Spectral distribution calculations of the level density and spin-cutoff parameters of ^{28}Si

S. M. Grimes*
University of California, Lawrence Livermore National Laboratory, Livermore, California 94550 and Ohio University, Athens, Ohio 45701

S. D. Bloom and H. K. Vonach†
University of California, Lawrence Livermore National Laboratory, Livermore, California 94550

R. F. Hausman, Jr.
University of California, Los Alamos National Laboratory, Los Alamos, New Mexico 87544

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Spectral distribution calculations of the separate positive and negative parity level densities and spin cutoff parameters for ^{28}Si have been made in an *sdf* basis. A configuration expansion utilizing higher moments of the Hamiltonian is found to be important in improving agreement between calculation and experiment at low excitation energies. The Hamiltonian moments were corrected for the effects of spurious c.m. motion. The present results suggest that spectral distribution calculations can yield level densities which agree with experimental information on the number of levels, spin cutoff factors, and parity ratios over an energy range of over 20 MeV.

[NUCLEAR STRUCTURE ^{28}Si; calculated level density and spin cutoff parameter 0–25 MeV.]

I. INTRODUCTION

Spectral distribution methods have been shown[1] to be a powerful tool in calculating the properties of many-body systems with the full inclusion of two-body forces. Calculations of level densities for a number of nuclei have already been presented. Some unresolved problems remain, however.

Most of the calculations[2–8] made to date have used propagator approaches[9] to calculate the needed parameters for level density expansions. These techniques work effectively for operators of low rank but not so efficiently for higher rank operators. For this reason, most of the calculations have utilized Gaussian functions to represent the level density. Only first and second moments of the Hamiltonian are required for such an expansion and these, as well as their product with J_z^2 (the z projection of the angular momentum), can be easily calculated using propagator techniques.

Unfortunately, there are indications that the two moment expansions are not always adequate if one focuses on low excitation energies. Chang and Zucker[3] have shown that inclusion of third and fourth moments of the Hamiltonian can improve the fit to a spectrum obtained by diagonalization. In comparisons[5,7] with data it has been found that the spin cutoff factor for ^{28}Si is overestimated at low energies by a two-moment expansion. An extension[8] of the calculation to include configuration expansion of the level density (but only with two moments) produced some improvement, but did not bring the results into agreement with the data.[8]

More recently, an alternative approach to[10–12] the calculation of energy moments in large systems was developed. This technique is based on Monte Carlo evaluation of the enormous sums needed in calculating moments. Because of unitarity, we may evaluate the moments (traces) in any basis including one, such as the shell model, in which the Hamiltonian is not diagonal. These sums would involve a number of terms equal to the number of levels in the basis, and the calculation of each term is lengthy. Thus, exact evaluation of the sum directly would be extremely difficult, even with modern computers.

Such sums can be evaluated[10–12] by choosing terms at random and then correcting the sum for the fraction of terms not sampled. Direct application of

this technique would result in slow convergence. The efficiency of the sampling technique can be improved enormously by biasing[10] the sample appropriately. Each term in the sum corresponds to the application of H (or H^n) to a vector consisting of a randomly chosen group of Slater determinants. If we first adjust the amplitude of the Slater determinants so as to make the vector have the correct value of $\langle H \rangle$ (the proper value of the first power of the Hamiltonian), the convergence of the higher moments will be greatly improved. For example, the dispersion of the value of $\langle H^4 \rangle$ calculated with a given set of Slater determinants is reduced by a factor of about 30 when the amplitudes of the Slater determinants are determined so as to have the correct first moment of the Hamiltonian instead of being assigned at random. This results in a saving of a factor of about $(30)^2$ or 900 in computer time. Such a reduction clearly could mean the difference between a calculation which is feasible and one which is virtually impossible.

Note that this same technique could also be used to calculate higher moments ($\langle H^n \rangle$, $n \geq 5$), as was done in Ref. 10. The fact that the rate of convergence decreases rapidly with moment order means that converged results will be difficult to obtain for $n \geq 5$. The present analysis is based on use of moments of order below five. Finally, the present method allows the calculation of the separate negative and positive parity moments (and hence level densities) as well as the correction for contributions to the moments of spurious states with center-of-mass excitations. Both of these features were utilized in this work.

