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Unified theory of meson and nucleon scattering by nuclei

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In the meson-nucleon model of nuclei, the decomposition of the meson and nucleon field operators into internal and external parts provides a way of treating orthogonality effects in the scattering of hadrons by a nucleus. Under this decomposition, the Hamiltonian of the system splits into an internal part and terms that couple external modes. The internal part of the Hamiltonian is the Hamiltonian of the meson-nucleon shell model, and its eigenstates are approximations to the bound nuclear states. The rest of the Hamiltonian describes the external quanta of the fields and their interactions with the internal states. The internal mode functions are chosen so as to minimize the coupling of the external modes to the low-lying internal states. The Green's functions for the external fields and the diagrammatic representation of the perturbation series are presented and used to describe the treatment of hadron scattering in the one-external-hadron sector.

[NUCLEAR STRUCTURE π-nucleus and nucleon-nucleus scattering theory.]

I. INTRODUCTION

There are various models of the nucleus that can be used as bases for computing nuclear properties. In the nucleon-nucleon model of the nucleus, the nucleus is regarded as consisting of nucleons interacting via two-body forces or potentials; this is the oldest and most thoroughly explored of the nuclear models. The more recent meson-nucleon model treats the nucleus as composed of nucleons and meson field, with the fundamental interactions being the virtual emission or absorption of a meson by a nucleon. A third model in which the nucleus is treated as consisting of quarks and nonlinear meson field is still in its early stages. Previously, the meson-nucleon model was used to treat bound states of nuclei. In this paper it is shown that the meson-nucleon framework is useful also in treating scattering problems and that it provides a unified picture of hadron scattering by nuclei.

In the meson-nucleon model the treatment of meson fields that can have an expectation value in the ground state of the nucleus is straightforward. In that case the ground-state expectation value of the meson field can be treated as a classical field; this meson mean field then acts as a potential in which the nucleons move. In order to treat fields like the pion field that do not have a ground-state expectation value, the mean-field technique has recently been extended[1,2] to a meson-nucleon shell model (MNSM) in which the nucleon field operator is expanded, as is usual in the nuclear shell model (NSM), in terms of a set of orthonormal single-particle wave functions and, in addition, the meson field operator is expanded in terms of a set of orthonormal mode functions. The mean-field method amounts to keeping just the single mode of the meson field that corresponds to its ground-state expectation value and making some simplifying assumptions about the nucleon part of the state vector. In the case where there is a ground-state expectation of the meson field, the MNSM provides an interesting extension of the mean-field procedure; for the case of vanishing ground-state expectation it is the only nonperturbative treatment available at present. As in all shell models, the state vectors are not eigenstates of the total momentum operator.

In the MNSM the set of single-particle functions and meson mode functions is restricted, as in the NSM, to a finite single-particle shell-model subspace (SPSMS). Then a field operator, for example, the annihilation operator $a(\vec{k})$ for a meson of momentum $\vec{k}$, is resolved in the form

$$\begin{aligned} a(\vec{k}) &= \sum A_i\phi_i(\vec{k})+a_\perp(\vec{k}) \\ &= a_{\parallel}(\vec{k})+a_\perp(\vec{k}) , \\ a_{\parallel}(\vec{k}) &= \sum A_i\phi_i(\vec{k}) , \end{aligned} \tag{1}$$

where the sum over i runs over the finite set of orthonormal meson SPSMS functions $\phi_i(\vec{k})$, and the "external" meson operator $a_\perp(\vec{k})$ is orthogonal to the meson SPSMS functions:

$$\int \phi_i^*(\vec{k}) a_\perp(\vec{k}) d\vec{k} = 0 , \quad (2)$$

for all ϕ_i in the meson SPSMS. The SPSMS functions ϕ_i and operators A_i will be called internal meson functions and internal meson annihilation operators, respectively; $a_{||}(\vec{k})$ will be called the internal meson field operator. Similarly, the nucleon field operator is the sum of an internal and an external part:

$$\begin{aligned} &\widetilde{\Psi}(\vec{p}) = \widetilde{\Psi}_{||}(\vec{p}) + \widetilde{\Psi}_\perp(\vec{p}) , \\ &\widetilde{\Psi}_{||}(\vec{p}) = \sum B_j \widetilde{f}_j(\vec{p}) , \\ &\int \widetilde{f}_j^\dagger(\vec{p}) \widetilde{\Psi}_\perp(\vec{p}) d\vec{p} = 0 , \end{aligned} \quad (3)$$

