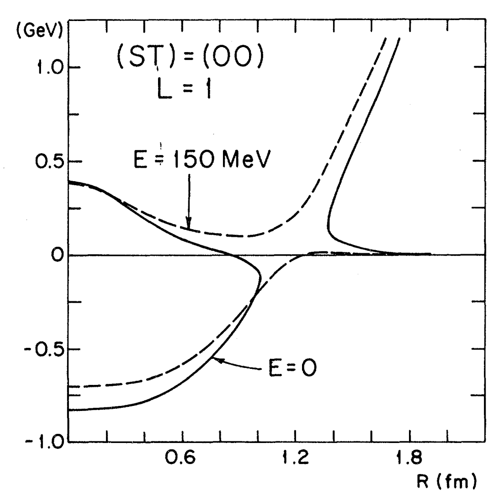

FIG. 7. The solution domain for $U(R)$ for the $(ST)=(00)$ $L=1$ channel (Oka-Yazaki).

TABLE II. The spin-isospin coefficients C_{ST}. The spin-isospin factors $C_{ST}^{(i)}$ are the matrix elements of $\mathcal{O}P_{36}$, where $\mathcal{O}=1$ $(i=0)$; $\mathcal{O}=\vec{\sigma}_4\cdot\vec{\sigma}_5$ $(i=1)$; $\mathcal{O}=\vec{\sigma}_5\cdot\vec{\sigma}_6$ $(i=2)$; $\mathcal{O}=\vec{\sigma}_2\cdot\vec{\sigma}_6$ $(i=3)$; $\mathcal{O}=\vec{\sigma}_2\cdot\vec{\sigma}_5$ $(i=4)$; $\mathcal{O}=\vec{\sigma}_3\cdot\vec{\sigma}_6$ $(i=5)$. If bra/ket interchange of $NN/\Delta\Delta$ is made then $C_{ST}^{(2)}$ and $C_{ST}^{(3)}$ must be interchanged.

(ST)	$C_{ST}^{(0)}$	$C_{ST}^{(1)}$	$C_{ST}^{(2)}$	$C_{ST}^{(3)}$	$C_{ST}^{(4)}$	$C_{ST}^{(5)}$
			NN/NN			
(00)	$\frac{7}{9}$	$-\frac{11}{9}$	$-\frac{5}{9}$	$-\frac{5}{9}$	0	$-\frac{1}{9}$
(01)	$-\frac{1}{27}$	$\frac{17}{27}$	$-\frac{7}{27}$	$-\frac{7}{27}$	0	$\frac{31}{27}$
(10)	$-\frac{1}{27}$	$\frac{17}{27}$	$-\frac{7}{27}$	$-\frac{7}{27}$	$\frac{2}{27}$	$\frac{19}{27}$
(11)	$\frac{31}{81}$	$-\frac{59}{81}$	$-\frac{17}{81}$	$-\frac{17}{81}$	$\frac{10}{81}$	$\frac{59}{81}$
			$\Delta\Delta/\Delta\Delta$			
(00)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$-\frac{1}{3}$	$-\frac{7}{9}$
(01)	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$-\frac{1}{9}$	$-\frac{7}{27}$
(02)	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$\frac{1}{3}$	$\frac{7}{9}$
(03)	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	1	$\frac{7}{3}$
(10)	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$\frac{1}{27}$	$-\frac{19}{27}$
(11)	$\frac{1}{81}$	$\frac{1}{81}$	$\frac{1}{81}$	$\frac{1}{81}$	$\frac{1}{81}$	$-\frac{19}{81}$
(12)	$-\frac{1}{27}$	$-\frac{1}{27}$	$-\frac{1}{27}$	$-\frac{1}{27}$	$-\frac{1}{27}$	$\frac{19}{27}$
(13)	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$\frac{19}{9}$
(20)	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$\frac{1}{3}$	$-\frac{5}{9}$
(21)	$-\frac{1}{27}$	$-\frac{1}{27}$	$-\frac{1}{27}$	$-\frac{1}{27}$	$\frac{1}{9}$	$-\frac{5}{27}$
(22)	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$	$-\frac{1}{3}$	$\frac{5}{9}$
(23)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	-1	$\frac{5}{3}$
(30)	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$
(31)	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$	$-\frac{1}{9}$
(32)	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
(33)	1	1	1	1	1	1
			$NN/\Delta\Delta$			
(00)	$\frac{4}{9}$	$\frac{4}{9}$	$-\frac{8}{9}$	$\frac{4}{9}$	$-\frac{2}{3}$	$-\frac{4}{9}$
(01)	$\frac{4\sqrt{5}}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{8\sqrt{5}}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{2\sqrt{5}}{9}$	$-\frac{4\sqrt{5}}{27}$
(10)	$\frac{4\sqrt{5}}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{8\sqrt{5}}{27}$	$\frac{4\sqrt{5}}{27}$	$-\frac{2\sqrt{5}}{27}$	$-\frac{4\sqrt{5}}{27}$
(11)	$\frac{20}{81}$	$\frac{20}{81}$	$-\frac{40}{81}$	$\frac{20}{81}$	$-\frac{10}{81}$	$-\frac{20}{81}$

