Spin observables in inelastic proton-nucleus scattering

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Two different prescriptions for the spin dependent amplitudes in inelastic proton-nucleus scattering have been put forward in the literature. We compare their predictions. The correct one is essentially the eikonal approximation to the distorted-wave Born approximation. The other one gives radically different predictions for many spin observables.

[NUCLEAR REACTIONS Spin observables in (p,p') reactions with a numerical illustration for 800 MeV proton-^{54}Fe scattering.]

Spin observables in the excitation of nuclear vibrational states by 800 MeV protons have recently been studied in some detail. A reasonable amount of data has been collected and several theoretical investigations have appeared. The theoretical studies are based on the Glauber model or the distorted-wave Born approximation (DWBA). In the present work we would like to point to a major difference in the Glauber models that have been employed.

The basic input to the Glauber model is the proton-nucleon amplitude. We keep only two terms

$$F_{NN}(\vec{q}) = f(q) + \vec{\sigma}\cdot\vec{n}g(q) \ ,$$

where $\vec{\sigma}$ is the spin operator of the incident proton and $\vec{n} = \vec{q}\times\hat{\kappa}$ with $\vec{\kappa} = \frac{1}{2}(\vec{k}+\vec{k}')$ and $\vec{q} = \vec{k}-\vec{k}'$. For simplicity we assume the transition density to be given by the collective model of Bohr and Mottelson. The nuclear density is assumed to have a Woods-Saxon shape.

The theory for inelastic scattering within this framework was first developed by Fäldt and Osland[1] and has been further investigated by Fäldt and Ingemarsson.[2] The scattering amplitude is decomposed as

$$\mathfrak{M}_{LM} = F_{LM}(q) + \tfrac{1}{2}[\sigma_+ G^{(-)}_{LM}(q) + \sigma_- G^{(+)}_{LM}(q)] + \sigma_z G^{(0)}_{LM}(q) \ , \tag{1}$$

with the coordinate axes chosen as $(\hat{x},\hat{y},\hat{z}) = (\hat{n},\hat{q},\hat{\kappa})$.

The expressions for the transition amplitudes are particularly simple when the nucleon amplitudes $f(q)$ and $g(q)$ are treated in the zero range approximation. Retaining only linear terms in the spin parameter $\lambda_s = 2\pi g(0)/ik$ we get

$$F_{LM}(q) = -\frac{\beta_L R}{\sqrt{2L+1}} ik \int_0^\infty b\,db\, J_M(qb) e^{-\lambda_c T(b)} \lambda_c Q_{LM}(b) \ , \tag{2a}$$

$$G^{(\mp)}_{LM}(q) = \pm\frac{\beta_L R}{\sqrt{2L+1}} ik \int_0^\infty b\,db\, J_{M\pm1}(qb) e^{-\lambda_c T(b)} \lambda_s \left[\frac{d}{db}Q_{LM}(b) - \lambda_c T'(b) Q_{LM}(b) \mp \frac{M}{b}Q_{LM}(b)\right] \ , \tag{2b}$$

$$G^{(0)}_{LM}(q) = 0 \ , \tag{2c}$$

where $\lambda_c = 2\pi f(0)/ik$ and

$$Q_{LM}(b) = \int dz\, P_{LM}\left(\frac{z}{\sqrt{b^2+z^2}}\right)\frac{\partial n(r)}{\partial r} \tag{3}$$

with $Y_{LM}(\Omega) = P_{LM}(\cos\theta)e^{im\phi}$. The remaining notation follows Ref. 1. For simplicity the Coulomb phase has been left out.

A partial integration in Eq. (2b) gives[1]

$$G^{(\mp)}_{LM}(q) = q\frac{\lambda_s}{\lambda_c}F_{LM}(q) \ , \tag{4}$$

i.e., the same relation as for elastic scattering.[3] Consequently, the polarization (or analyzing power) in inelastic scattering can be interpreted in a two-component model similar to that for elastic scattering.[4]

In Ref. 2 we investigate the relation between the Glauber model and the DWBA. We show that the Glauber model is the eikonal approximation to the DWBA, except for minor differences in the treatment of the excitation mechanism. In particular, the Glauber model as developed in Ref. 1 automatically reproduces the full Thomas form of the spin-orbit interaction[5] as used in DWBA calculations. This is an important aspect of our version of the Glauber

model.

Historically, the first attempt to deform the spin-orbit term within the optical model was done by the Oak Ridge group.[6] The spin amplitudes $G_{LM}^{(\pm)}$ obtained with their prescription differ slightly from those of Eq. (2b). The difference can be traced to a gradient operator acting on the transition density. In the Oak Ridge prescription it acts exclusively on its radial part but in the full Thomas prescription also on its angular part. As a result the last term of Eq. (2b), the one proportional to M, is left out and the first one is modified according to

$$\frac{d}{db}Q_{LM}(b) \rightarrow \int dz\, P_{LM}\left(\frac{z}{\sqrt{b^2+z^2}}\right)\frac{\partial}{\partial b}\left(\frac{d}{dr}n(r)\right)_{r=(b^2+z^2)^{1/2}} . \tag{5}$$

The missing terms in the Oak Ridge prescription are important when deriving Eq. (4) and naturally for the compatibility with the full Thomas form of the spin-orbit interaction. Consequently, it is of some interest to investigate where the differences between the two formulations show up.