where the functions $\widetilde{f}_j$ are in the nucleon SPSMS. If the meson field $a_\nu(\vec{k})$ has a symmetry index ν, such as a component of isospin, then the resolution of Eq. (1) takes the form

$$a_\nu(\vec{k}) = \sum A_{i\nu} \phi_i(\vec{k}) + a_{\nu\perp}(\vec{k}) , \quad (4)$$

where it is important that the functions ϕ_i do not depend on the degeneracy index ν.

The overall shell-model subspace (SMS) is generated by the SPSMS creation operators $A_i^\dagger$ and $B_j^\dagger$ acting on the particle vacuum. In the usual NSM of the nucleon-nucleon model, the SMS is finite; since the operators $A_i^\dagger$ are Bose creation operators, the SMS is infinite in the meson-nucleon model. The shell-model idea is that strong-coupling effects can be treated within the SMS by diagonalizing the matrix of the Hamiltonian over all the states in the SMS. It is assumed that the effects of the external parts of the field operators can be treated by perturbation theory.

The treatment of the internal or $||$ parts of the field operators has been considered in Refs. 1 and 2. The present paper aims to establish a framework for treating the $\perp$ or external parts of the operators; in particular, scattering states start out as external field creation operators acting on SMS states. If the external parts of the fields can really be treated perturbatively, then it makes sense to expand state vectors in the order of the total number of external hadron creation operators; that is, the zeroth term has no external hadrons (0EH) and consists of components within the SMS, each component of the first term is a 1EH state consisting of a single external hadron creation operator acting on a SMS state, and each component of the nth term is an nEH state consisting of n external hadron creation operators acting on a SMS state. The nEH sector is defined as consisting of all state vectors with up to n external hadrons.

The 1EH sector is particularly interesting since it is the simplest subspace that provides a nontrivial unified unitary picture of hadron scattering by the nucleus. For the case of a single internal mode of a meson field interacting with a static source, the 1EH sector has already been studied.[3] The effects that arise from the requirement that the external field be orthogonal to the internal field functions are especially interesting in that case, and it was suggested in Ref. 3 that the smallness of the P_{11} pion-nucleon phase shift is such an orthogonality effect. Similar orthogonality effects are of current interest in the five- and six-nucleon systems.[4] In deuteron stripping reactions, it has long been known that the simple direct reaction theory of stripping[5] gives too large a cross section by a considerable factor and that distorted waves must be used to obtain agreement with experimental results. It will be argued below that the simple direct matrix element is in fact zero in lowest order, so that the dominant influence of wave distortion has a rather simple explanation.

The splitting of the meson field into internal and external parts also provides an understanding of the double-counting problem in meson interactions. This problem arises because the same mesons that are scattered and virtually scattered by nucleons are also responsible for at least some of the nucleon-nucleon interactions that are used to bind the nucleons in the nucleus. In the present terms, the binding is due largely to the internal meson field, while it is the external meson field that is scattered and virtually scattered in a reaction. The resolution of the double-counting problem lies in treating these two meson fields consistently while maintaining their mutual orthogonality or commutativity.

All of the above statements about the internal and external fields are only meaningful within a particular shell model that specifies a particular set of internal and external fields; that is, there is no obvious way of defining internal and external fields in terms of the actual wave functions of the nucleus. The question of just where to establish the boundary between the internal and external fields is open.

Section II of this paper describes the Hamiltonian of the meson-nucleon model and how it looks when the substitutions of Eqs. (1)–(4) are made. Section III considers the particular terms that are important for choosing the form of the internal mode functions for the mesons and the single-particle functions for the nucleons. A suitable choice of the meson functions was given in Ref. 2; the analogous choice of functions for the nucleons is discussed in Sec. III. In Sec. IV the correspondence between terms in the Hamiltonian and diagrams is described, and the Green's functions for the external hadron fields are exhibited. The 1EH sector is worked out in Sec. V in terms of the diagrams of Sec. IV. Section VI has some further remarks about applications of the technique of splitting the field operators into internal and external parts. Section VII summarizes the work.

