Low-order variational calculations for model nuclear matter with the healing condition

B. Kargas*
Department of Theoretical Physics, University of Thessaloniki, Thessaloniki, Greece

E. Mavrommatis
Department of Physics, University of Athens, Panepistimiopolis, Athens, Greece
and Tandem Accelerator Laboratory, Nuclear Research Center "Demokritos,"
Aghia Paraskevi, Attiki, Greece

(Received 10 August 1982)

Low order calculations for the energy and density of model nuclear matter are carried out which make use of correlation functions $f(r)$ obtained by means of a differential equation. This is derived by variation of the two-body energy functional subject to the well known healing condition ($\rho \int [1-f(r)]^2 D(r) d\vec{r}=c$, c a small constant). The problem of the determination of the corresponding Lagrangian multiplier has been investigated and we have concluded to determine it from the less deep minimum of the ratio $|E^{(3)}/E^{(2)}|$ of the third to the second term of the factorized Iwamoto-Yamada expansion of the energy. In addition, the corresponding condition for the minimum has been studied. Results for the values of the energy (which has been approximated by the first three terms of the factorized Iwamoto-Yamada expansion), the density, and several convergence quantities have been obtained with the test potentials of Iwamoto and Yamada and Ohmura, Morita, and Yamada. Compared to other results from low and high order calculations, it is seen that those of the density are similar, but those for the energy are smaller in our case, suggesting that correlation functions quite constrained have been obtained. The possibility of improvements is finally considered.

[NUCLEAR STRUCTURE Correlation function, binding energy, and Fermi momentum of nuclear matter. Low order calculation, healing condition.]

I. INTRODUCTION

In this paper we perform low order variational calculations of the energy, the density, and other interesting quantities for model nuclear matter, using state-independent correlation functions $f(r)$ obtained by variation of the two-body energy functional with the help of the healing condition. Owing to the complications that high order calculations present, especially for state-dependent correlation functions,[1,2] an investigation of low order calculations is quite useful.[3] Moreover, investigation of ways to obtain $f(r)$ by solving corresponding Euler equations is desirable, since, on one hand, use of parametric forms restricts the variational space and becomes very cumbersome in the case of state dependent correlations due to the large number of parameters involved, and, on the other hand, there is not as yet a generally accepted procedure to derive $f(r)$ from Euler equations.[4]

Aiming for simplicity, we have chosen to obtain $f(r)$ from the simplest Euler equation, a differential equation. Correlation functions derived in some way by means of the differential equation, which results from the functional variation of the two-body energy functional, have appeared in the literature before. There are mainly two methods. In the first method[5] the term $\lambda f(r)D(r)$ [λ is considered as an external potential added to make possible the satisfaction of the boundary conditions, or as the contribution of the potential $V(r)$ to the average field, or as the Lagrangian multiplier for the constraint

$$\rho \int_0^d f^2(r)D(r)d\vec{r}=1 ,$$

which is imposed on the variation] is added to the differential equation. The correlation function $f(r)$ is then determined by solving the resulting equation from $r=0$ or c (c is the hard core radius of the potential) up to the healing distance d at which $f(r)$ satisfies the conditions $f(d)=1$ and $f'(d)=0$. Satisfaction of the last condition is achieved by suitably adjusting the parameter λ. The parameter d at the beginning of application of the method had a prescribed value [e.g., $d \approx 1.2r_0$, where r_0 is the unit radius, $r_0=(\frac{3}{4}\pi\rho)^{1/3}$]. Subsequently, d was determined by imposing the above-mentioned constraint on $f(r)$ (Ref. 5) and later by minimizing the energy

in a given approximation ($E \approx E^{(1,2)}$ or $E^{(1,2,3)}$ or E^{FHNC}), where FHNC denotes Fermi-hypernetted chain.[6,7] In the second method[8,9] the normalization condition in first order,

$$I_N = \rho \int [1 - f^2(r) D(r)] d\vec{r} = 1 ,$$

is imposed on the variation of the two-body energy functional. The corresponding Lagrangian multiplier λ is initially set equal to zero, and the equation is solved up to the distance d, which is chosen as large as possible, but such that $f(r<d) < h(d)$ and $f'(d) = h'(d)$ [$h(r)$ is a certain *a priori* specified asymptotic behavior]. The solution is inserted in the normalization integral I_N [see Eq. (10)]. If this is not less than or equal to one, the procedure is repeated by assigning different values to λ until the integral equals one.

In our approach we use the healing condition

$$H = \rho \int [f(r) - 1]^2 D(r) \vec{dr} = c \qquad (c \text{ a small constant}) \quad (1)$$

[$D(r)$ is given by Eq. (5)] to constrain the variation of the two-body energy functional and obtain $f(r)$ from the resulting differential equation. Constraint (1) was first proposed by Jastrow.[10] Its left-hand side, the healing integral H, is the Jastrow version of the wound parameter k_{ab} of Brueckner's theory averaged over all pairs $a < b$ in the Fermi sea. [Note that

$$k_{ab} = (\varphi_{ab} - \psi_{ab}, \varphi_{ab} - \psi_{ab}) ,$$

where ϕ_{ab} and ψ_{ab} are, respectively, the uncorrelated and correlated wave function of the pair occupying orbitals a and b inside the Fermi sea.[11]] Indeed, it can be shown that it is equal to the first cluster term of the wound integral $(\Psi - \Phi \mid \Psi - \Phi)$ with Ψ and Φ the wave functions of the interacting and noninteracting system, respectively. The healing integral is expected to give a rather good measure of the rate of convergence and it has been used extensively as a convergence criterion.[4] A possible relation of H to the higher order terms has been given by Arenhövel,[12] and other authors have used it together with the orthogonality condition to derive corresponding Euler equations in comparisons of the Brueckner and Jastrow theories.[13,14] However, owing to its unknown value, limited use of it as a constraint in low order calculations has been made.[14–17]

In this paper no specific value is given to H, but the Lagrangian multiplier involved is determined by the minimum of the ratio of the three body energy to the two body energy $|E^{(3)}/E^{(2)}|$. The $f(r)$ we obtain in this way is then used in our low order calculation. Details of our method of calculation as well as the derivation of the minimum condition are given in Sec. II. In Sec. III, we present our results, discuss them, and compare them with other results.

II. METHOD OF CALCULATIONS AND THE CONDITION FOR THE MINIMUM

We have obtained our Jastrow correlation functions $f(r)$ through functional variation. As the energy expression we have used $E^{(1,2)}$, that is, the sum of the Fermi energy E_F and the first cluster term $E^{(2)}$,

$$E^{(1,2)} = E_F + E^{(2)} , \qquad (2)$$

which for central state dependent potentials V,

$$V = \sum_i Vi(r) \hat{P}_i , \qquad (3)$$

($\hat{P}_i$ being the projection operator to the spin-isospin state i) and for spherically symmetric state independent correlation functions $f(r)$ is given by[18]:

$$E^{(2)} = 2\pi\rho \int_{0(c)}^{\infty} \left[\frac{\hbar^2}{m} \left(\frac{df}{dr} \right)^2 D(r) + \sum_i f^2(r) Vi(r) D_i(r) \right] r^2 dr , \qquad (4)$$

where $D(r)$ and $D_i(r)$ are defined as follows:

$$D(r) = 1 - \frac{1}{s} l^2(k_F r) , \qquad (5)$$

$$D_i(r) = \frac{1}{s^2} [\Delta_i^{(d)} - \Delta_i^{(e)} l^2(k_F r)] , \qquad (6)$$

where $l(x) = 3j_1(x)/x$; $j_1(x)$ is the spherical Bessel function of first order; k_F is the Fermi wave number, which is related to the nuclear matter density ρ by the formula $\rho = (2/3\pi^2) k_F{}^3$; s is the degeneracy of each orbital state; and $\Delta i^{(d)}$ and $\Delta i^{(e)}$ are the direct and exchange statistical factors, respectively, for the i orbital state. As is well known,[14] variation of $E^{(1,2)}$ yields an equation with long-ranged solutions [$f(r) = 1 + (\tilde{C}_1/r)$, where $\tilde{C}_1$ is a constant], which in turn make the energy cluster expansion converge badly. We have therefore incorporated in the variation the healing condition [Eq. (1)] with a Lagrangian multiplier λ, trying in some way to avoid the consequences of unconstrained variation.