II. CALCULATIONS

The level density and spin cutoff parameter were calculated for ^{28}Si using a basis consisting of $d_{5/2}$, $s_{1/2}$, $d_{3/2}$, and $f_{7/2}$ orbitals. The two-body matrix elements were calculated with the potential of Petrovich *et al.*[13] Single particle energies were inferred from the energies of the states of appropriate spin and parity in the $A=17$ system with one exception. Energies of -4.15, -3.29, and 0.93 MeV were assigned to the $d_{5/2}$, $s_{1/2}$, and $d_{3/2}$ states, respectively, on the basis of the level positions in the $A=17$ system. A value of $+1.55$ MeV would be assigned to the $f_{7/2}$ state on the basis of the energy of the lowest $f_{7/2}$ state in ^{17}O. This state, however, has less than 20% of the strength in the (d,p) reaction which would be expected if this is a single particle state; this fact, together with the lack of a shell gap between this energy and that of the $d_{3/2}$ state, suggests that the centroid of $f_{7/2}$ strength may lie nearer to the next $\frac{7}{2}$ state, which is 2 MeV higher in excitation. Thus, the $f_{7/2}$ state was placed at 3.55 MeV.

Level density and spin cutoff parameter calculations were made for ^{28}Si using two approaches: a single expansion for the entire basis and two alternative configuration expansions. In the first case, the moments $\langle H \rangle$, $\langle H^2 \rangle$, $\langle H^3 \rangle$, and $\langle H^4 \rangle$ as well as $\langle HJ_z^2 \rangle$, $\langle H^2J_z^2 \rangle$, $\langle H^3J_z^2 \rangle$, and $\langle H^4J_z^2 \rangle$ were calculated for all states in the basis. This enables us to express the state density and spin cutoff parameter in terms of a Hermite polynomial expansion of order four. The configuration expansions result in state densities and spin cutoff parameters which are the sums of a number of such four term Hermite polynomial expansions. Since the constituent expansions are expressed relative to their centroids, the resultant sum is not necessarily expressible in terms of one four-term expansion. Thus, the configuration expansion in principle carries some information about higher moments of the entire distribution.

The first of the two configuration expansions utilized a partition of the space into two parts: the *sd* shell and the $f_{7/2}$ shell. Each configuration can be labeled with parameters (n_1,n_2) which express the number of particles in the *sd* orbitals and the $f_{7/2}$ orbital, respectively. Moments were calculated for configurations from (12,0) to (8,4) using this partition. The average energy increases as particles are promoted to the $f_{7/2}$ orbital. Higher configurations were ignored because the last configuration included caused no change in the level density at energies below 25 MeV. Since the primary focus of this paper is on the level density in the low energy region, we did not invest the computer time required to calculate the parameters for configurations (7,5) through (0,12).

The dimensionalities and $\langle J_z^2 \rangle$ for some typical configurations are listed in Table I. Note that the partitioning results in breaking down the information on level densities and spin cutoff parameters to rather small parts of the entire space.

A second partitioning leading to three groups was also utilized. The $d_{5/2}$ and $s_{1/2}$ orbitals comprised the first group, with the $d_{3/2}$ and $f_{7/2}$ orbitals as the second and third groups, respectively. With this partitioning a total of 81 configurations are produced, ranging from (12,0,0) to (0,0,12). For this expansion the lowest nine positive parity groups and the lowest seven negative parity groups were calculated. Note that both configuration expansions allow a separate calculation of the level density for even parity levels from that for odd parity, based on whether an even or odd number of particles is in the $f_{7/2}$ orbital.

A further advantage of the configuration expan-

TABLE I. Configuration dimensionalities.

	Approximate energy span (MeV)	Number of states	$\langle J_z^2 \rangle$
Whole Basis	150	1.502×10^9	28.74
Two-component configurations			
(12,0)	70	8.54×10^5	12.5
(11,1)	80	1.17×10^7	17.6
(10,2)	85	6.58×10^7	21.8
(9,3)	90	1.98×10^8	25.1
(8,4)	90	3.43×10^8	28.2
(7,5)	90	4.01×10^8	29.6
(6,6)	90	2.8×10^8	32.7
Three-component configurations			
(12,0,0)	50	7.68×10^2	7.71
(11,1,0)	65	1.25×10^4	9.93
(10,2,0)	65	7.369×10^4	11.6
(9,3,0)	70	2.007×10^5	12.0
(11,0,1)	70	2.509×10^4	13.9
(11,1,1)	70	3.261×10^5	15.9

sion is that it allows us to correct for center-of-mass energy shifts. Our calculations, precisely as conventional shell model calculations, do not impose the condition that the center-of-mass of the nucleus be in its ground state. Thus, "spurious" states, corresponding to a real state at lower energy coupled to a center-of-mass excitation, are introduced into the calculations. The present configuration expansion calculations have been corrected for this problem by evaluating the expectation value of the center-of-mass Hamiltonian and making the appropriate correction to the average energy of each configuration. This correction cannot be made for the single expansion calculation, because the various components of the spectrum require different shifts.