II. PIECES OF H

As in Refs. 1 and 2, the Hamiltonian describes a system of nucleons and mesons with Yukawa interaction of the fields. The formulation is noncovariant. The meson field $\Phi_\nu(\vec{x})$, with associated annihilation operator $a_\nu(\vec{k})$, is assumed to be invariant under space rotations; in covariant terminology it can be a scalar or pseudoscalar field or the zeroth component of a vector or pseudovector field; the index ν is the isospin or other nonrotational symmetry index of the meson field and will be understood to be subject to the usual summation convention in the following. The Hamiltonian is

$$\begin{aligned} H &= T_F + T_B + H_I \ , \\ T_F &= \int \widetilde{\Psi}^\dagger(\vec{p})t(p)\widetilde{\Psi}(\vec{p})d\vec{p} \ , \\ T_B &= \int \omega(k)a_\nu^\dagger(\vec{k})a_\nu(\vec{k})d\vec{k} \ , \\ H_I &= -\int [a_\nu^\dagger(\vec{k})+a_\nu(-\vec{k})]Y(k)\widetilde{\Psi}^\dagger(\vec{p}) \\ &\quad \times W_\nu\left[\vec{k},\frac{\vec{p}+\vec{q}}{2}\right]\widetilde{\Psi}(\vec{q})\delta(\vec{k}+\vec{p}-\vec{q}) \\ &\quad \times d\vec{p}\,d\vec{q}\,d\vec{k} \ , \end{aligned} \tag{5}$$

where $t(p)$ and $\omega(k)$ are the energies of a free nucleon of momentum $\vec{p}$ and a free meson of momentum $\vec{k}$, respectively, and $Y(k)$ and $W_\nu(\vec{k},\vec{K})$ are the form factors that characterize the particular Yukawa interaction; W_ν represents the nucleon current that interacts with the field Φ_ν, while Y is the factor that comes from the relation between $\Phi_\nu(\vec{x})$ and $a_\nu(\vec{k})$; representative forms for W and Y are given in Ref. 6. It is assumed that

$$W_\nu^\dagger\left[\vec{k},\frac{\vec{p}+\vec{q}}{2}\right]=W_\nu\left[-\vec{k},\frac{\vec{p}+\vec{q}}{2}\right] . \tag{6}$$

In the case of vector or pseudovector meson fields there is an additional nucleon-nucleon interaction term in the Hamiltonian; this term has been discussed in Ref. 6 and will be omitted here.

Clearly, H is a functional of the field operators $\widetilde{\Psi}^\dagger(\vec{p})$, $\widetilde{\Psi}(\vec{p})$, $a_\nu^\dagger(\vec{k})$, and $a_\nu(\vec{k})$. Let $H_{||}$ denote the same functional of the corresponding $||$ operators; $H_{||}$ is just the MNSM Hamiltonian of Refs. 1 and 2, and the SMS is just the $||$ subspace generated by the $||$ operators acting on the vacuum. The Hamiltonian $H_{||}$ has discrete eigenvalues ϵ_α with corresponding eigenstates $|\alpha\rangle$,

$$H_{||}\,|\alpha\rangle=\epsilon_\alpha\,|\alpha\rangle \ . \tag{7}$$

When the resolution of the field operators into $||$ and $\perp$ parts of Eqs. (1) and (3) is substituted into the Hamiltonian, it is clear that the Hamiltonian has terms with varying numbers of $\perp$ annihilation and creation operators; let $H_{n,m}$ have n $\perp$ creation operators and m $\perp$ annihilation operators. In general, $H_{n,m}^\dagger=H_{m,n}$, and with the Hamiltonian of Eqs. (5) it follows that $m+n\leq 3$. Clearly, $H_{0,0}$ is just $H_{||}$. Similarly $T_{F,n,m}$, $T_{B,n,m}$, and $H_{I,n,m}$ can be defined corresponding to the parts of the Hamiltonian defined in Eq. (5).