independent of baryon P-wave properties, and the underlying exchange kernels may not be very reliable for the odd-L partial waves. The difficulties associated with the lack of a solution for the low energy ($E=0$) limit may be associated with the breakdown of the local momentum approximation. (A similar difficulty is encountered by Aoki and Horiuchi[16] in some cases of nuclear scattering.) However, no such difficulties are encountered for the $(ST)=(10)$ and (01) channels. Insofar as the concept of an effective local potential can be used, we expect the WKB approximation to be a reasonable approximation in the ~ 300 MeV range for these important channels.

IV. SUMMARY

Since the intrinsically nonlocal character of the color interaction among the quark constituents of colorless nucleons makes it difficult to interpret the nature of the quark exchange kernels, an attempt has been made to construct equivalent local potentials for the short range part of the *NN* interaction. Calculations have been carried out in the framework of the resonating group method since the RGM exchange kernels can be given in analytic form and can be converted to equivalent potentials without recourse to the long range (mesonic) part of the interaction. Owing to the uncertainties associated with the use of nonrelativistic dynamics and the phenomenology associated with the quark confining terms in the quark-quark interaction, the WKB approximation based on the use of the Wigner transform of the nonlocal exchange kernels has been chosen as the simplest method of constructing an equivalent local potential for the short range part of the *NN* interaction. The resulting potentials for the $(ST)=(10)$ and (01) channels are in a form which can be compared with the phenomenological short range terms of the Paris potential.[8] The core heights at $E=0$ are considerably larger than those for the Paris potential, whereas the rates of increase with E are smaller by 30—60% than the corresponding phenomenological terms. The results seem to indicate that the overall strength of the effective repulsive core in the $(ST)=(10)$ and (01) channels may be set largely by the contributions of the color magnetic part of the exchange kernel and that the energy dependence of the repulsive core is matched by the energy dependence of the color magnetic contribution. However, the large cancellations of the color Coulombic and quark confining contributions make it clear that the exact nature of equivalent effective local potentials may be very sensitive to the details of the parametrization of the underlying quark-quark interaction. We may thus be only at the beginning stage of a fundamental understanding of the nature of the repulsive core in the *NN* interaction.

ACKNOWLEDGMENTS

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APPENDIX

Since the needed kernels are available only in GCM form[5,7] or have been given in RGM form only for specialized terms,[4,22] explicit expressions are given in this appendix together with the needed Wigner transforms. The RGM kernels have been evaluated through their Bargmann-Segal transforms.[21] The evaluation of the color and spin-isospin parts of the coefficients is illustrated in Ref. 19. Since the kernels for the $\Delta\Delta$ channel and the overlaps between *NN* and $\Delta\Delta$ states differ only in these color, spin-isospin factors, they are included in the tabulations. Similar coefficients needed for the coupling to hidden color states, as well as the kernels for tensor and spin-orbit terms, will be given in a future publication.