The Euler equation that results from the variation of the functional $E^{(1,2)} + \lambda H$ is

$$\frac{d^2 f_0}{dr^2} + \left[\frac{2}{r} + \frac{1}{D(r)} \frac{dD(r)}{dr} \right] \frac{df_0}{dr} + \left[-\frac{m}{\hbar^2} \sum_{i=1}^{4} Vi(r) \frac{D_i(r)}{D(r)} - \beta^2 \right] f_0 = -\beta^2 , \qquad (7)$$

where $\beta^2=(m/\hbar^2)\lambda$. f_0 must satisfy the boundary conditions $\lim_{r\to 0} r f_0(r)=0$ or $f_0(c)=0$ (according to whether the potential has soft or hard core), and $f_0(\infty)=1$. Equation (7) is a linear inhomogeneous differential equation of second order. Its solution for large r,

$$f_0(r) \underset{r\to\infty}{\to} 1-\frac{Cce^{-\beta r}}{r} ,$$

(where C is a constant), although different from the known truly optimum one,[19]

$$f(r) \underset{r\to\infty}{\to} 1+\frac{nr^{-2}}{4\pi^2} \quad (n\neq 0) ,$$

is short ranged and therefore suitable for low order calculations. For each potential and value of k_F and β, $f(r)$ has been found by solving Eq. (7) numerically. However, for a given V and k_F the determination of β is not at all an easy task, since the healing integral does not have an *a priori* known value.

There are several ways that one can consider to determine β. One is to use the corresponding value of the averaged smallness parameter in Brueckner theory.[20] Another is to use that value of β for which the sum of the first three terms of the energy expansion, $E^{(1,2,3)}$, or the ratio of the third to the second term of the energy expansion $|E^{(3)}/E^{(2)}|$ is minimum, as was done before in the case of a Λ particle in nuclear matter.[21,22] An alternative way is to use that value of β for which the normalization condition, I_N [Eq. (10)] equals 1, or the orthogonality condition, I_0 [Eq. (12)] equals 0, is satisfied in first order. All methods except the first have been investigated, and we have chosen to determine β from the minimum of the ratio $|E^{(3)}/E^{(2)}|$. This choice seems reasonable, since the magnitude of $|E^{(3)}/E^{(2)}|$ as well as of H gives an indication of how rapidly the energy cluster expansion converges. For the potentials and densities we have used, the ratio $|E^{(3)}/E^{(2)}|$ had two minima (a primary one which we will call the first minimum and a secondary one which we will call the second minimum) within the range of β values considered, and we have performed calculations with each of them. Determination of β from the minimum of $E^{(1,2,3)}$ was not possible, since no minimum existed in our range of β values, and determination from the satisfaction of $I_N=1$ or $I_0=0$ is not expected to lead to less constrained results, as we shall mention in the next section.

Having determined the correlation function, we have proceeded to calculate the energy E. This has been approximated by its first three cluster terms,

$$E\simeq E^{(1,2)}+E^{(3)} . \tag{8}$$

Besides formula (4) we have also used for $E^{(2)}$ the expression

$$E^{(2)}=2\pi\rho\frac{\hbar^2}{m}\beta^2\int_{0(c)}^{\infty} f_0[1-f_0(r)]D(r)r^2dr , \quad (\beta\neq 0) \tag{9}$$

to check our computation. For $E^{(3)}$ we have used the third term of the factorized Iwamoto-Yamada (FIY) cluster expansion,[23] which has been given explicitly in Ref. 24.

Concerning the magnitude of the neglected higher order terms, that is, convergence, an estimate has been obtained by calculating the ratio $|E^{(3)}/E^{(2)}|$ and the healing integral H, as well as the normalization integral in first order[20,24]

$$I_N=\rho\int[1-f^2(r)D(r)]d\vec{r} , \tag{10}$$

the so-called smallness parameter ξ averaged over the Fermi sea[25]

$$\xi=\rho\int[f^2(r)-1]D(r)d\vec{r} , \tag{11}$$

(notice that $\xi=-I_N+1$), and in some cases the orthogonality integral in first order[26,20]

$$I_0=\rho\int[1-f(r)]D(r)d\vec{r} . \tag{12}$$

As is well known, good convergence is expected when H is small, $I_N\approx 1$, and $\xi\approx 0$ or $I_0\approx 0$.

Finally, we come to the discussion of the condition for $E^{(1,2)}(f_0)$ to be a minimum with respect to functional variation. It is an interesting feature of the present approach that such a condition can be derived. The procedure which will be described here is similar to the ones given in the cases of finite nuclei[27(a)] and Λ-hypernuclear matter,[22] which were based on analogous studies of functional variation.[27(b)]

We start calculating

$$\Delta E^{(1,2)}=E^{(1,2)}(f_1)-E^{(1,2)}(f_0)$$
$$=E^{(2)}(f_1)-E^{(2)}(f_0) \tag{13}$$

with

$$f_1(r)=f_0(r)+n(r) . \tag{14}$$

Since $f_1(r)$ too must satisfy the boundary conditions, it follows that $n(r)$ must satisfy the following ones for the case of hard core potentials:

$$n(c)=0 \text{ and } n(\infty)=0 . \tag{15}$$

By substituting (14) into $E^{(2)}$ [Eq. (4)] and taking into account the boundary conditions on $n(r)$, we end up with the following expression for $E^{(2)}(f_1)$:

$$E^{(2)}(f_1)=E^{(2)}(f_0)+2\pi\rho\left[2\int_c^\infty n(r)\left\{\left[-\frac{\hbar^2}{m}\right]\left[\frac{d^2f_0}{dr^2}+\left[\frac{2}{r}+\frac{1}{D(r)}\frac{dD(r)}{dr}\right]\frac{df_0}{dr}\right]+\sum_i V_i(r)\frac{D_i(r)}{D(r)}f_0\right\}D(r)r^2dr\right.$$

$$\left.+\int_c^\infty n(r)\left\{\left[-\frac{\hbar^2}{m}\right]\left[\frac{d^2n}{dr^2}+\left[\frac{2}{r}+\frac{1}{D(r)}\frac{dD(r)}{dr}\right]\frac{dn}{dr}\right]+\sum_i V_i(r)\frac{D_i(r)}{D(r)}n\right\}D(r)r^2dr\right]. \tag{16}$$

The first integral in the above expression can be reduced further by taking into account that f_0 is the solution of Eq. (7) and that f_1 too must satisfy the healing condition, since the variation is performed within the class of functions which comply with the constraint. From this last requirement we have the equality

$$\int_c^\infty [f_1(r)-1]^2D(r)r^2dr=\int_c^\infty [f_0(r)-1]^2D(r)r^2dr\ , \tag{17}$$

which with the help of (14) reduces to:

$$-2\int_c^\infty n(r)[f_0(r)-1]D(r)r^2dr=\int_c^\infty n^2(r)D(r)r^2dr\ . \tag{18}$$