The "unperturbed" Hamiltonian H_U corresponding to the division of the field operators into internal and external parts is

$$\begin{aligned} H_U &= H_{||}+T_{F,1,1}+T_{B,1,1} \ , \\ T_{F,1,1} &= \int \widetilde{\Psi}_\perp^\dagger(\vec{p})t(p)\widetilde{\Psi}_\perp(\vec{p})d\vec{p} \ , \\ T_{B,1,1} &= \int \omega(k)a_{\nu\perp}^\dagger(\vec{k})a_{\nu\perp}(\vec{k})d\vec{k} \ . \end{aligned} \tag{8}$$

The 0EH states are the eigenstates $|\alpha\rangle$ of $H_{||}$; these are also eigenstates of H_U with the same eigenvalue ϵ_α. The 1EH states are the one-external-meson states $a_{\nu\perp}^\dagger(\vec{k})\,|\alpha\rangle$ with eigenvalues $\epsilon_\alpha+\omega(k)$ and the one-external-fermion states $\widetilde{\Psi}_\perp^\dagger(\vec{p})\,|\alpha\rangle$ with eigenvalues $\epsilon_\alpha+t(p)$; the nEH states are constructed analogously.

III. SOURCES AND INTERNAL MODE FUNCTIONS

Consider now $H_{1,0}$, which must be of the form

$$\begin{aligned} H_{1,0} &= \int a_{\nu\perp}^\dagger(\vec{k})\hat{J}_\nu(\vec{k})d\vec{k} \\ &\quad +\int \widetilde{\Psi}_\perp^\dagger(\vec{p})\hat{K}(\vec{p})d\vec{p} \ , \end{aligned} \tag{9}$$

where $\hat{J}_\nu$ and $\hat{K}$ are operators within the SMS; $\hat{J}_\nu$ is the source operator for the $\perp$ mesons and $\hat{K}$ is a similar source operator for $\perp$ fermions. From Eq. (5) it is clear that

$$\hat{J}_\nu(\vec{k})=\omega(\vec{k})a_{\nu||}(\vec{k})-Y(k)\int \widetilde{\Psi}_{||}^\dagger(\vec{p})W_\nu\left[\vec{k},\frac{\vec{p}+\vec{q}}{2}\right]\widetilde{\Psi}_{||}(\vec{q})\delta(\vec{k}+\vec{p}-\vec{q})d\vec{p}\,d\vec{q}=\frac{\delta H_{||}}{\delta a_{\nu||}^\dagger(\vec{k})} \ . \tag{10}$$

Because of Eq. (2), $\hat{J}_\nu$ in Eq. (9) can be replaced by $\hat{J}_{\nu\perp}$, where

$$\hat{J}_{\nu\perp}(\vec{k})=\hat{J}_\nu(\vec{k})-\hat{J}_{\nu||}(\vec{k}) , \quad (11)$$

$$\hat{J}_{\nu||}(\vec{k})=\sum_{i\in||}\phi_i(\vec{k})\int\phi_i^*(\vec{k}')\hat{J}_\nu(\vec{k}')d\vec{k}' = [a_{\nu||}(\vec{k}),H_{||}] ,$$

and similarly for $\hat{K}$ and $\hat{K}_\perp$, so that

$$H_{1,0}=\int a_{\nu\perp}^\dagger(\vec{k})\hat{J}_{\nu\perp}(\vec{k})d\vec{k} + \int\tilde{\Psi}_\perp^\dagger(\vec{p})\hat{K}_\perp(\vec{p})d\vec{p} . \quad (12)$$

Similar treatments can be formulated for the || subspace operators analogous to $\hat{J}_\nu$ and $\hat{K}$ that appear in each of the operators $H_{n,m}$.

The operators $\hat{J}_{\nu||}$ and $\hat{K}_{||}$ appear in $H_{||}$ and are expected to have large matrix elements between the low-lying eigenstates of $H_{||}$, reflecting the strong-coupling nature of the MNSM. On the other hand, it is quite possible for the $\perp$ parts of the $\hat{J}$ and $\hat{K}$ operators to have small matrix elements between the low-lying eigenstates of $H_{||}$, and the internal functions must be chosen so that the external modes are weakly coupled to the internal states. In Ref. 2 it was shown that there is a particular choice of the internal meson functions that makes all the diagonal matrix elements of $\hat{J}_{\nu\perp}$ vanish; it seems likely that the off-diagonal matrix elements are also small for this choice of the meson functions.