The kernels are split into direct and exchange parts; see Eqs. (7) and (8). It is convenient to express them in terms of dimensionless coordinates $\vec{a}$, $\vec{a}\,'$ rather than $\vec{R}$, $\vec{R}\,'$

$$\vec{a}=[\tfrac{3}{2}]^{1/2}(\vec{R}/b), \quad b=[\hbar/m\omega]^{1/2}\,, \tag{A1}$$

(similarly for $\vec{a}\,'$).

The physical kernels $K_{\mathscr{O}}(\vec{R},\vec{R}\,')$ of the text are related to the dimensionless kernels $K_{\mathscr{O}}(\vec{a},\vec{a}\,')$ of this appendix by a factor of dimension L^{-3}.

$$K_{\mathscr{O}}(\vec{R},\vec{R}\,')=\left[\left[\frac{3}{2}\right]^{1/2}\frac{1}{b}\right]^{3}K_{\mathscr{O}}(\vec{a},\vec{a}\,')\,. \tag{A2}$$

The norm kernels are

$$K_1^{(D)}(\vec{a},\vec{a}\,')=\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')] , \tag{A3}$$

$$K_1^{(E)}(\vec{a},\vec{a}\,')= -9(\tfrac{1}{3})C_{ST}^{(0)}\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)]$$

$$\times\tfrac{1}{2}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')] , \tag{A4}$$

where the spin-isospin factors $C_{ST}^{(0)}$ are given in Table II. The kinetic energy kernels are

$$K_T^{(D)}(\vec{a},\vec{a}\,')=\tfrac{1}{2}\hbar\omega[-\nabla_{\vec{a}}{}^2+6]\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')] , \tag{A5}$$

$$K_T^{(E)}(\vec{a},\vec{a}\,')=-9(\tfrac{1}{3})C_{ST}^{(0)}(\tfrac{1}{2}\hbar\omega)\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)]$$

$$\times\tfrac{1}{2}\{[\tfrac{45}{4}-\tfrac{3}{2}(\vec{a}-\vec{a}\,')^2-\tfrac{3}{16}(\vec{a}+\vec{a}\,')^2]\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')$$

$$-(-1)^{S+T}[\tfrac{45}{4}-\tfrac{3}{2}(\vec{a}+\vec{a}\,')^2-\tfrac{3}{16}(\vec{a}-\vec{a}\,')^2]\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')\} . \tag{A6}$$

The interaction kernels for

$$v_{ij}=(\lambda_i\cdot\lambda_j)f(r_{ij}) \text{ and } v_{ij}=(\lambda_i\cdot\lambda_j)(\vec{\sigma}_i\cdot\vec{\sigma}_j)g(r_{ij}) \tag{A7}$$

are the following:

(1) For the Gaussian interaction $f(r)$ [or $g(r)$]$=\exp[-(\beta/2)(r/b)^2]$,

$$K_V^{(D)}(\vec{a},\vec{a}\,')=-\tfrac{8}{3}p(6)(1+\beta)^{-3/2}\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')] . \tag{A8}$$

For f-type interactions $p=+1$ for both NN and $\Delta\Delta$; for g-type interactions, $p=-1$ for NN and $p=+1$ for $\Delta\Delta$.