With the use of Eqs. (7) and (18), the first integral in Eq. (16) with the factor 2 included is written:

$$-2\lambda\int_c^\infty n(r)[f_0(r)-1]D(r)r^2dr=\lambda\int_c^\infty n^2(r)D(r)r^2dr\ , \tag{19}$$

and Eq. (13) for $\Delta E^{(1,2)}$ with the help of (16) becomes

$$\Delta E^{(1,2)}=2\pi\rho\left[\int_c^\infty n(r)\left\{-\frac{\hbar^2}{m}\left[\frac{d^2n}{dr^2}+\left[\frac{2}{r}+\frac{1}{D(r)}\frac{dD(r)}{dr}\right]\frac{dn}{dr}\right]+\sum_i V_i(r)\frac{D_i(r)}{D(r)}n(r)+\lambda^2n(r)\right\}D(r)r^2dr\right]. \tag{20}$$

Since $D(r)>0$, it is evident from (20) that in order that $\Delta E^{(1,2)}$ be positive, and therefore $E^{(1,2)}(f_0)$ minimum, the following eigenvalue problem,

$$-\frac{\hbar^2}{m}\left[\frac{d^2n}{dr^2}+\left[\frac{2}{r}+\frac{1}{D(r)}\frac{dD(r)}{dr}\right]\frac{dn}{dr}\right]+\left[\sum_i V_i(r)\frac{D_i(r)}{D(r)}+\lambda^2\right]n(r)=Kn(r)\ ,$$

$$n(c)=0,\ n(\infty)=0\ , \tag{21}$$

must have only positive eigenvalues.

III. RESULTS AND DISCUSSION

The results which will be given in this section were obtained with the well-known hard core central potentials of Iwamoto and Yamada[28] (IY) and Ohmura, Morita, and Yamada[29] (OMY_6), which have been used in other model nuclear matter calculations. First, for each potential and value of the Fermi wave number k_F, the values of $E^{(2)}$, $E^{(1,2)}$, $E\approx E^{(1,2,3)}$, and $|E^{(3)}/E^{(2)}|$, as well as the values of the healing integral H, the normalization integrals I_N and ξ, and in some cases the orthogonality integral I_0, were computed as functions of β. We have started the calculations with the IY potential and $k_F=1.3767$ fm^{-1} (a value of saturation density

TABLE I. Various quantities as functions of β for the IY potential at $k_F = 1.3767$ fm^{-1}.

β	$E^{(2)}$	$E^{(1,2)}$	$E^{(3)}$	$E = E^{(1,2,3)}$					
(fm^{-1})	(MeV)	(MeV)	(MeV)	(MeV)	$\left\lvert\frac{E^{(3)}}{E^{(2)}}\right\rvert$	H	I_N	ξ	I_0
0.6	−35.07	−11.49	−4.72	−16.21	0.135	0.431	−7.99	8.961	−4.266
0.7	−34.45	−10.87	−0.57	−11.44	0.016	0.360	−5.46	6.422	−3.031
0.8	−33.94	−10.36	1.52	−8.84	0.045	0.316	−3.83	4.800	−2.242
1.0	−33.12	−9.54	3.01	−6.53	0.091	0.266	−1.96	2.929	−1.332
2.0	−30.83	−7.25	2.28	−4.97	0.074	0.206	0.43	0.537	−0.166
2.5	−29.89	−6.31	1.94	−4.37	0.065	0.197	0.70	0.264	−0.034
3.0	−28.83	−5.26	1.81	−3.45	0.063	0.190	0.81	0.119	0.036
3.5	−27.62	−4.04	1.81	−2.23	0.066	0.185	0.93	0.033	0.076
4.0	−26.24	−2.66	1.91	−0.75	0.073	0.180	0.95	−0.022	0.101
4.5	−24.69	−1.11	2.06	0.95	0.083	0.176	0.99	−0.058	0.117
5.0	−22.99	0.59	2.25	2.84	0.098	0.172	[a]	−0.084	0.128
5.5	−21.16	2.42	2.47	4.89	0.117	0.169	[a]	−0.102	0.135
6.0	−19.20	4.37	2.71	7.08	0.141	0.166	[a]	−0.115	0.141

[a]Values with less accuracy than that of the preceding values.

we obtained in earlier investigations[24]). Our detailed results for the various quantities as functions of β are presented in Table I. We realize that by increasing β, $E^{(2)}$ (and therefore $E^{(1,2)}$) decreases absolutely, whereas $E^{(3)}$ decreases more rapidly than $E^{(2)}$ up to zero and then fluctuates in the positive domain. As a result, $E^{(1,2,3)}$ increases continuously without having a minimum, whereas the ratio $|E^{(3)}/E^{(2)}|$ has two minima, a primary one at $\beta = 0.7$ fm^{-1}, and a secondary one at $\beta = 3$ fm^{-1}. As far as the integrals related to convergence are concerned, H decreases as β increases, whereas I_N, ξ, and I_0 tend to their values 1, 0, and 0, respectively. This behavior can be easily explained from the corresponding behavior of the correlation function $f(r)$ as a function of β, which is deduced from the plots in Figs. 1 and 2. We observe that by increasing β (that is, by considering more of the constraint in the variation), the overshoot and the range of $f(r)$ are reduced, and this results in a reduction of the magnitude of $E^{(2)}$ and generally in better convergence.

The above behavior of the different quantities as functions of β is systematically shown in Figs. 3–8 for both potentials and for the following three values of k_F: 1.1 fm^{-1} (low); 1.7 fm^{-1} (high); and 1.34 fm^{-1} (IY) or 1.515 fm^{-1} (OMY_6) (saturation). In particular, in Figs. 3 and 4, $E^{(2)}$, $E^{(3)}$, and E ($\approx E^{(1,2,3)}$) are plotted as functions of β for the above values of k_F and for the IY and OMY_6 potentials, respectively. In Figs. 5 and 6 the same is done for the ratio $|E^{(3)}/E^{(2)}|$. Finally, in Figs. 7 and 8 the healing integral H and the normalization integrals I_N and ξ are plotted. Aside from the dependence on β, we can also draw conclusions from these figures about the dependence of the different quantities on k_F for constant β. We observe that by increasing k_F, $E^{(2)}$ increases absolutely, whereas $E^{(3)}$ increases except for small values of β ($\beta \lesssim 1$ fm^{-1}). As a result, $E^{(1,2,3)}$ and $|E^{(3)}/E^{(2)}|$ increase for β larger than a certain value, but for smaller values do not have a systematic dependence on k_F. Regarding the convergence integrals, the healing integral H increases with k_F, whereas I_N decreases below $\beta \sim 2.5$ fm^{-1} and increases above it. As expected from Eq. (11), the opposite is true for ξ. This behavior of the various quantities can be understood from their defining formulas if one considers, on one hand, the dependence of ρ and $D(r)$ on k_F, and on the other hand, the dependence of $f(r)$ on k_F, which depends on the value of β. This latter for the case of the IY potential and $\beta = 0.7$ fm^{-1} is shown in the plot of Fig. 9. We realize that for $\beta = 0.7$ fm^{-1} an increase in k_F results in smaller overshoot and shorter range.