Table I lists some of the parameters characterizing the various level density expansions. Note the enormous span of energy corresponding to the whole basis expansion. Although the use of a finite basis causes the level density to decrease with energy beyond the centroid energy, it can be argued[14] that this behavior is physically more reasonable than that of the traditional Fermi gas which has a level density which increases indefinitely. The fact that nuclei have finite binding energies makes it unreasonable to consider "levels" at very high energies as legitimate excited states of the nucleus. It has also been shown,[15] however, that over a range of energy of more than 10 MeV the Gaussian form and a traditional Fermi gas form are virtually indistinguishable. Indeed, if the Fermi gas level density is evaluated in a finite basis, the level density is found to be Gaussian.[16]

As previously indicated, three separate spectral distribution calculations were made: one utilizing the whole basis (SD1) and two utilizing either two- (SD2) or three-group configurations (SD3). Two fundamental problems complicate the use of the SD1 results. First, the use of the third and fourth moments of the Hamiltonian can produce negative values for the level density, because the third and fourth Hermite polynomials are negative in some regions. For SD1, the negative overshoot in the low energy region was as large as 50 levels per MeV. This is not extraordinarily large in a basis of size 10^9, but it does suggest that mathematical convergence problems will compromise the validity of this calculation for energies where the density of levels is comparable to this value. The corresponding problems for SD2 and SD3 are much reduced. Although negative overshoots occur for these as well, they are below 2 levels per MeV for both expansions. This is the result for four moment expansions; both SD2 and SD3 have excursions as large as 10 levels per MeV when a three moment expansion is used.

A second problem with the SD1 calculation is the presence in the traces of contributions from spurious states. These states correspond to real states at lower energy with the center-of-mass of the nucleus in an excited state. Thus, all moments of the Hamiltonian as well as the number of states must be corrected for these states. For the configuration expansions, the number of spurious states was calculated first and then the expectation value of the center-of-mass Hamiltonian was evaluated for the random vector. Comparison of the value of the center-of-mass energy with the value with the center-of-mass in its ground state directly gives the value by which the centroid energy of each configuration must be shifted. Higher moment corrections were made by assuming that the spurious states had the same distribution relative to their centroid as do the (12,0), (SD2) or (12,0,0) (SD3) configurations, both of which are free of spurious states. Such a correction procedure obviously cannot be applied to SD1.

For this reason, the only use made of SD1 was to check one type of convergence of the configuration expansions. Comparison of SD2 and SD3, before corrections for spurious states, with SD1 indicated that the number of configurations used was adequate to describe the level density to about 25 MeV. If values of the level density at higher energies are desired, more configurations must be included in the

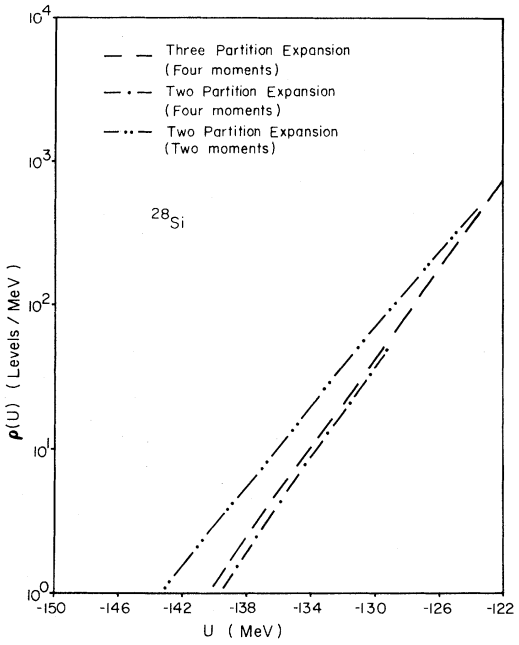

FIG. 1. Level densities calculated for ^{28}Si using two partitions for the configurations with two and four moments of the Hamiltonian and three partitions for the configurations with four moments for the Hamiltonian. The energy scale is based on a zero of energy for the ground state of ^{16}O.

expansion.

Figure 1 shows the level densities resulting from the SD2 and SD3 expansions. The energy scale is based on zero for the energy of the ^{16}O ground state; in Sec. III we will describe the energy scale normalization used for comparison with measured level densities. Very similar results are obtained with SD2 and SD3, with the largest discrepancies ($\sim$20%) found in the tail region. In addition to four moment calculations with SD2 and SD3, we also show a two-moment expansion for SD2. This differs from the four moment calculations by as much as a factor of 2 in the tail region, but agrees to within 10% with the four moment calculations 25 MeV above the end point.