$$K_V^{(E)}(\vec{a},\vec{a}\,')= -9\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)](\tfrac{1}{2})$$

$$\times\Bigg\{-\tfrac{8}{9}C_{ST}^{(1)}(2)(1+\beta)^{-3/2}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')]$$

$$-\tfrac{8}{9}C_{ST}^{(2)}(4)(1+\tfrac{5}{8}\beta)^{-3/2}\exp\left[-\frac{3\beta(a^2+9a'^2)}{8(8+5\beta)}\right]$$

$$\times\left[\exp\left[\frac{6+6\beta}{8+5\beta}\vec{a}\cdot\vec{a}\,'\right]-(-1)^{S+T}\exp\left[-\frac{6+6\beta}{8+5\beta}\vec{a}\cdot\vec{a}\,'\right]\right]$$

$$-\tfrac{8}{9}C_{ST}^{(3)}(4)(1+\tfrac{5}{8}\beta)^{-3/2}\exp\left[-\frac{3\beta(9a^2+a'^2)}{8(8+5\beta)}\right]$$

$$\times\left[\exp\left[\frac{6+6\beta}{8+5\beta}\vec{a}\cdot\vec{a}\,'\right]-(-1)^{S+T}\exp\left[-\frac{6+6\beta}{8+5\beta}\vec{a}\cdot\vec{a}\,'\right]\right]$$

$$+\tfrac{4}{9}C_{ST}^{(4)}(4)(1+\tfrac{4}{8}\beta)^{-3/2}\exp\left[-\frac{3\beta(a^2+a'^2)}{2(8+4\beta)}\right]$$

$$\times\left[\exp\left[\frac{3}{4+2\beta}\vec{a}\cdot\vec{a}\,'\right]-(-1)^{S+T}\exp\left[-\frac{3}{4+2\beta}\vec{a}\cdot\vec{a}\,'\right]\right]$$

$$+\tfrac{16}{9}C_{ST}^{(5)}(1)\exp\left[-\frac{3\beta}{4}(a^2+a'^2)\right]$$
$$\times\left[\exp\left[\frac{3+6\beta}{4}\vec{a}\cdot\vec{a}\,'\right]-(-1)^{S+T}\exp\left[-\frac{3+6\beta}{4}\vec{a}\cdot\vec{a}\,'\right]\right]\Bigg]\Bigg\}\,. \tag{A9}$$

The $C_{ST}^{(1)}-C_{ST}^{(5)}$ needed for g-type interactions are given in Table II. For f-type interactions $C_{ST}^{(i)}$ are replaced by $C_{ST}^{(0)}$ for all i.

(2) For the special case of the color Coulombic term $f(r)=b/r$,

$$K_V^{(D)}(\vec{a},\vec{a}\,')=-\tfrac{8}{3}(6)\left[\frac{2}{\pi}\right]^{1/2}\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')]\,, \tag{A10}$$

$$K_V^{(E)}(\vec{a},\vec{a}\,')=-9C_{ST}^{(0)}\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)](\tfrac{1}{2})$$
$$\times\Bigg\{-\tfrac{8}{9}(2)\left[\frac{2}{\pi}\right]^{1/2}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')]$$
$$-\tfrac{8}{9}(4)\frac{2}{\sqrt{5}}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{40})^{1/2}|\vec{a}-3\vec{a}\,'|)$$
$$-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{40})^{1/2}|\vec{a}+3\vec{a}\,'|)]$$
$$-\tfrac{8}{9}(4)\frac{2}{\sqrt{5}}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{40})^{1/2}|3\vec{a}-\vec{a}\,'|)$$
$$-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{40})^{1/2}|3\vec{a}+\vec{a}\,'|)]$$
$$+\tfrac{4}{9}(4)[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{8})^{1/2}|\vec{a}+\vec{a}\,'|)$$
$$-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')h((\tfrac{3}{8})^{1/2}|\vec{a}-\vec{a}\,'|)]$$
$$+\tfrac{16}{9}(1)(\tfrac{2}{3})^{1/2}\left[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')\frac{1}{|\vec{a}-\vec{a}\,'|}-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')\frac{1}{|\vec{a}+\vec{a}\,'|}\right]\Bigg\}\,, \tag{A11}$$

where

$$h(x)=erf(x)/x=\frac{2}{\sqrt{\pi}}\int_0^1 e^{-x^2t^2}dt\,.$$

(3) For the quadratic confining potential $f(r)=r^2/b^2$,

$$K_V^{(D)}(\vec{a},\vec{a}\,')=-\tfrac{8}{3}(6)(3)\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')]\,, \tag{A12}$$

$$K_V^{(E)}(\vec{a},\vec{a}\,')=-9C_{ST}^{(0)}(-16)\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)]\tfrac{1}{2}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')]\,. \tag{A13}$$