Having studied in detail the dependence on β for several k_F, we have proceeded to derive the equilibrium values for the interesting quantities of our

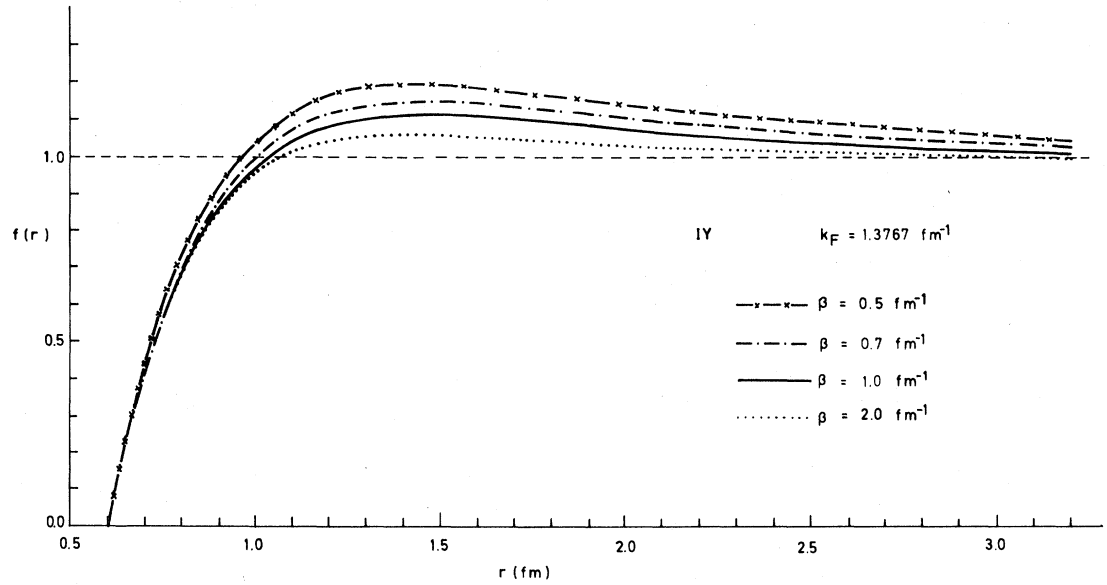

FIG. 1. Correlation functions obtained with the IY potential and several values of β at $k_F=1.3767$ fm^{-1}.

model nuclear matter, choosing for each k_F those values of β for which $|E^{(3)}/E^{(2)}|$ is minimum. First, we performed calculations with the first minimum choice of β, and we present the results in Tables II and III for the IY and OMY_6 potentials, respectively. In addition, we have drawn the corresponding saturation curves on Fig. 10. One sees, as expected, that by increasing k_F, $|E^{(2)}|$, H, $|I_N|$, and ξ increase. This dependence on k_F is almost the same as that discussed previously for constant β smaller than 2.5 fm^{-1}, since the corresponding values of β for each k_F are less than one and do not differ significantly. Regarding the values of $E^{(3)}$ and $|E^{(3)}/E^{(2)}|$, they are almost equal to zero, and therefore $E^{(1,2)}$ is almost equal to $E^{(1,2,3)}$ and both saturate.

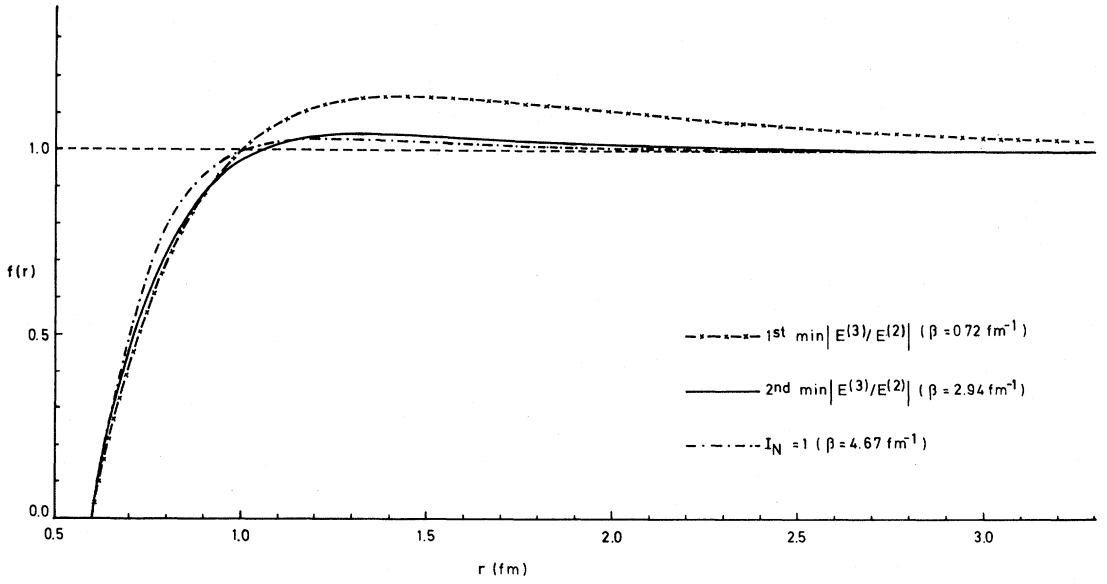

FIG. 2. Correlation functions obtained with the IY potential and the three choices of β at $k_F=1.3767$ fm^{-1}.

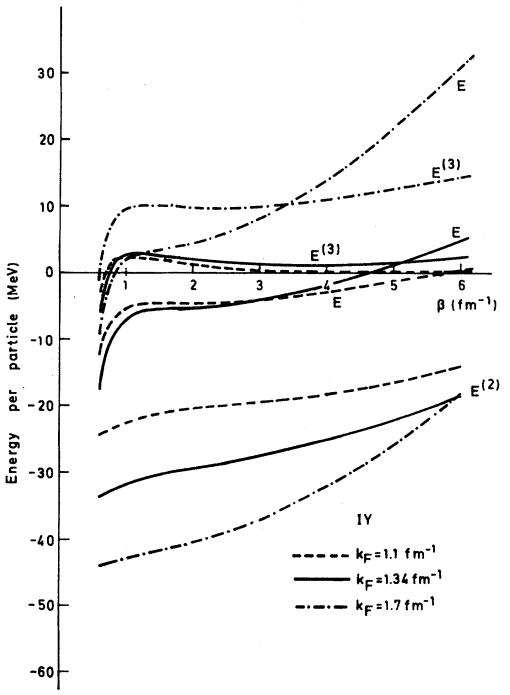

FIG. 3. $E^{(2)}$, $E^{(3)}$, and E $(=E_F+E^{(2)}+E^{(3)})$ as functions of β for the IY potential at $k_F=1.1$ fm^{-1}, 1.34 fm^{-1} (second minimum), and 1.7 fm^{-1}.

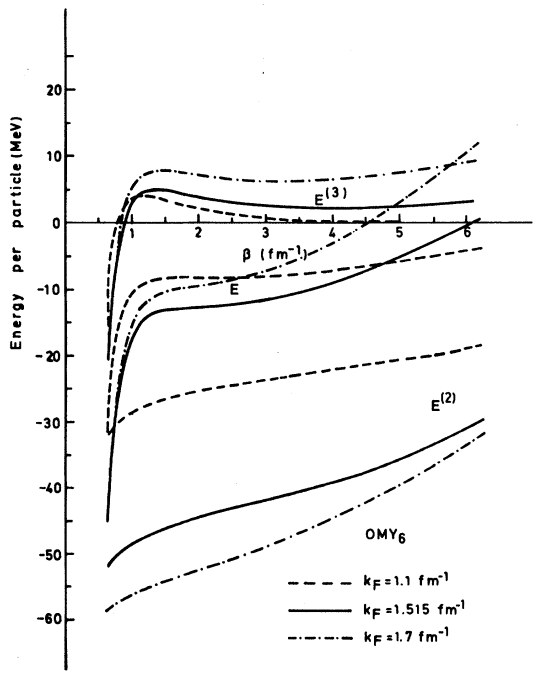

FIG. 4. $E^{(2)}$, $E^{(3)}$, and $E(=E_F+E^{(2)}+E^{(3)})$ as functions of β for the OMY_6 potential at $k_F=1.1$ fm^{-1}, 1.515 fm^{-1} (second minimum), and 1.7 fm^{-1}.