It is difficult to ensure that the expansion has converged mathematically without having the higher moments available. As mentioned previously, fifth and sixth moments could in principle be calculated with the random vector technique. Unfortunately, achieving convergence for $\langle H^4 \rangle$ requires roughly ten times as long as for $\langle H^3 \rangle$, which in turn needs about three times as long as for $\langle H^2 \rangle$. The total computer time needed for each configuration expansion was about an hour and a half, so obtaining convergence on $\langle H^5 \rangle$ is effectively out of the question with the present technique using current computers. These times are based on the CDC 7600 computers at Los Alamos and Livermore, which were used for the calculations.

Two arguments, in addition to the smallness of the undershoot, could be advanced in support of the conclusion that these expansions have converged. First, French[17] performed an analysis of the convergence of the moment method in a situation where exact eigenvalues are known. Thus, not only can moments to arbitrary order be evaluated, but the exact "answer" is known. He found significant improvement in the fit when the third and fourth moments were included, but much less effect from the fifth and sixth moment. Moreover, good convergence with the four moment expansion was seen. A second reason is that the two separate four moment expansions lead to similar results. This is an argument not only for the adequacy of a four moment expansion, but also for statistical convergence of the two separate calculations. The two configuration distributions are not as similar when truncated after two or three moments.

III. COMPARISON WITH DATA AND OTHER CALCULATIONS

The calculations were compared with values of the level density and spin cutoff parameter inferred from the tabulation of low-lying levels presented in Endt and van der Leun.[18] In addition, the level density points inferred by Beckerman[19] from ^{27}Al(p,p) and ^{27}Al(p,γ) measurements (as tabulated by Endt and van der Leun as well as Ericson fluctuation measurements of the level density listed by Beckerman but based on the measurements of Singh *et al.*,[20] Shaw, Katsanos, and Vanderbosch,[21] and Eberhard and Mayer-Böricke,[22]) were also included. These measurements provide information on the level density at excitation energies up to 28 MeV. Finally, level density parameters for ^{28}Si have been determined by Roeders *et al.*[23] from measurements of Ericson fluctuations by Put *et al.*,[24] Allardyce *et al.*,[25] and Eberhard.[26] The tabulation of Endt and van der Leun[18] is probably complete below 10 MeV, but may only represent lower limits beyond this point. Beckerman,[19] for example, has estimated that a correction of about 35% is needed at 12.5 MeV based on the observed spin distribution. The fluctuation measurements lead to level densities which are uncertain to about a factor of 2. Since these values depend on the reliability of the level density parameters assumed for the residual nuclei at lower energies, it is possible that a reanalysis of these data using current information about the residual level densities might yield level densities with reduced error bars. Spin cutoff parameters for ^{28}Si

have been inferred by Grimes *et al.*[8]; these values and those calculated from discrete levels tabulated by Endt and van der Leun were used in the spin cutoff parameter comparison.

To provide a basis for evaluating the quality of agreement achieved by the spectral distribution calculations, we have also used the microscopic Fermi gas level density code described in Ref. 27 to calculate level densities and spin cutoff factors for ^{28}Si. Finally, we also compare our results with the level density calculated with the parameters proposed by Beckerman.[19]

Figure 2 presents the tabulated level density results in comparison to the microscopic Fermi gas calculations based on the single particle energy sets proposed by Nilsson,[28] Seeger and Perisho,[29] and Seeger and Howard.[30] Clearly, the Beckerman parameters provide a better representation of the data than any of the microscopic Fermi gas calculations; this is not surprising since it is a fit to the data, and the other curves are calculations. The Seeger-Perisho levels are obviously the best of the three calculated level sets, with the Seeger-Howard