(4) For the color-magnetic interaction $g(r)=\delta(\vec{r}/b)$,

$$K_V^{(D)}(\vec{a},\vec{a}\,')=-\tfrac{8}{3}p(6)(2\pi)^{-3/2}\tfrac{1}{2}[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')]\,, \tag{A14}$$

where $p=-1$ for NN and $p=+1$ for $\Delta\Delta$.

$$K_V^{(E)}(\vec{a},\vec{a}\,')=-9\left[\frac{9}{8\pi}\right]^{3/2}\exp[-\tfrac{5}{8}(a^2+a'^2)](\tfrac{1}{2})$$

$$\times\left\{-\tfrac{8}{9}C_{ST}^{(1)}(2)(2\pi)^{-3/2}[\exp(\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{3}{4}\vec{a}\cdot\vec{a}\,')]\right.$$

$$-\tfrac{8}{9}C_{ST}^{(2)}(4)\left[\frac{5\pi}{4}\right]^{-3/2}\exp\left[-\frac{3(a^2+9a'^2)}{40}\right][\exp(\tfrac{6}{5}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{6}{5}\vec{a}\cdot\vec{a}\,')]$$

$$-\tfrac{8}{9}C_{ST}^{(3)}(4)\left[\frac{5\pi}{4}\right]^{-3/2}\exp\left[-\frac{3(9a^2+a'^2)}{40}\right][\exp(\tfrac{6}{5}\vec{a}\cdot\vec{a}\,')-(-1)^{S+T}\exp(-\tfrac{6}{5}\vec{a}\cdot\vec{a}\,')]$$

$$+\tfrac{4}{9}C_{ST}^{(4)}(4)\pi^{-3/2}\exp[-\tfrac{3}{8}(a^2+a'^2)][1-(-1)^{S+T}]$$

$$\left.+\tfrac{16}{9}C_{ST}^{(5)}(1)(\tfrac{3}{2})^{-3/2}\exp(\tfrac{3}{4}a^2)[\delta(\vec{a}-\vec{a}\,')-(-1)^{S+T}\delta(\vec{a}+\vec{a}\,')]\right\}. \qquad \text{(A15)}$$

The Wigner transforms for the corresponding exchange kernels [see Eq. (15)] are collected below. They are expressed in terms of the dimensionless coordinate $\vec{a}$ and the dimensionless momentum coordinate $\vec{q}$

$$\vec{q}=[2/3]^{1/2}b\vec{P}/\hbar\,. \qquad \text{(A16)}$$

Our kernels are of the form

$$K^{(E)}(\vec{a},\vec{a}\,')=\tfrac{1}{2}[\tilde{K}(\vec{a},\vec{a}\,')-(-1)^{S+T}\tilde{K}(\vec{a},-\vec{a}\,')]\,, \qquad \text{(A17)}$$

where $\tilde{K}(\vec{a},\vec{a}\,')$ is of a Wigner or A type, using the language of Ref. 15, whereas $\tilde{K}(\vec{a},-\vec{a}\,')$ is of a Majorana or B type. The B type kernel is first transformed to A type and then converted to the Wigner transform (see Sec. 4 of Ref. 15). Because we have identical three-quark fragments, cf. Eq. (9), effectively only the Wigner transforms of $\tilde{K}(\vec{a},\vec{a}\,')$ are needed; and these are the Wigner transforms $K_W^{(E)}$ given below.

