Charge multipole contributions to 180° electron scattering by complex nuclei

E. Moya de Guerra
Departamento de Física Teórica, Universidad Autónoma de Madrid,
Canto Blanco, Madrid-34, Spain
(Received 9 September 1982)

An analysis of the differential cross section for electron scattering at 180° from complex nuclei is made in the distorted-wave Born approximation to see the conditions in which charge multipoles may contribute to such cross sections, questioning the usual interpretation of the data in terms of pure transverse form factors.

[NUCLEAR REACTIONS Electron scattering; complex nuclei. Charge multipole contributions to 180° (e,e') in DWBA.]

It is well known[1] that in PWBA (plane-wave Born approximation) the differential cross section for electron scattering at 180° is directly proportional to the so-called nuclear transverse form factor, i.e., involves only transverse multipoles of the nuclear vector current density. For sufficiently large nuclear charges the Coulomb distortion of the electron wave functions is important and an analysis of the data in terms of DWBA (distorted-wave Born approximation) is mandatory. Indeed, this approximation is now commonly used to interpret the experimental data on longitudinal form factors.[2] However transverse form factors are still analyzed on a PWBA basis, under the assumption that a change of scale in momentum transfer ($q \rightarrow q_{\rm eff}$) (Refs. 1 and 3) is sufficient to take into account distortion effects. Recently experiments at 180° on ^{181}Ta have been performed[3] (and other rare earth nuclei are under investigation) to explore current distributions in rotational nuclei. In the case of ^{181}Ta large discrepancies were found at low momentum transfer between theoretical and experimental results[3,4] and calculations improving the theoretical description of transverse multipoles in rotational nuclei are now under way. However, the comparison of these calculations to experimental data strongly suggests that distortion effects are not properly taken into account and that in order to extract information on current distributions in heavy deformed nuclei a thorough analysis of the differential cross sections in DWBA is also required in this case.

The widespread idea that charge multipoles do not contribute to electron scattering at 180° is usually based on a change of electron spin polarization arguments.[1] However, to our knowledge, no analytic proof in favor of or against this assumption has been presented on the basis of DWBA. The purpose of this Brief Report is to provide such an analysis. To this end, an explicit expression of the differential cross section in DWBA for $\theta = \pi$ is given in which current conservation has been used to explicitly separate longitudinal and transverse contributions [as is done in PWBA (Ref. 1)], and the conditions under which longitudinal contributions vanish are investigated. Numerical results will be presented elsewhere; here we will strictly focus on the question of whether charge multipoles may or may not contribute to such cross sections.

Using the continuity equation to separate contributions from longitudinal and transverse multipoles the differential cross section for electron scattering at 180° in DWBA is given by[5]

$$\left.\frac{d\sigma(\epsilon_i,\theta=\pi)}{d\Omega}\right|_{I_i\rightarrow I_f} = \frac{\alpha^2 64\pi\epsilon_f^{\,2}}{1+2\epsilon_i/M_T}\,\frac{1}{2(2I_i+1)}\sum_{\lambda\geq 1,\mu=\pm 1}\left|\sum_{\tau}\mathcal{H}^{\tau}_{\lambda\mu}\right|^2 , \tag{1}$$

where ϵ_i and ϵ_f denote the initial and final electron energies, respectively, and $\mathcal{H}^{\tau}_{\lambda\mu}$ stand for the individual reduced amplitudes corresponding to the contributions of longitudinal $(\tau=C)$, transverse electric $(\tau=E)$, and magnetic $(\tau=M)$ multipoles of the nuclear current in the transition from state I_i to state I_f $(E_{I_f}-E_{I_i}=\epsilon_i-\epsilon_f=\omega)$. The latter are given by

$$\mathcal{H}^{\tau}_{\lambda\mu=1} = \sum_{\kappa_i,\kappa_f\geq 1} e^{i(\delta_{\kappa_i}+\delta_{\kappa_f})}\, i^{\kappa_i+\kappa_f+\lambda}\,\kappa_i\kappa_f(-1)^{j_i+j_f}\begin{pmatrix} j_f & j_i & \lambda\\ \frac{1}{2} & \frac{1}{2} & -1\end{pmatrix}\begin{pmatrix} j_f & j_i & \lambda\\ -\frac{1}{2} & \frac{1}{2} & 0\end{pmatrix}[\mathcal{R}^{\tau}_{\lambda}(\kappa_i,\kappa_f)+i\mathcal{R}^{\tau}_{\lambda}(-\kappa_i,\kappa_f)] \tag{2}$$