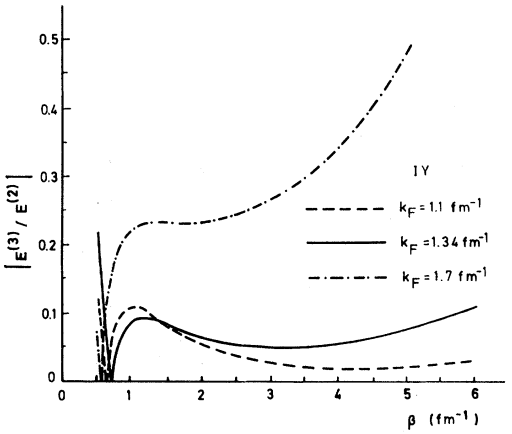

FIG. 5. Ratio $|E^{(3)}/E^{(2)}|$ as a function of β for the IY potential at $k_F=1.1$ fm^{-1}, 1.34 fm^{-1} (second minimum), and 1.7 fm^{-1}.

Next, results were obtained with the second minimum choice of β and are displayed in Tables IV and V for the IY and OMY_6 potentials, respectively. The corresponding saturation curves are plotted in Fig. 10. We realize that by increasing k_F the various quantities behave in the expected way. In particular, $E^{(3)}$ and $|E^{(3)}/E^{(2)}|$ increase, and $E^{(1,2,3)}$ has a minimum within the relevant density region, whereas $E^{(1,2)}$ does not. Concerning the dependence of the second minimum value of β on k_F, we notice that it decreases with increasing k_F and this is shown schematically in Fig. 11 for both potentials.

Let us summarize the results obtained for both potentials and choices of β:

IY. First minimum: $k_F=1.42$ fm^{-1}, $E=-10.8$ MeV, $|E^{(3)}/E^{(2)}|\approx 0$, $H=0.371$, $I_N=-5.42$, $\xi=6.391$. Second minimum: $k_F=1.34$ fm^{-1}, $E=-3.6$ (629) MeV, $|E^{(3)}/E^{(2)}|=0.054$, $H=0.174$, $I_N=0.83$, $\xi=0.101$.

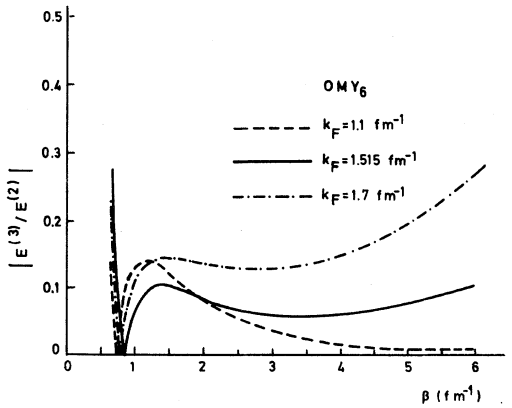

FIG. 6. Ratio $|E^{(3)}/E^{(2)}|$ as a function of β for the OMY_6 potential at $k_F=1.1$ fm^{-1}, 1.515 fm^{-1} (second minimum), and 1.7 fm^{-1}.

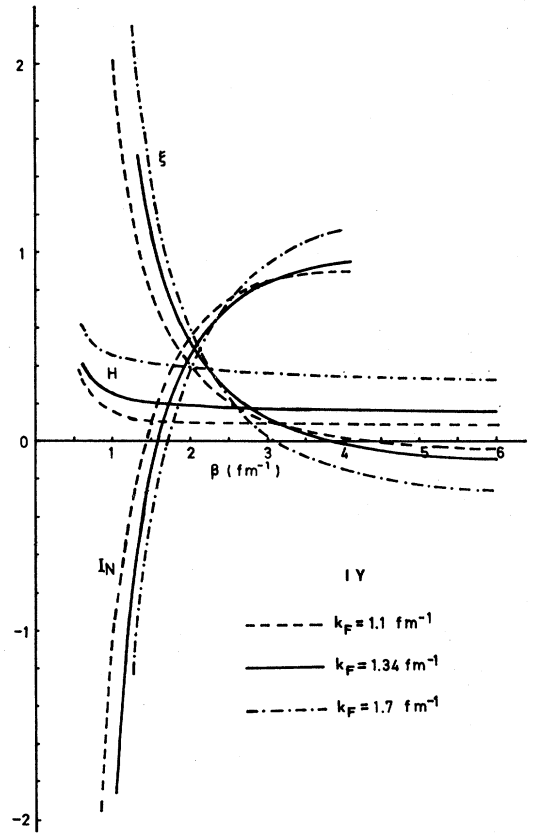

FIG. 7. Healing integral H and the normalization integrals I_N and ξ as functions of β for the IY potential at k_F=1.1 fm^{-1}, 1.34 fm^{-1} (second minimum), and 1.7 fm^{-1}.

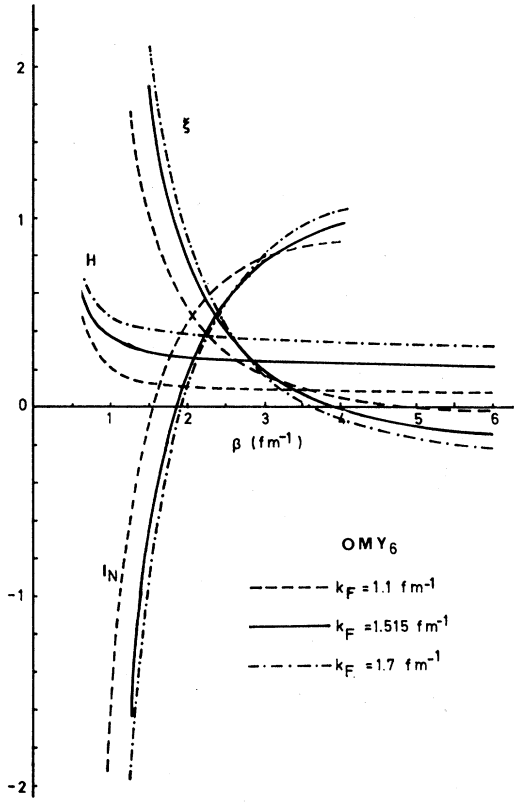

FIG. 8. Healing integral H and the normalization integrals I_N and ξ as functions of β for the OMY$_6$ potential at k_F=1.1 fm^{-1}, 1.515 fm^{-1} (second minimum), and 1.7 fm^{-1}.

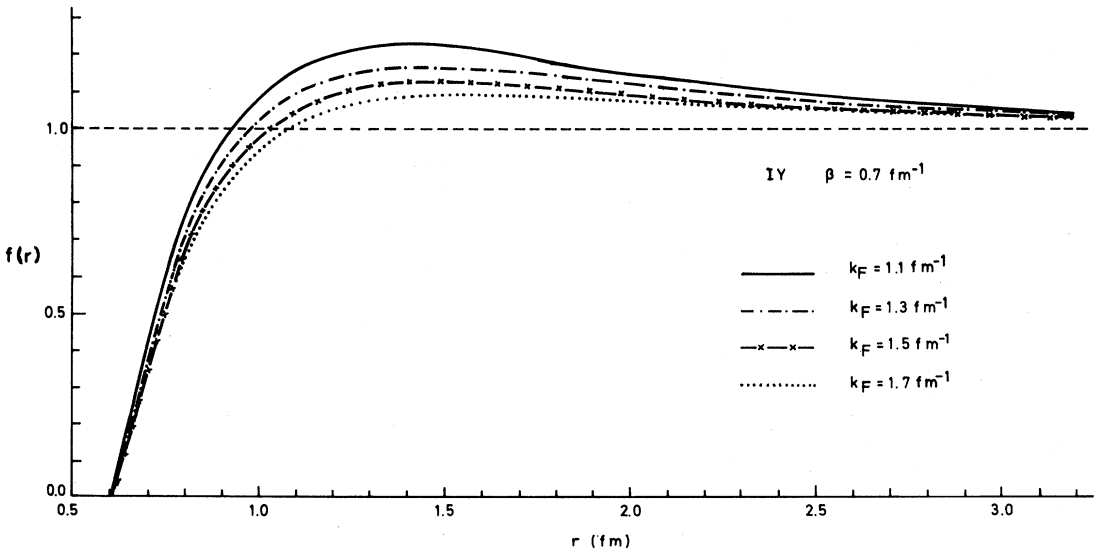

FIG. 9. Correlation functions obtained with the IY potential and β=0.7 fm^{-1} for several values of k_F.