$$K_W^{(E)}(\text{norm})=-9(\tfrac{1}{3})C_{ST}^{(0)}(\tfrac{3}{2})^3\exp[-\tfrac{1}{2}(a^2+q^2)]\,, \qquad \text{(A18)}$$

$$K_W^{(E)}(\text{kinetic})=-9(\tfrac{1}{3})C_{ST}^{(0)}(\tfrac{1}{2}\hbar\omega)(\tfrac{3}{2})^3(\tfrac{27}{4}-\tfrac{3}{4}a^2+\tfrac{3}{2}q^2)\exp[-\tfrac{1}{2}(a^2+q^2)]\,, \qquad \text{(A19)}$$

$$K_W^{(E)}(\text{Gaussian})=-9(\tfrac{3}{2})^3(1+\beta)^{-3/2}\exp[-\tfrac{1}{2}(a^2+q^2)]$$

$$\times\left\{-\tfrac{8}{9}C_{ST}^{(1)}(2)-\tfrac{8}{9}C_{ST}^{(2)}(4)\exp\left[-\frac{3\beta}{16(1+\beta)}(\vec{a}+i\vec{q})^2\right]\right.$$

$$-\tfrac{8}{9}C_{ST}^{(3)}(4)\exp\left[-\frac{3\beta}{16(1+\beta)}(\vec{a}-i\vec{q})^2\right]$$

$$+\tfrac{4}{9}C_{ST}^{(4)}(4)\left[\frac{1+\beta}{1+\frac{1}{2}\beta}\right]^{3/2}\exp\left[-\frac{3\beta}{4+2\beta}a^2\right]$$

$$\left.+\tfrac{16}{9}C_{ST}^{(5)}(1)\left[\frac{1+\beta}{1+\frac{3}{2}\beta}\right]^{3/2}\exp\left[\frac{3\beta}{4+6\beta}q^2\right]\right\}, \qquad \text{(A20)}$$

$$K_W^{(E)}\left[\text{Coulomb};\frac{b}{r}\right]=-9(\tfrac{3}{2})^3C_{ST}^{(0)}\left[\frac{2}{\pi}\right]^{1/2}\exp[-\tfrac{1}{2}(a^2+q^2)]$$
$$\times\left\{-\tfrac{8}{9}(2)-\tfrac{8}{9}(4)(2)\int_0^1 dt\exp[-\tfrac{3}{16}(a^2-q^2)t^2]\cos[\tfrac{3}{8}(\vec{a}\cdot\vec{q})t^2]\right.$$
$$\left.+\tfrac{4}{9}(4)\left[\frac{\pi}{2}\right]^{1/2}h((\tfrac{3}{2})^{1/2}a)+\tfrac{16}{9}(1)(\tfrac{2}{3})^{1/2}\int_0^1 dt\exp(\tfrac{1}{2}q^2t^2)\right\}, \quad \text{(A21)}$$

$$K_W^{(E)}\left[\text{quadratic confining};\frac{r^2}{b^2}\right]=-9C_{ST}^{(0)}(-16)(\tfrac{3}{2})^3\exp[-\tfrac{1}{2}(a^2+q^2)]\ , \quad \text{(A22)}$$

$$K_W^{(E)}\left[\text{magnetic};\delta\left[\frac{\vec{r}}{b}\right]\right]=-9(\tfrac{3}{2})^3(2\pi)^{-3/2}\exp[-\tfrac{1}{2}(a^2+q^2)]$$
$$\times\{-\tfrac{8}{9}C_{ST}^{(1)}(2)-\tfrac{8}{9}C_{ST}^{(2)}(4)\exp[-\tfrac{3}{16}(\vec{a}+i\vec{q})^2]$$
$$-\tfrac{8}{9}C_{ST}^{(3)}(4)\exp[-\tfrac{3}{16}(\vec{a}-i\vec{q})^2]$$
$$+\tfrac{4}{9}C_{ST}^{(4)}(4)\sqrt{2}\exp(-\tfrac{3}{2}a^2)[1-(-1)^{S+T}]$$
$$+\tfrac{16}{9}C_{ST}^{(5)}(1)(\tfrac{2}{3})^{3/2}\exp(\tfrac{1}{2}q^2)\}\ . \quad \text{(A23)}$$

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