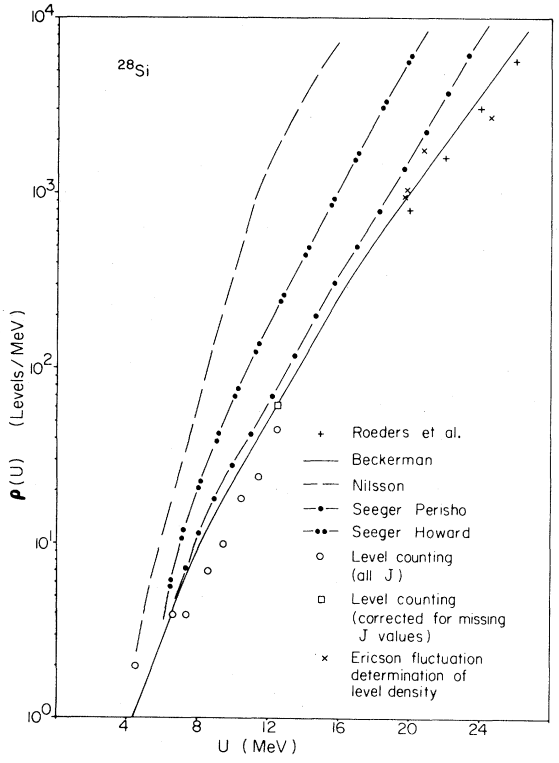

FIG. 2. Comparison of Fermi gas level densities calculated with single particle schemes proposed by Nilson (Ref. 28), Seeger and Perisho (Ref. 29), and Seeger and Howard (Ref. 30) with experimental values. Also shown is a Fermi gas fit due to Beckerman. Data are from level counting and Ericson fluctuation measurements.

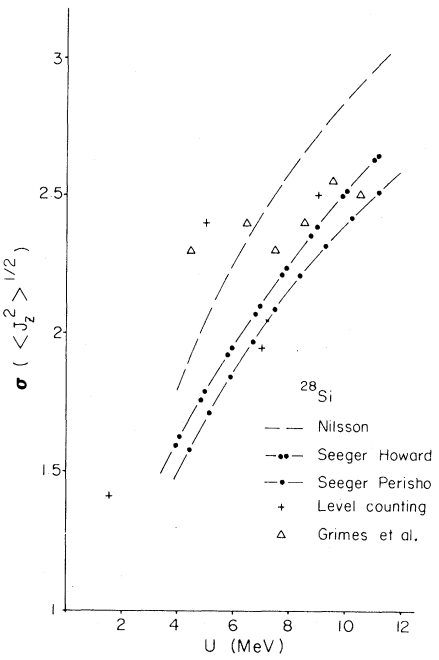

FIG. 3. Comparison of calculated and experimental values for the spin cutoff parameter of ^{28}Si. The + values are inferred from level counting, while the Δ values result from an analysis of (α,n) angular distribution measurements by Grimes *et al.* The Fermi gas calculations utilize the level schemes of Nilsson, Seeger, and Perisho, and of Seeger and Howard.

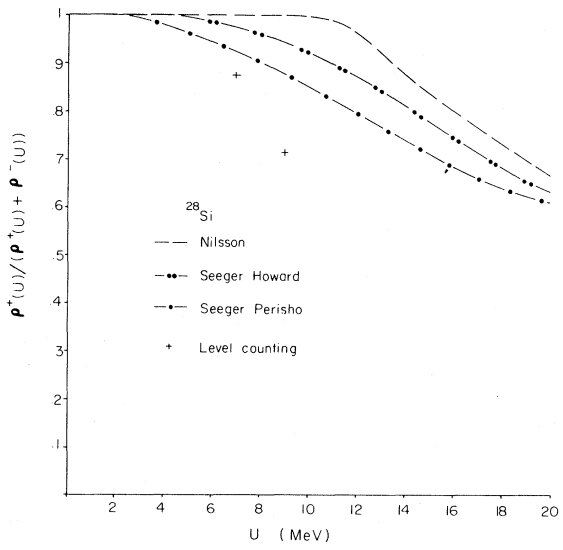

FIG. 4. Comparison of calculated and measured level density parity ratios. The three calculations are based on a Fermi gas model with input described in the caption of Fig. 2. Data points are from known low-lying levels.

and the Nilsson levels providing much worse fits. The fact that discrepancies are observed even for the fit suggests that the errors on some of the experimental points must be substantial if we believe that the level density is smooth at high excitations; Beckerman estimates the uncertainty in the high energy values to be of the order of 50%. The irregularity at low excitation energy is presumably due to the statistics associated with low values of the level density.

In Figs. 3 and 4 we show the calculated spin cutoff parameters and parity ratios for these level density calculations. Relative to the measured points, all three calculations of the spin cutoff parameter have too rapid an energy dependence, although this finding is not as definitive as would be the case if data were available over a wider energy range. Evaluation of the positive parity-negative parity ratio is difficult because of the lack of experimental data. From the trend of the points obtained from level counting and from the fraction of levels near 13 MeV whose parities are known, it appears that the parity ratio should be about 0.6 at this energy. This is lower than any of the Fermi gas calculations. Correcting this energy dependence within the framework of a Fermi gas calculation would require raising the energy of the $p_{1/2}$ and $p_{3/2}$ hole states or reducing the energy of the $f_{7/2}$ level. Either of these changes would increase the level density, possibly destroying the good agreement with experiment found for the Seeger-Perisho level set. This discrepancy in the parity ratio is opposite to that found in heavy nuclei[15] (especially deformed nuclei), where it was found that the Fermi gas level density predicted a too-rapid approach to the asymptotic value 0.5.