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and $\mathcal{H}^{\tau}_{\lambda\mu=-1}=\mathcal{H}^{\tau}_{\lambda\mu=1}$ for $\tau=C,E$; $\mathcal{H}^{\tau}_{\lambda\mu=-1}=-\mathcal{H}^{\tau}_{\lambda\mu=1}$ for $\tau=M$. The notation of Ref. (6) for initial (i) and final (f) distorted electron wave functions has been used $(j_{i(f)}=\kappa_{i(f)}-\frac{1}{2})$. The relations for phase-shifts (δ_κ) and radial solutions (g_κ, f_κ) of the Dirac equation for the electron in the electrostatic field $V(r)$,[6]

$$\begin{aligned}&\delta_{-\kappa}=\delta_\kappa\ ,\\&g_{-\kappa}(r)=f_\kappa(r)\ ,\\&f_{-\kappa}(r)=-g_\kappa(r)\end{aligned}\tag{3}$$

with $\kappa>0$, valid in the high energy limit $(m_e/\epsilon_e<<1)$, have been enforced to reduce the summations in Eq. (2) to be carried over positive κ_i, κ_f values. Likewise, from the symmetry properties of the Clebsch-Gordan coefficients in Eq. (2), we see that for even-parity transitions ($\lambda=$ even for $\tau=C,E$; $\lambda=$ odd for $\tau=M$) the sums over κ_i, κ_f in $\mathcal{H}^C_{\lambda\mu},\mathcal{H}^E_{\lambda\mu}$ are restricted to κ_f different from κ_i values, whereas for odd-parity transitions the above restriction holds for $\mathcal{H}^M_{\lambda\mu}$. The explicit expressions of the radial integrals $\mathcal{R}^\tau_\lambda(\pm\kappa_i,\kappa_f)$ are as follows:

$$\mathcal{R}^C_\lambda(\kappa_i,\kappa_f)=\frac{\delta_{\kappa_i+\kappa_f+\lambda,\,\text{even}}}{2\lambda+1}\int\int\frac{r_<^\lambda}{r_>^{\lambda+1}}\rho_\lambda^{(r)}G^+_{\kappa_i\kappa_f}(r')\ ,\tag{4}$$

$$\mathcal{R}^M_\lambda(\kappa_i,\kappa_f)=\delta_{\kappa_i+\kappa_f+\lambda,\,\text{odd}}\frac{(\kappa_i+\kappa_f)\omega}{[\lambda(\lambda+1)]^{1/2}}\int\int K_\lambda(r)K_\lambda(r')\rho_{\lambda,\lambda}(r)F^+_{\kappa_i\kappa_f}(r')\ ,\tag{5}$$

$$\begin{aligned}\mathcal{R}^E_\lambda(\kappa_i,\kappa_f)=\;&\delta_{\kappa_i+\kappa_f+\lambda,\,\text{even}}(-i\omega)[\lambda(\lambda+1)]^{1/2}(2\lambda+1)^{-3/2}\\&\times\int\int[\sqrt{\lambda+1}\rho_{\lambda\lambda-1}(r)K_{\lambda-1}(r)+\sqrt{\lambda}\rho_{\lambda\lambda+1}(r)K_{\lambda+1}(r)]\\&\times\left[(\kappa_i-\kappa_f)F^+_{\kappa_i\kappa_f}(r')\left(\frac{K_{\lambda-1}(r')}{\lambda}-\frac{K_{\lambda+1}(r')}{\lambda+1}\right)+F^-_{\kappa_i\kappa_f}(r')(K_{\lambda-1}+K_{\lambda+1})\right]\ .\end{aligned}\tag{6}$$

In the above equations the double integral symbol stands for $\int r^2dr\int r'^2dr'$; $r_>$ $(r_<)$ is the larger (smaller) of r and r'. The products of $K_L(r)$ and $K_L(r')$ denote products of spherical Bessel functions of the first (j_L) and third $(h_L^{(1)})$ kind[7]:

$$K_L(r)K_L(r')=j_L(\omega r_<)h_L^{(1)}(\omega r_>),\quad L=\lambda,\lambda\pm 1\tag{7}$$

$$K_{\lambda+1}(r)K_{\lambda-1}(r')=\theta(r'-r)j_{\lambda+1}(\omega r)h^{(1)}_{\lambda-1}(\omega r')+\theta(r-r')\left[h^{(1)}_{\lambda+1}(\omega r)j_{\lambda-1}(\omega r')+\frac{i(2\lambda+1)r'^{\lambda-1}}{\omega^3r^{\lambda+2}}\right]\ ,\tag{8}$$