TABLE II. Results with the IY potential and the "first minimum" choice of β as functions of k_F.

k_F (fm^{-1})	β (fm^{-1})	$E^{(1)}=E_F$ (MeV)	$E^{(2)}$ (MeV)	$E^{(1,2)}$ (MeV)	$E^{(3)}$ (MeV)	$E=E^{(1,2,3)}$ (MeV)	$\left\|\frac{E^{(3)}}{E^{(2)}}\right\|$	H	I_N	ξ
1.10	0.654	15.1	−24.3	−9.2	0.0	−9.2	0.0	0.283	−4.23	5.184
1.20	0.694	17.9	−27.9	−10.0	0.0	−10.0	0.0	0.294	−4.34	5.295
1.30	0.718	21.0	−31.6	−10.6	0.0	−10.6	0.0	0.319	−4.62	5.582
1.42	0.716	25.1	−35.9	−10.8	0.0	−10.8	0.0	0.371	−5.42	6.391
1.50	0.691	28.0	−38.7	−10.7	0.0	−10.7	0.0	0.423	−6.42	7.391
1.60	0.639	31.8	−42.1	−10.3	0.0	−10.3	0.0	0.507	−8.45	9.417
1.70	0.572	36.0	−45.2	−9.2	0.0	−9.2	0.0	0.615	−11.71	12.684

TABLE III. Results with the OMY$_6$ potential and the "first minimum" choice of β as functions of k_F.

k_F (fm^{-1})	β (fm^{-1})	$E^{(1)}=E_F$ (MeV)	$E^{(2)}$ (MeV)	$E^{(1,2)}$ (MeV)	$E^{(3)}$ (MeV)	$E=E^{(1,2,3)}$ (MeV)	$\left\|\frac{E^{(3)}}{E^{(2)}}\right\|$	H	I_N	ξ
1.20	0.794	17.9	−35.4	−17.5	0.0	−17.5	0.0	0.340	−4.12	5.081
1.30	0.834	21.0	−40.0	−19.0	0.0	−19.0	0.0	0.351	−4.24	5.200
1.40	0.862	24.4	−44.5	−20.1	0.0	−20.1	0.0	0.375	−4.44	5.407
1.50	0.870	28.0	−48.9	−20.9	0.0	−20.9	0.0	0.414	−4.85	5.819
1.60	0.851	31.8	−53.1	−21.3	0.0	−21.3(260)	0.0	0.472	−5.63	6.601
1.65	0.831	33.9	−55.2	−21.3	0.0	−21.3(280)	0.0	0.509	−6.22	7.195
1.75	0.772	38.1	−59.0	−20.9	0.0	−20.9	0.0	0.600	−7.97	8.944
1.80	0.736	40.3	−60.8	−20.5	0.0	−20.5	0.0	0.655	−9.21	10.181

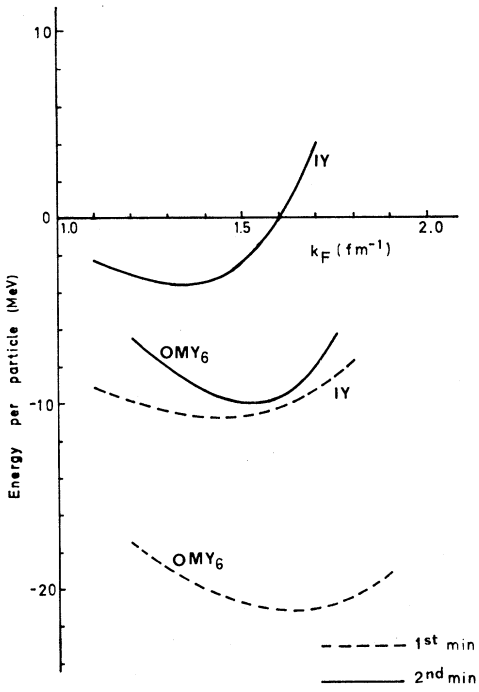

FIG. 10. Saturation curves for the IY and OMY$_6$ potentials obtained with the "first minimum" and "second minimum" choices of β.

OMY$_6$. First minimum: $k_F=1.65$ fm^{-1}, $E=-21.3$ (280) MeV, $|E^{(3)}/E^{(2)}|\approx 0$, $H=0.509$, $I_N=-6.22$, $\xi=7.195$. Second minimum: $k_F=1.52$ (515) fm^{-1}, $E=-10.0$ (027) MeV, $|E^{(3)}/E^{(2)}|=0.058$, $H=0.241$, $I_N=0.85$, $\xi=0.087$.

First, we observe that for each choice of β the values of k_F and E at equilibrium for the IY and OMY$_6$ potentials are related in the same way as in all previous references: k_F as well as $|E|$ are larger in the case of OMY$_6$. Second, comparing results obtained with the two choices of β we realize that for each potential the values of k_F, $|E|$, H, $|I_N|$, and ξ are larger in the case of the first minimum choice, which means that in this case the corresponding values of the equilibrium density are further or closer, respectively, to the empirical ones but convergence is poorer. Since good convergence is required for reliable results, we consider as our final results the ones obtained with the aid of the second minimum (the less deep) of $|E^{(3)}/E^{(2)}|$. These give values for k_F quite close to the empirical one ($k_{F_{\text{emp}}}\simeq 1.36$ fm^{-1}), but small values for E that differ from the empirical one ($E_{\text{emp}}\simeq -16$ MeV).[30] Since in the first minimum $|E^{(3)}/E^{(2)}|$ is smaller and H larger than in the second one, the above calculations show that $|E^{(3)}/E^{(2)}|$ is not always proportional to H, in agreement with previous observations.[24,31] One should add that choosing the second

TABLE IV. Results with the IY potential and the "second minimum" choice of β as functions of k_F.

k_F (fm^{-1})	β (fm^{-1})	$E^{(1)}=E_F$ (MeV)	$E^{(2)}$ (MeV)	$E^{(1,2)}$ (MeV)	$E^{(3)}$ (MeV)	$E=E^{(1,2,3)}$ (MeV)	$\left\|\frac{E^{(3)}}{E^{(2)}}\right\|$	H	I_N	ξ
1.10	4.220	15.1	−17.7	−2.6	0.3	−2.3	0.019	0.090	0.91	0.006
1.20	3.740	17.9	−21.6	−3.7	0.6	−3.1	0.029	0.120	0.89	0.029
1.30	3.275	21.0	−25.8	−4.8	1.2	−3.6(587)	0.045	0.157	0.85	0.075
1.34	3.100	22.3	−27.4	−5.1	1.5	−3.6(629)	0.054	0.174	0.83	0.101
1.40	2.845	24.4	−29.9	−5.5	2.1	−3.4	0.069	0.202	0.78	0.154
1.50	2.444	28.0	−34.0	−6.0	3.6	−2.4	0.105	0.257	0.64	0.292
1.60	2.070	31.8	−37.9	−6.1	6.0	−0.1	0.157	0.323	0.44	0.530
1.70	1.720	36.0	−41.6	−5.6	9.6	4.0	0.232	0.403	0.02	0.952

TABLE V. Results with the OMY_6 potential and the "second miminum" choice of β as functions of k_F.