The spectral distribution calculations are based on an energy scale which has its zero at the ^{16}O core energy. Thus, the calculated energies are all less than zero, and this scale must be renormalized to give the parameter values in terms of excitation energy relative to the ground state.

Ratcliff[2] originally suggested determining the energy zero (ground state) by integrating the distribution to an energy where the integral was 0.5; this was taken to be the ground state. Because the ground state of ^{28}Si is well separated from the point where the level density begins to rise smoothly, this procedure would have given poor results here. Instead, the normalization point was chosen to be 8 MeV, i.e., the first 12 levels. This does not guarantee that the level density is correct at 8 MeV, but only that the integral is correct. Clearly, if the predicted slope is incorrect, any choice of normalization point will yield poor results, while if the slope

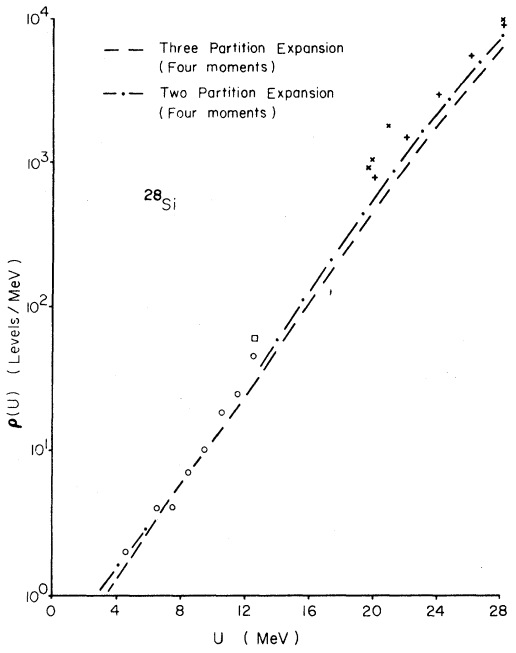

FIG. 5. Comparison of two spectral distribution calculations of the level density for ^{28}Si with experiment. The two curves represent two- and three-partition configuration expansions, each carried out to four moments. Data points are explained in the caption to Fig. 2.

is approximately correct, the quality of fit will be good regardless of the normalization point. In cases where the slope is somewhat incorrect, this will make agreement at 20 MeV poor, while the normalization to the integral over a wider range of energy would balance the discrepancy between high and low energy regions. In particular, the two four moment expansions agree to within 5% over most of the range of energy, except only in the tail of the distribution, where the discrepancy is 15%. However, since the normalization is made in the low energy region, the energy shifts are different for the two, leaving them in agreement at low energies but with a 15% discrepancy at 20 MeV. This difference is, of course, smaller than the uncertainty in either calculation or data in this region. A somewhat better fit over the entire energy region can be obtained by normalizing the integral over the first 12 MeV, but this worsens the fit below 8 MeV somewhat.

Figure 5 shows the comparison between the four moment fit from SD2 and SD3 and the data. The argeement is comparable to that obtained with Beckerman's fitted parameters. Both SD2 and SD3 fit the lower energy region better than Beckerman but are not as good as the latter parametrization at higher energies. It is interesting that the present results agree well in shape with Beckerman, differing

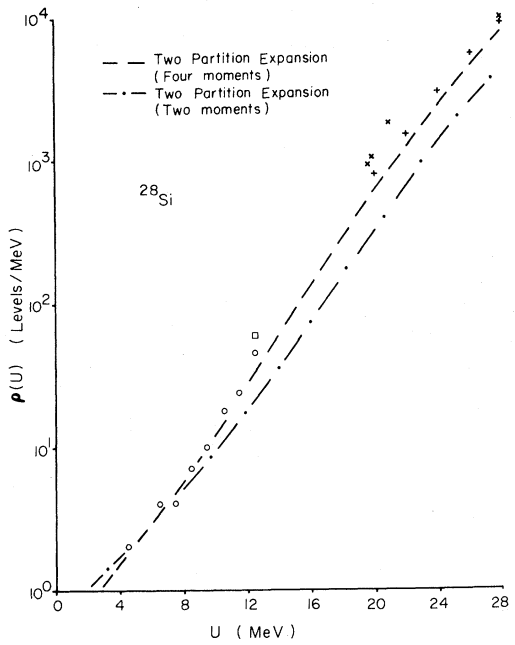

FIG. 6. Same as Fig. 5, except that the calculated curves are for two- and four-moment expansions for the two-partition configuration calculations. Data points are explained in the caption to Fig. 2.