where θ is the step function and $K_{\lambda-1}(r)K_{\lambda+1}(r')$ is obtained from Eq. (8) interchanging r and r'. The radial functions G and F are defined as

$$\begin{aligned}&G^\pm_{\kappa_i\kappa_f}(r)=f_{\kappa_i}(r)f_{\kappa_f}(r)\pm g_{\kappa_i}(r)g_{\kappa_f}(r)\ ,\\&F^\pm_{\kappa_i\kappa_f}(r)=f_{\kappa_i}(r)g_{\kappa_f}(r)\pm g_{\kappa_i}(r)f_{\kappa_f}(r)\ .\end{aligned}\tag{9}$$

The radial integrals $\mathcal{R}^C_\lambda(-\kappa_i,\kappa_f)$, $\mathcal{R}^M_\lambda(-\kappa_i,\kappa_f)$, $\mathcal{R}^E_\lambda(-\kappa_i,\kappa_f)$ are obtained from Eqs. (4), (5), and (6), respectively, by changing $\delta_{\kappa_i+\kappa_f+\lambda,\,\text{even (odd)}}$ into $\delta_{\kappa_i+\kappa_f+\lambda,\,\text{odd (even)}}$, $F^\pm_{\kappa_i\kappa_f}(r')$ by $\pm G^\mp_{\kappa_i\kappa_f}(r')$, $G^+_{\kappa_i\kappa_f}(r')$ by $F^-_{\kappa_i\kappa_f}(r')$, and κ_i by $-\kappa_i$. Finally the longitudinal (ρ_λ) and transverse $(\rho_{\lambda L=\lambda,\lambda\pm1})$ multipoles, purely dependent on the nulcear structure, are defined as reduced matrix elements of the nuclear charge $[\hat\rho(\vec r)]$ and vector current $[\hat{\vec J}(\vec r)]$ density operators as follows:

$$\rho_\lambda(r)=\frac{1}{e}\sum_{M_iM_f\mu}(-1)^{I_f-M_f}\begin{pmatrix}I_f&\lambda&I_i\\-M_f&\mu&M_i\end{pmatrix}\left\langle I_fM_f\left|\int d\Omega\, i^\lambda Y^\mu_\lambda(\Omega)\hat\rho(\vec r)\right|I_iM_i\right\rangle\ ,\tag{10}$$

$$\rho_{\lambda L}(r)=\frac{1}{e}\sum_{M_iM_f\mu}(-1)^{I_f-M_f}\begin{pmatrix}I_f&\lambda&I_i\\-M_f&\mu&M_i\end{pmatrix}\left\langle I_fM_f\left|\int d\Omega\, i^L\vec Y^\mu_{\lambda L}(\Omega)\cdot\hat{\vec J}(\vec r)\right|I_iM_i\right\rangle\ .\tag{11}$$

With the above definitions, the (nuclear) selection rules for the amplitudes $\mathcal{H}^C_{\lambda\mu},\mathcal{H}^E_{\lambda\mu},\mathcal{H}^M_{\lambda_\mu}$ are identical to those for the usual longitudinal (or Coulomb) and transverse electric and magnetic form factors, respectively, defined in Ref. (1). In particular, it is apparent that $\mathcal{H}^E_{\lambda\mu}$ is zero for elastic scattering and that the differential cross section at 180° is zero for $I_i=I_f=0$. The usual PWBA (Refs. 1 and 4) expression for the differential cross section can be obtained from Eq. (1) by taking the plane wave limit

$[V(r)\to 0]$, i.e., null phase shifts and spherical Bessel functions for the radial solutions[6]:

$$g_\kappa(r)=f_{\kappa+1}(r)=j_\kappa(K_e r)\quad(\kappa>0)\ .$$

Let us now analyze the conditions under which the cross section in Eq. (1) does not receive contributions from charge multipoles, i.e., $\mathcal{H}^C_{\lambda\mu=\pm1}=0$. Inspection of Eqs. (2) and (4) leads to the following conditions.

(i) The first obvious condition is that the nuclear charge density involved in the transition be spherically symmetric: $\rho_\lambda(r)=\rho_0(r)\delta_{\lambda,0}$.

(ii) The second condition is that $\omega=0$ (elastic scattering). In this case $(K_i=K_f)$ the radial solutions $[f_\kappa(r),g_\kappa(r)]$ for initial and final electron states are identical and since the terms in the sum over κ_i,κ_f in $\mathcal{H}^C_{\lambda\mu}$ change sign under the interchange of κ_i with κ_f, it follows that $\mathcal{H}^C_{\lambda\mu=\pm1}=0$ for $\omega=0$.