Briefly, the choice of meson mode functions goes as follows. First, it is possible to write $\hat{J}$ of Eq. (10) in the form

$$\hat{J}_\nu(\vec{k})=\omega(\vec{k})a_{\nu||}(\vec{k})-\sum_{\alpha,\beta}\{B_\alpha^\dagger,B_\beta\}_\nu J^{\alpha,\beta}(\vec{k}) , \quad (13)$$

where the c-number function $J^{\alpha,\beta}(\vec{k})$ is an integral over internal single-nucleon functions and the curly braces indicate vector coupling of the isospins of the $B^\dagger$ and B operators (see Ref. 2 for a few more details of the coupling of symmetries). Thus, the operator $\hat{J}_\nu(\vec{k})$ involves the sets of functions $\omega(k)\phi_i(\vec{k})$ and $J^{\alpha,\beta}(\vec{k})$. These are the same sets of functions if[7] the functions ϕ_i are chosen to be an orthonormal set that spans the finite set of functions $J^{\alpha,\beta}/\omega$. With this choice $\hat{J}$ becomes

$$\hat{J}_\nu(\vec{k})=\sum_i\left[A_{i\nu}-\sum_{\alpha\beta}\{B_\alpha^\dagger,B_\beta\}_\nu g_{\alpha,\beta,i}\right]\omega(k)\phi_i(\vec{k}) = \sum_i(A_{i\nu}-\rho_{i\nu})\omega(k)\phi_i(\vec{k}) . \quad (14)$$

With this same choice of the functions ϕ_i, the terms in $H_{||}$ with

$$\int\omega(k)a_{\nu||}^\dagger(\vec{k})a_{\nu||}(\vec{k})d\vec{k}$$

and with

$$\int a_{\nu||}^\dagger(\vec{k})\hat{J}_{\nu||}(\vec{k})d\vec{k}$$

and its adjoint all involve the same integrals over momenta, so that $H_{||}$ takes the form

$$H_{||}=T_{F,0,0}+\sum_{ij}\omega_{ij}(A_{i\nu}^\dagger A_{j\nu}-A_{i\nu}^\dagger\rho_{j\nu}-\rho_{i\nu}^\dagger A_{j\nu}) ,$$
$$\omega_{ij}=\int\omega(k)\phi_i^*(\vec{k})\phi_j(\vec{k})d\vec{k} , \quad (15)$$

and therefore

$$[A_{i\nu},H_{||}]=\sum_j\omega_{ij}(A_{j\nu}-\rho_{j\nu}) . \quad (16)$$

Since ω_{ij} is positive definite, it follows that all diagonal matrix elements of $\hat{J}_\nu(\vec{k})$ vanish in the SMS.

Now what is the corresponding procedure for treating the operator $\hat{K}$ and determining an appropriate set of nucleon functions f_i? The operator $\hat{K}$ changes the number of nucleons in a || state; it has only nondiagonal elements. Explicitly,

$$\hat{K}(\vec{p})=\frac{\delta H_{||}}{\delta\tilde{\Psi}_{||}^\dagger(\vec{p})} = t(p)\tilde{\Psi}_{||}(\vec{p}) - \sum_i Q_{i\nu}\int\tilde{V}_{i\nu}(\vec{p},\vec{q})\tilde{\Psi}_{||}(\vec{q})d\vec{q} , \quad (17)$$

where

$$\tilde{V}_{i\nu}(\vec{p},\vec{q})=\sqrt{2}\int Y(k)\phi_i(\vec{k})W_\nu\left[-\vec{k},\frac{\vec{p}+\vec{q}}{2}\right]\times\delta(\vec{p}-\vec{q}-\vec{k})d\vec{k} , \quad (18)$$

$$Q_{i\nu}=(A_{i\nu}+A_{i\nu}^\dagger)/\sqrt{2} ,$$

and it has been assumed that

$$\phi_i^*(\vec{k})=\phi_i(-\vec{k}) . \quad (19)$$

Let the number of fermion functions f_i in the SPSMS be n_F. If a particular SMS matrix element of $\hat{K}$ is required to satisfy the condition

$$\langle\alpha|\hat{K}(\vec{p})|\beta\rangle=\mu_{\alpha\beta}\langle\alpha|\tilde{\Psi}(\vec{p})|\beta\rangle = \mu_{\alpha\beta}\langle\alpha|\tilde{\Psi}_{||}(\vec{p})|\beta\rangle , \quad (20)$$