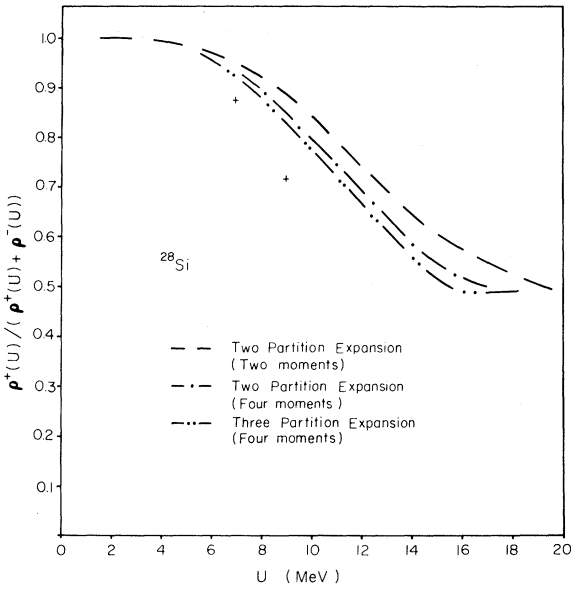

FIG. 8. Comparison of calculated and experimental values for the parity ratio for ^{28}Si. The calculated values are for the expansions listed in the caption to Fig. 7. Data points are obtained as described in Fig. 4.

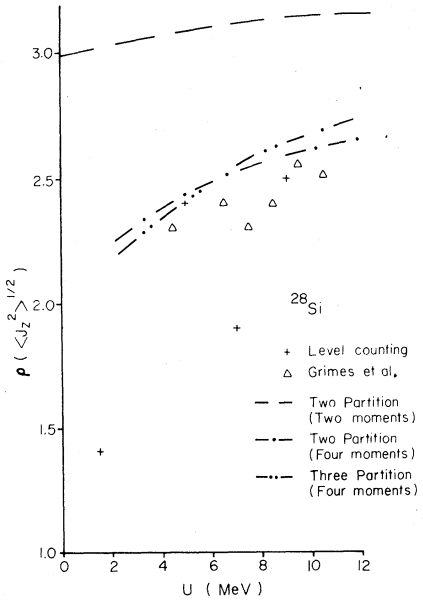

FIG. 7. Comparison of the calculated and experimental values of the spin cutoff parameter for ^{28}Si. The calculated values are for a four-moment three-partition expansion and both a two- and four-moment two-partition expansion.

in the ratio of $\rho(20)/\rho(8)$ by only 20%. Both our calculation and Beckerman's fit have difficulty fitting the points below 12 and those near 20 MeV simultaneously; this does not necessarily mean, of course, that some of the data are wrong. Use of configuration expansions can modify the shape slightly from the Fermi gas or Gaussian form, and perhaps another choice of input parameters might have produced a slope in the calculation more consistent with the data.

Figure 6 shows the level density results for SD2 when the expansion is truncated at two and four moments, respectively. The agreement is substantially poorer for either a three or two moment expansion than for a four moment expansion.

Figure 7 presents the SD2 and SD3 results for the spin cutoff parameter and compares the predictions with data. Also shown is the SD2 result with two moments. A tendency to overestimate the experimental values is barely evident, but the energy dependence appears to be in better agreement with data than that predicted by the Fermi gas calculations. The inclusion of higher moments again can be seen to improve the agreement with data.

Calculated parity ratios are displayed in Fig. 8.

TABLE II. Level density parameters for ^{28}Si.

Fitted values		
Beckerman[a]	$a=2.168$	$c=0.001\,87$
Roeders *et al.*[b]	$a=3.11$	$\delta=3.5$
Spectral distribution calculations		
Wong and Lougheed[a]	$a=1.3$	
Present results[b]	$a=2.87$	$\delta=2.57$
Fermi gas calculations		
Nilsson levels[b]	$a=5.7$	$\delta=3.1$
Seeger-Perisho levels[b]	$a=2.45$	$\delta=2.4$
Seeger-Howard levels[b]	$a=3.55$	$\delta=2.8$

[a]Based on the form $ce^{2\sqrt{au}}$.
[b]Based on the form

$$\frac{e^{2\sqrt{a(u-\delta)}}}{24\sqrt{2}\sigma a^{1/4}(u-\delta)^{5/4}}.$$

As was the case for the Fermi gas calculations, the comparison with data is handicapped by the limited information available. It appears as if the parity ratio predicted by these calculations goes somewhat too slowly to the asymptotic value, but the agreement is better than for the Fermi gas calculations.