k_F (fm^{-1})	β (fm^{-1})	$E^{(1)}=E_F$ (MeV)	$E^{(2)}$ (MeV)	$E^{(1,2)}$ (MeV)	$E^{(3)}$ (MeV)	$E=E^{(1,2,3)}$ (MeV)	$\left\|\frac{E^{(3)}}{E^{(2)}}\right\|$	H	I_N	ξ
1.30	4.510	21.0	−29.8	−8.8	0.6	−8.2	0.022	0.144	0.93	−0.003
1.40	4.012	24.4	−35.0	−10.6	1.2	−9.4	0.034	0.184	0.91	0.025
1.50	3.540	28.0	−40.2	−12.2	2.2	−10.0(003)	0.054	0.233	0.86	0.077
1.52(515)	3.470	28.6	−41.0	−12.4	2.4	−10.0(027)	0.058	0.241	0.85	0.087
1.60	3.094	31.8	−45.3	−13.5	3.8	−9.7	0.083	0.290	0.77	0.166
1.65	2.886	33.9	−47.7	−13.8	4.9	−8.9	0.102	0.323	0.71	0.230
1.70	2.683	36.0	−50.1	−14.1	6.3	−7.8	0.126	0.358	0.63	0.314
1.75	2.490	38.1	−52.4	−14.3	8.0	−6.3	0.153	0.396	0.53	0.418

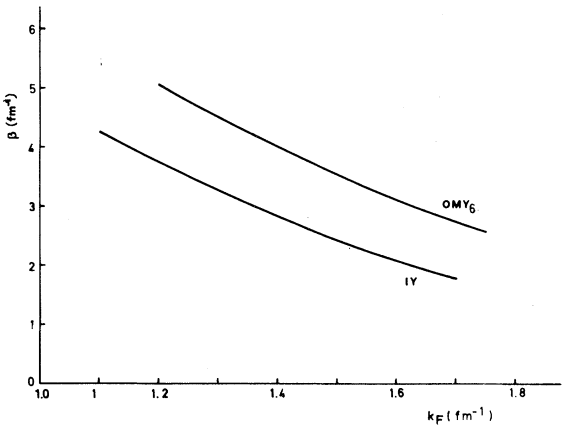

FIG. 11. "Second minimum" value of β as a function of k_F for the IY and OMY_6 potentials.

minimum of $|E^{(3)}/E^{(2)}|$ to specify the value of β seems equivalent to imposing an additional constraint besides the minimization of $|E^{(3)}/E^{(2)}|$ (e.g., that β must be such that $I_N \geq 0$).

Let us now compare our results with those derived by other authors. First, we compare with those results extracted by means of correlation functions obtained from differential Euler equations. Krotscheck[16] used a modified healing constraint to derive $f(r)$ from the variation of the two-body energy functional in the case of the OMY_6 potential. He suggests that the corresponding Lagrangian multiplier could be specified with the aid of the condition $\xi < 1$. In this way he restricted the range of its values, but did not give final results. Dabrowski *et al.*[17] used the healing condition to derive an $f(r)$ in a study of spin and isospin stability of dense nuclear matter. The Lagrangian multiplier is fixed by the minimization of the first cluster terms of the IY cluster expansion of the energy, but results have been obtained only for the pure hard core potential. Also, results obtained with the first method mentioned in the Introduction and developed by Pandharipande *et al.* do not exist for these test potentials, but results have been obtained by Owen *et al.*[9] with the second method. They are given in Table VI and are extracted from the corresponding figures of Ref. 9. We see that the values of the density are almost the same as ours, but their values of the energy E $(=E^{(1,2)})$ lie closer to the empirical one. An explicit estimate of convergence of their calculation is not available. The corresponding correlation function together with ours is plotted in Fig. 12 for the IY potential. We realize that their long-range behavior is the same but they differ in their short-range behavior.

Second, let us compare our results with those extracted from other low-order calculations using

TABLE VI. Results for k_F, E, $|E^{(3)}/E^{(2)}|$, and H obtained with the IY and OMY_6 potentials and different methods.

Potential	Correlation function	Minimized quantity	Constraint	k_F (fm^{-1})	E[a] (MeV)	$\left\|\frac{E^{(3)}}{E^{(2)}}\right\|$	H	Ref.
IY	Differential equation	$E^{(1,2)}$	healing	1.34	−3.6	0.054	0.174	This work
	Differential equation	$E^{(1,2)}$	norm. and bound.	1.36	−7.3			9
	C_1	$E^{(1,2)}$		1.40	−8.2	0.105	0.237	32
	C_2	$E^{(1,2)}$	norm.	1.47	−6.6	0.003	0.262	24
	C_3	$E^{(1,2,3)}$	orth. and subs.	1.36	−7.5	0.037	0.224	34
	C_1	E_{FHNC}		1.34	−7.3			35
OMY_6	Differential equation	$E^{(1,2)}$	healing	1.52	−10.0	0.058	0.241	This work
	Differential equation	$E^{(1,2)}$	norm. and bound.	1.59	−15.5			9
	C_1	$E^{(1,2)}$		1.60	−16.3	0.093	0.319	32
	C_2	$E^{(1,2)}$	norm.	1.68	−14.2	0.014	0.370	24
	C_3	$E^{(1,2,3)}$	orth. and subs.	1.54	−16.1	0.046	0.300	34
	C_1	E_{FHNC}		1.50	−15.5			35

[a] E stands for $E^{(1,2,3)}$ except in the second and the last case for each potential where it is $E^{(1,2)}$ and E_{FHNC}, respectively.

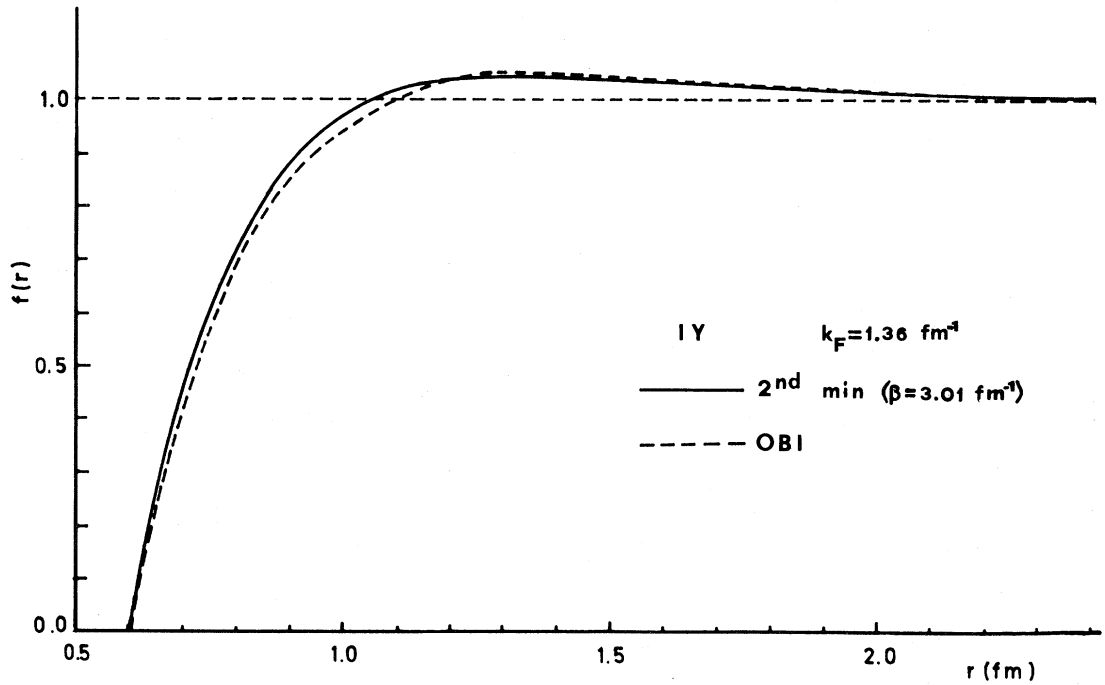

FIG. 12. Correlation function of this work ("second minimum" choice of β) and that of Ref. 9 for the IY potential at $k_F=1.36$ fm^{-1}.