then clearly, for that same pair $\alpha\beta$,

$$\langle \alpha | \hat{K}_{\perp}(\vec{p}) | \beta \rangle = 0 \ . \tag{21}$$

In order to have n_F equations that determine the n_F functions f_i, it is necessary to select n_F SMS matrix elements of the form of Eq. (20); of course, care must be taken that the set of n_F matrix elements gives a linearly independent consistent set of equations. From the form of the matrix element in Eq. (20), it is evident that the arbitrary parameters $\mu_{\alpha\beta}$ turn out to be fermion single-particle energies, and for this reason it seems indicated that the SMS matrix elements to be used for determining the fermion functions are the ones with large single-particle strength. Hence, the transitions with large single-particle strength have no $\hat{K}_{\perp}$ strength; there is no zeroth-order direct matrix element for a single-particle transition between the corresponding states. The observed single-particle transition strength results from iterations of the interaction Hamiltonian, that is, from distortion effects.

IV. DIAGRAMS AND GREEN'S FUNCTIONS

Instead of writing equations for such iterations, it is simpler to use the diagrams that correspond to the equations in the standard way. Figures 1 and 2 show the elementary vertex diagrams that represent the various parts of the interaction Hamiltonian. The heavy line in these diagrams represents the status of the SMS part of a state vector; it is labeled by the current eigenvector specification of the SMS vector. The dashed meson lines and wavy fermion lines both represent external fields. The vertex operators are all orthogonalized in all their external field momenta by using procedures like that given in Eq. (11). The vertex shown in Fig. 1(a) represents the meson part of $H_{1,0}$ of Eq. (12); the factor associated with it is the function

$$\langle \alpha | \hat{J}_{\nu\perp}(\vec{k}) | \beta \rangle \ , \tag{22}$$

where the $\perp$ subscript really belongs outside the matrix element, since it is the matrix element that must be orthogonalized to the internal mode functions. The factor associated with Fig. 1(b) is the complex conjugate of the factor associated with Fig. 1(a). Similarly, the factors associated with the figure pairs 1(c) and 1(d), 1(e) and 1(f), 2(a) and 2(b), and 2(c) and 2(d) are complex conjugates, and only the first of each will be given. Associated with Fig. 1(c) is the factor

$$\langle \alpha | \hat{K}_{\perp}(\vec{p}) | \beta \rangle \ . \tag{23}$$

For Fig. 1(e) the associated factor is

$$\langle \alpha | \hat{L}_{\nu\perp}(\vec{k},\vec{q}) | \beta \rangle \ , \tag{24}$$

where the SMS operator $\hat{L}_{\nu\perp}$ is defined by

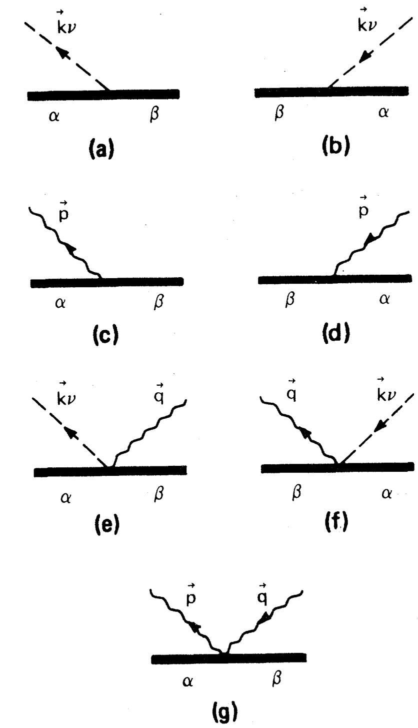

FIG. 1. Vertices that can act in the 1EH sector. The dashed lines represent the external meson field, and the wavy ones the external nucleon field. The heavy lines represent the status of the SMS or internal part of the state vector.