Previous spectral distribution calculations for ^{28}Si have been made by Ayik and Ginocchio[4] and Wong and Lougheed.[7] Neither of these calculations is precisely comparable to ours, in that both used the Kahanna-Lee-Scott (KLS) interaction[31] and both truncated the expansion at two moments. Given these differences, the fact that the calculated level density parameters differ somewhat is not surprising. We note that the claim made by Wong and Lougheed[7] that their results imply a very different Fermi gas parameter than is conventionally used is somewhat misleading. These authors, as does Beckerman, use an unconventional form of the Fermi gas form. In Table II, we tabulate level density parameters obtained from fits to our results, both from spectral distribution calculations and Fermi gas calculations, and those due to Beckerman[19] and Wong and Lougheed.[7] It can be seen that the parameters of Refs. 7 and 19 are considerably lower than those derived using the conventional formula. Our calculated level density has a slightly less rapid energy dependence than that of Beckerman, but the difference is not large. Moreover, the differences found in Ref. 7 may be due in part to the absence of third and fourth moment corrections.

Verbaarschot, Timmer, and Brussard[32] have calculated integrals of positive parity versus negative parity levels for a number of nuclei including ^{28}Si. They used a different interaction and only two moments, but they did make a correction for spurious states. Their results are in reasonable agreement with ours given the different input and procedures.

IV. DISCUSSION

Although the present calculations show good agreement with data, there are indications that the level density energy dependence is slightly too slow, and that the parity ratio approaches 0.5 too slowly. One possible explanation for both of these problems is the omission of the $p_{3/2}$ and $p_{1/2}$ orbitals from the calculations. The use of a large basis required us to renormalize the Petrovich-McManus-Madsen (PMM) interaction[13] to 65% of its original strength. This factor was determined by adjusting the strength of the interaction to match calculation to experiment. The length of the calculations made it impossible to repeat them often enough to make any fine adjustments in either this normalization factor or the $f_{7/2}$ single particle energy. Assessing the possible importance of p orbits will be difficult until all other parameters are optimized, and this is very difficult at present.

An obvious need is for techniques to speed up the calculations. We are investigating the effect of correcting our random vectors for both $\langle H\rangle$ and $\langle H^2\rangle$. If we first determine $\langle H^2\rangle$ from an ensemble of vectors constructed to have the correct value for $\langle H\rangle$, then a second smaller number could be made, which we could correct for both $\langle H\rangle$ and $\langle H^2\rangle$. This might allow fewer calculations of $\langle H^3\rangle$ and $\langle H^4\rangle$ to yield a given accuracy for these moments.

A second possibility would be to utilize the systematics of the Hermite polynomial expansion coefficients. As has already been pointed out,[10] these parameters have a very systematic dependence on the centroid energy of the configuration. This dependence could possibly be exploited to interpolate and obtain parameters of many configurations from direct evaluation of those from a few. Wong and Chang[33] have also commented on this systematic dependence but have found that it varies somewhat as a function of the matrix elements used.

The present results suggest that spectral distribution techniques can yield calculated level densities which agree fairly well with data over an energy range of about 20 MeV. These calculations also provide a fit to the spin cutoff parameter and the positive parity-negative parity ratio where data are available. The fact that the basis used is large makes possible a level density calculation to fairly high excitation energies; the large basis also causes a prob-

lem, in that the convergence at very low energies is not good enough to base the energy scale on the ground state position. For a nucleus for which a number of low-lying levels are known, the normalization can be based on these levels. Supplementing the moment calculations with level densities at low energies inferred from a Lanczos type of calculation[34] would improve the reliability of the energy scale determination. Much additional work is needed, both to speed up the calculations and to establish sets of appropriate input parameters.

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*Permanent address: Ohio University, Athens, OH 45701.

†Present and permanent address: Institut für Kernphysik and Radiumforschung, Boltzmanngasse 3, 1090 Vienna, Austria.

[1]See, for example, the papers presented in *Theory and Application of Moment Methods in Many-Fermion Systems,* edited by B. J. Dalton, S. M. Grimes, J. P. Vary, and S. A. Williams (Plenum, New York, 1980).

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