Unfortunately, the comparison we have in mind is not completely straightforward. In a microscopic model, such as that of Glauber, the nuclear density always appears folded with a nucleon amplitude. Since the deformed spin-orbit term of the Oak Ridge prescription cannot be derived on a microscopic basis its extension away from the zero range approximation is not unique. Our solution to this dilemma has been to fold the deformed zero-range spin-orbit term in analogy with the full Thomas prescription.

For pedagogical reasons, our discussion has hitherto been based on the amplitudes of Eq. (2), which account for the spin dependence only to leading order in the parameter λ_s. The numerical results presented below, however, employ the calculations of Ref. 2, where the spin dependence is treated to all orders, the difference in the slope parameters accounted for, and the Coulomb phase included. The corresponding Oak Ridge amplitudes are obtained from Eqs. (16) of Ref. 2, replacing $(d/db)Q_{LM}^{s}$ by the properly folded version of the right hand side of Eq. (5) and neglecting terms proportional to M. The second condition implies $G_M^{(0)}=0$. In order to illustrate the effects of each of the modifications we compare the full Thomas prescription with results obtained when (a) only the M term is left out and (b) the complete Oak Ridge prescription is employed, i.e., when in addition the modification according to Eq. (5) is performed.

An important reason for emphasizing the difference between the two formulations is that the Glauber model as employed by the Penn group[7] starts from the Oak Ridge prescription, Eq. (5) of their article. A number of approximations are then performed yielding approximate asymptotic expressions for the amplitudes. Clearly, in an approximation scheme differences between models might easily be washed out, even to leading order of λ_s. Nevertheless, we deemed it worthwhile to compare the two prescriptions in their original formulations, before any approximations are made. Indeed, we shall demonstrate that there are spin observables which are extremely sensitive to the way the spin-orbit term is deformed. This discovery, however, does not invalidate the results of the Penn group,[7] since the analyzing powers turn out to be almost identical as soon as we are beyond the first minimum.

First, we demonstrate how the two prescriptions for the spin amplitudes compare for the quantities σ_{++}, σ_{--}, and σ_{+-}. They are defined as in Ref. 2 and refer to the cross sections $\sigma_{S_f S_i}$, where S_f and S_i

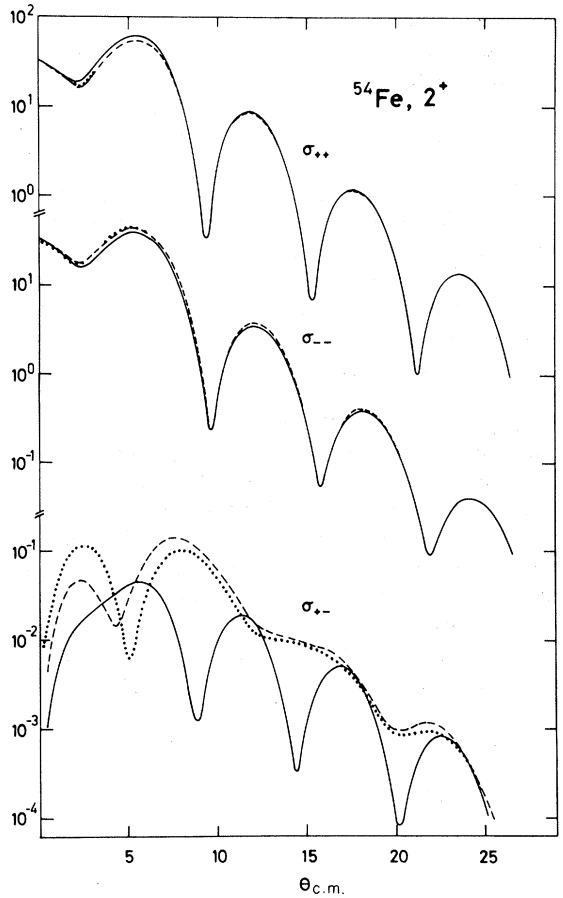

FIG. 1. Cross sections σ_{++} and σ_{--} for excitation of the 2^+ level in ^{54}Fe by 800 MeV protons. Solid curves are calculated with our (= full Thomas) prescription (Refs. 1 and 2), dashed curves without the M terms, and dotted curves with the Oak Ridge prescription.