parametric forms for $f(r)$. Some of them are given in Table VI. We remark that the values for k_F, E, as well as H obtained with correlation functions with one parameter, C_1

$$[f(r)=(1-e^{-\mu_1(r-c)}),\ r>c]$$

are larger than ours.[18,32] The same is true for those extracted by minimizing $E^{(1,2)}$ and using correlation functions with two parameters, C_2

$$[f(r)=(1-e^{-\mu_1(r-c)})(1+\nu e^{-\mu_1(r-c)}),\ r>c]$$

and the normalization condition in first order,[24,33] or by minimizing $E^{(1,2,3)}$ and using correlation functions with three parameters, C_3

$$[f(r)=(1-e^{-\mu_1(r-c)})(1+\nu e^{-\mu_2(r-c)}),\ r>c]\ ,$$

the orthogonality condition, and some subsidiary conditions.[26,34] In fact, we come to the same conclusion if we compare our results with all the ones mentioned in Ref. 24. Finally, regarding the relation of our results with those derived from an FHNC calculation with the use of C_1,[35] we realize that the values of equilibrium density are the same, but those for the binding energy are smaller in our case.

In concluding, results for the density and energy of model nuclear matter have been obtained by means of a method which makes use of correlation functions determined from the Euler differential equation with the aid of the well-known healing condition. However, it seems that our choice for the determination of the Lagrangian multiplier for the healing condition by means of the second minimum of $|E^{(3)}/E^{(2)}|$ leads to a rather overconstrained variation, and since determination of β from satisfaction of the conditions $\xi=0$ or $I_0=0$ is expected to lead to a more constrained or almost equally constrained variation, respectively [since the resulting values of β are larger or almost equal (see Table I)], it seems for the moment that the only possible way to improve the results in the framework of the present approach is to modify the healing constraint by suitably introducing a second parameter, which may lead to less constrained $f(r)$ and may even result in saturation of $E^{(1,2)}$ within the relative density region.

ACKNOWLEDGMENTS

We would like to thank Professor M. E. Grypeos for his constant interest in our work and useful comments on the manuscript. One of us (E.M.) also thanks him for partial financial support through a grant from the National Science Foundation. She also would like to thank Professor J. M. Irvine and Dr. J. C. Owen for useful discussions related to this work. This work was supported in part by the National Research Foundation through the Department of Theoretical Physics, University of Thessaloniki, Greece.

*Present address: Institute for Higher Technical Education, Serres, Greece.

[1]S. Fantoni and S. Rosati, Nuovo Cimento 43A, 413 (1978).

[2]V. R. Panharipande and R. B. Wiringa, Nucl. Phys. A266, 269 (1976); A299, 1 (1978); Rev. Mod. Phys. 51, 821 (1979).

[3]J. M. Irvine, Prog. Part. Nucl. Phys. 5, 1 (1981).

[4]J. W. Clark, Prog. Part. Nucl. Phys. 2, 89 (1979).

[5]V. R. Panharipande, Nucl. Phys. A174, 645 (1971); A178, 123 (1971); V. R. Panharipande and H. A. Bethe, Phys. Rev. C 7, 1312 (1973).

[6]E. g. I. E. Lagaris, V. R. Panharipande, and R. B. Wiringa, Nucl. Phys. A328, 113 (1979); O. Benhar, C. Ciofi degli Atti, S. Fantoni, and S. Rosati, *ibid.* A319, 127 (1979); R. Guardiola, A. Polls, S. Fantoni, and S. Rosati, Nuovo Cimento 44A, 414 (1978).

[7]J. Zabolitzky, Phys. Rev. A 16, 1258 (1977).

[8]J. C. Owen, R. F. Bishop, and J. M. Irvine, Ann. Phys. (N.Y.) 102, 170 (1976); Phys. Lett. 59B, 1 (1975); 61B, 247 (1976); 66B, 25 (1977).

[9]J. C. Owen, R. F. Bishop, and J. M. Irvine, Nucl. Phys. A274, 108 (1976).

[10]R. Jastrow, private communication to V. Emery [Nucl. Phys. 6, 585 (1958)].

[11]H. A. Bethe, Annu. Rev. Nucl. Sci. 21, 93 (1971).

[12]P. G. Reinhard and H. Arenhövel, Z. Phys. 264, 211 (1973).

[13]L. Schäfer, Nucl. Phys. A194, 497 (1972).

[14]K. Schäfer and G. Schütte, Nucl. Phys. A183, 1 (1972).

[15]P. G. Reinhard and H. Arenhövel, Z. Phys. 262, 1 (1973).

[16]E. Krotscheck, Ph.D. dissertation, Universität zu Köln, Köln, Federal Republic of Germany, 1974 (unpublished).

[17]J. Dabrowski, W. Piechocki, J. Rozynek, and P. Haensel, Acta Phys. Pol. B10, 83 (1979).

[18]P. Westhaus, Ph.D. thesis, Washington University, Washington, D.C., 1966 (unpublished).

[19]E. Krotscheck, Lett. Nuovo Cimento 14, 219 (1975); Phys. Rev. A15, 397 (1977); E. Krotscheck and W. Kundt, Phys. Lett. 71B, 19 (1977).

[20]J. W. Clark and M. L. Ristig, Phys. Rev. C 5, 1553 (1972).

[21]M. E. Grypeos, L. P. Kok, and S. Ali, Nucl. Phys. B4, 335 (1968).

[22]S. Ali, M. E. Grypeos, and B. Kargas, Phys. Rev. C 14, 285 (1976); B. Kargas and M. E. Grypeos, Science Annals, Faculty of Physics and Mathematics, University of Thessaloniki, 1977, Vol. 17, p. 13.

[23]J. W. Clark and M. L. Ristig, in *The Nuclear Many Body Problem,* edited by F. Calogero and C. Ciofi degli Atti (Editrice, Bologna, 1973), Vol. 2, p. 273.

[24]E. Mavrommatis, Nucl. Phys. A301, 65 (1978).

[25]J. W. Clark and M. L. Ristig, Phys. Rev. C 7, 1792 (1973).

[26]S. O. Bäckman, D. A. Chakkalakal, and J. W. Clark, Nucl. Phys. A130, 635 (1969).

[27](a) M. E. Grypeos, Phys. Lett. 34B, 481 (1971); (b) D. M. Brink and M. E. Grypeos, Nucl. Phys. A97, 81 (1967).

[28]F. Iwamoto and M. Yamada, Prog. Theor. Phys. (Kyoto) 18, 345 (1957).

[29]T. Ohmura, M. Morita, and M. Yamada, Prog. Theor. Phys. (Kyoto) 15, 222 (1956).

[30]Not much emphasis should be attributed to the comparison with empirical values, since the potentials are semirealistic.

[31]G. P. Mueller, Nucl. Phys. A210, 544 (1973).

[32]J. W. Clark, P. M. Lam, and W. J. Ter. Louw, Nucl. Phys. A255, 1 (1975).

[33]M. E. Grypeos and E. Mavrommatis, Lett. Nuovo Cimento 5, 369 (1972).

[34]S. O. Bäckman, J. W. Clark, W. J. Ter Louw, D. A. Chakkalakal, and M. L. Ristig, Phys. Lett. 41B, 247 (1972).

[35]E. Campani, S. Fantoni, and S. Rosati, Lett. Nuovo Cimento 15, 217 (1976).