(iii) The third condition is that the radial solutions satisfy the relation

$$e^{i\delta_\kappa}g_\kappa(r)=e^{i\delta_{\kappa+1}}f_{\kappa+1}(r) \tag{12}$$

for any $\kappa>0$. For then, with rearrangement of the terms of the sum in $\mathcal{H}^C_{\lambda\mu}$ and use of the identity[8]

$$\begin{pmatrix} j_f & j_i & \lambda\\ \frac12 & \frac12 & -1\end{pmatrix}=-[\lambda(\lambda+1)]^{1/2}\begin{pmatrix} j_f & j_i & \lambda\\ -\frac12 & \frac12 & 0\end{pmatrix}[(j_f+\tfrac12)+(-1)^{j_i+j_f+\lambda}(j_i+\tfrac12)]\ ,$$

$\mathcal{H}^C_{\lambda\mu=\pm1}$ can be written as

$$\begin{aligned}\mathcal{H}^C_{\lambda\mu=\pm1}&=[(2\lambda+1)^2\lambda(\lambda+1)]^{-1/2}\\
&\times\int\int\frac{r_<^\lambda}{r_>^{\lambda+1}}\rho_\lambda(r)\sum_{\kappa_i,\kappa_f\geq1}e^{i(\delta_{\kappa_i}+\delta_{\kappa_f})}g_{\kappa_i}(r')g_{\kappa_f}(r')(-1)^{(3\kappa_i+3\kappa_f+\lambda)/2}\delta_{\kappa_i+\kappa_f+\lambda,\ \mathrm{even}}\\
&\times\Bigg\{(\kappa_f-\kappa_i)\left[\kappa_i\kappa_f\begin{pmatrix} j_f & j_i & \lambda\\ -\frac12 & \frac12 & 0\end{pmatrix}^2-(\kappa_i+1)(\kappa_f+1)\begin{pmatrix} j_f+1 & j_i+1 & \lambda\\ -\frac12 & \frac12 & 0\end{pmatrix}^2\right]\\
&+(\kappa_i+\kappa_f+1)\left[\kappa_f(\kappa_i+1)\begin{pmatrix} j_f & j_i+1 & \lambda\\ -\frac12 & \frac12 & 0\end{pmatrix}^2-\kappa_i(\kappa_f+1)\begin{pmatrix} j_f+1 & j_i & \lambda\\ -\frac12 & \frac12 & 0\end{pmatrix}^2\right]\Bigg\}\ ,\end{aligned}$$

and with the use of the recurrence relations for Clebsch-Gordan coefficients[8] the coefficient within large curly brackets can be shown to be identically zero.

This third condition is only met in the plane wave limit $[V(r)\to 0]$, but for $V(r)\neq 0$ the two members of Eq. (12) become more and more different as the charge increases, as can be shown by using the analytical solutions[6] of a pure Coulomb potential $[V(r)=-Z\alpha/r]$. No other conditions have so far been found to exclude the possibility of charge multipole contributions to the differential cross section at 180° in DWBA.

Taking into consideration these conditions we may conclude that for inelastic scattering on heavy nuclei at $\theta=\pi$, involving nonspherically symmetric transition charge distributions, charge multipoles with $\lambda\geq 2$ $(\lambda\geq1)$ for even (odd) parity transitions are expected to contribute to the cross section, provided that the excitation energy (ω) is not neglected against the incident energy (ϵ_i). On the other hand, since for the type of scattering under consideration the momentum transfer is $q=K_i+K_f\simeq 2\epsilon_i-\omega$, it is clear that, for a given ω value, the larger q is, the more one approaches the situation where condition (ii) applies, and consequently, charge multipoles would contribute more for smaller q values. Thus a quantitative analysis of such contributions may also be important for the correct interpretation of transverse form factors corresponding to natural parity excitations[9] in even-even nuclei as well as to transitions in odd-A nuclei, in which more than one multipolarity may enter.

Preliminary results[10] of $\lambda=2$ charge multipole contributions at 180° show that they are different from zero for inelastic scattering and grow with ω for fixed incident energy. In particular, this contribution appears to be negligible, as compared to the contributions from magnetic multipoles, in the transition $\frac{7}{2}^+\to\frac{9}{2}^+$ in ^{181}Ta due to the small excitation energy involved.

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