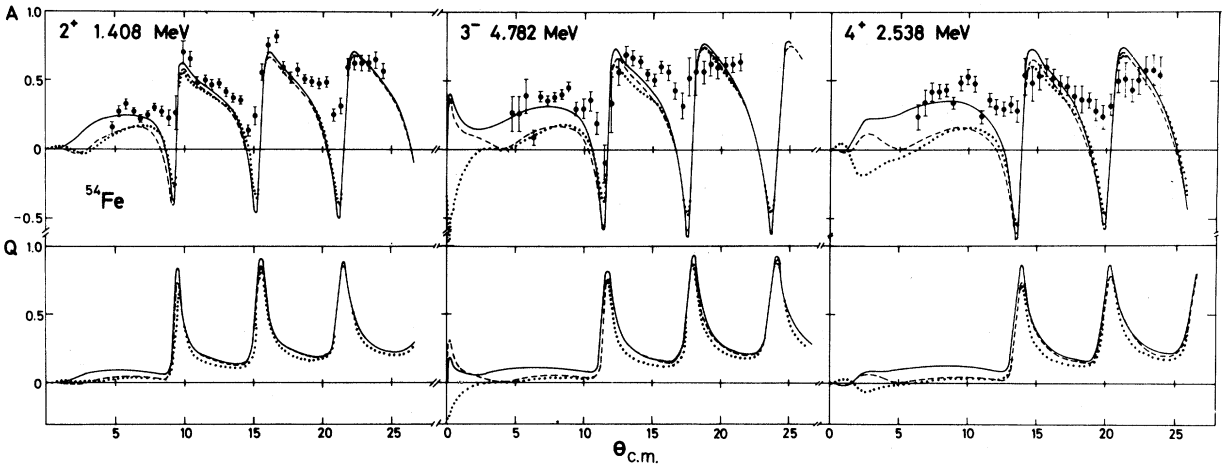

FIG. 2. Analyzing power and spin rotation quantity for excitation of the 2^+, 3^-, and 4^+ levels in ^{54}Fe by 800 MeV protons. Solid curves are calculated with our prescription, dashed curves without the M terms, and dotted curves with the Oak Ridge prescription. Experimental data from Ref. 9.

denote the spin projections of the final and initial protons along the normal $\hat{n} = \hat{q} \times \hat{\kappa}$. The spin-flip cross section σ_{+-} directly measures the strength of the spin amplitudes, and is given by

$$\sigma_{+-} = \sum_M |\tfrac{1}{2}(G_{LM}^{(-)} - G_{LM}^{(+)}) - G_{LM}^{(0)}|^2 \ . \qquad (6)$$

Clearly, since to leading order in λ_s, the spin-flip amplitudes $G_{LM}^{(+)}$ and $G_{LM}^{(-)}$ are equal in the full Thomas prescription, the spin-flip cross section σ_{+-} becomes very small. On the other hand, in the Oak Ridge prescription Eq. (4) is not fullfilled and consequently the spin-flip amplitudes $G_{LM}^{(+)}$ and $G_{LM}^{(-)}$ differ already in leading order of λ_s. Hence, we expect in this case a very different σ_{+-}.

In Fig. 1 we display the numerical results for the excitation of the 2^+ level of ^{54}Fe. The solid curves are obtained with our own prescription, the dashed curves with neglect of the M terms, and the dotted curves with the Oak Ridge prescription. The main contributions to σ_{++} and σ_{--} come from the spin independent amplitudes F_{LM}. Consequently, the plotted quantities are quite insensitive to the precise prescription for the spin amplitudes. Moreover, modifications of the deformed spin-orbit term enter with opposite signs in σ_{++} and σ_{--} implying substantial cancellations in the unpolarized cross section. The spin-flip cross section σ_{+-}, however, is as expected very sensitive. Our own prescription yields a cross section with extrema in the neighborhood of those of the cross sections σ_{++} and σ_{--}. The Oak Ridge prescription, on the contrary, produces a curve with a completely different shape. We also conclude that the main difference between the the prescriptions is due to the M terms.

Next, we compare the predictions for the analyzing power A (equal to the polarization in our approximation) and the spin rotation quantity Q.[8] In Fig. 2 the results for the excitation of the 2^+, 3^-, and 4^+ levels of ^{54}Fe are shown. Clearly, the differences between the full Thomas and the Oak Ridge prescriptions increase with increasing multipolarity but it is only below the first minimum that the difference is dramatic. The effect on Q is much smaller. We also observe that the M terms alone account for a large fraction of the difference.

The differences between the full Thomas and the Oak Ridge prescriptions as displayed for ^{54}Fe are very similar for other nuclei, though stronger for light ones such as ^{12}C.

We conclude that it is important to use the correct analytic expression for the spin-flip amplitude, Eq. (2b), in inelastic proton-nucleus scattering. Spin-flip cross sections, such as σ_{+-}, are extremely sensitive to the precise expressions used. The sensitivity is also sizable for the polarization and spin rotation observables, which measure the interference between the spin dependent and spin independent amplitudes.

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