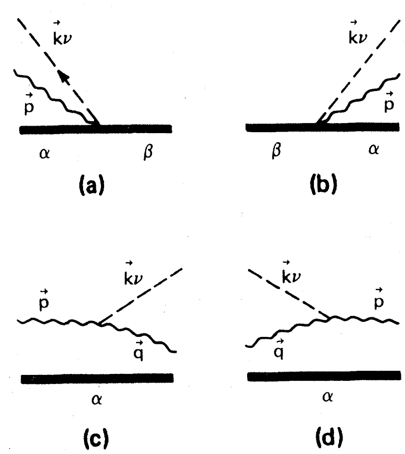

FIG. 2. Vertices not shown in Fig. 1. The correspondences between lines and fields are as in Fig. 1.

$$\hat{L}_{\nu\perp}(\vec{k},\vec{q})=-\left[\int Y(k)W_\nu\left[\vec{k},\frac{\vec{p}+\vec{q}}{2}\right]\delta(\vec{k}+\vec{p}-\vec{q})d\vec{p}\right]_\perp , \tag{25}$$

and the $\perp$ subscript indicates that the operator inside the brackets is orthogonalized so that

$$\begin{aligned}&\int \phi_i^*(\vec{k})\hat{L}_{\nu\perp}(\vec{k},\vec{q})d\vec{k}=0 ,\\ &\int \hat{L}_{\nu\perp}(\vec{k},\vec{q})\tilde{f}_i(\vec{q})d\vec{q}=0 .\end{aligned} \tag{26}$$

The orthogonalization procedure is similar to that given in Eq. (11). The derivation of the vertex functions for the diagrams of Figs. 1(g) and 2(a)—2(d) is analogous and will be left to the interested reader.

A general diagram is obtained in the usual way by putting together various vertices. The connecting external-field lines in a general diagram represent external-field Green's functions, which propagate the external fields while annihilating the internal fields. If the vertex operators are all orthogonalized as specified above, then the external-field Green's functions can be replaced by ordinary free-particle Green's functions, since the vertices can generate only external fields. Or, alternatively, if the Green's functions and external-line wave functions are all suitably chosen, the vertex functions need not be orthogonalized. However, in order to ensure that internal modes do not vitiate the results of a particular approximation procedure, it is much safer to remove the internal modes from both vertices and propagators. The external-field Green's function $g_\perp(\vec{p},\vec{q};\lambda)$ for the fermion field satisfies the equation

$$[\lambda-t(p)]g_\perp(\vec{p},\vec{q};\lambda)+\sum f_i(\vec{p})\int \tilde{f}_i^\dagger(\vec{s})t(s)g_\perp(\vec{s},\vec{q};\lambda)d\vec{q}=\delta(\vec{p}-\vec{q})-\sum \tilde{f}_i(\vec{p})\tilde{f}_i^\dagger(\vec{q}) , \tag{27}$$

where λ is the (complex) energy parameter. The solution to Eq. (27) is

$$g_\perp(\vec{p},\vec{q};\lambda)=\frac{\delta(\vec{p}-\vec{q})}{\lambda-t(p)}-\sum_{i,j}\frac{\tilde{f}_i(\vec{p})}{\lambda-t(p)}[I(\lambda)^{-1}]_{ij}\frac{\tilde{f}_j^\dagger(\vec{q})}{\lambda-t(q)} , \tag{28}$$

where the matrix $I(\lambda)$ is given by

$$I_{ij}(\lambda)=\int \frac{\tilde{f}_i^\dagger(\vec{p})\tilde{f}_j(\vec{p})}{\lambda-t(p)}d\vec{p} . \tag{29}$$

The Green's function of Eq. (28) annihilates internal single-particle functions. The meson external-field Green's function is similar to the fermion Green's function of Eq. (28); note that it is not useful to unify the forward- and backward-going external-meson Green's functions. In Ref. 3, it was shown that the irreducible self-energy part has a factor $I(\lambda)^{-1}$; this can also be understood in terms of the Green's function of Eq. (28).

V. 1EH SECTOR

As is well known, the set of diagrams up to a particular order in the interaction Hamiltonian does not give a unitary approximation to scattering amplitudes. On the other hand, summing all diagrams that remain entirely within a particular subspace of the full vector space of the system does give a unitary approximation for scattering amplitudes. The nEH sector is a suitable subspace within which to work out a unitary approximation to the scattering matrix. The 0EH sector consists of the eigenstates of H_U. The next simplest sector is the 1EH sector, where only the vertex diagrams shown in Fig. 1 are allowed. Note that the nEH sector differs from the n-hadron sector in that it contains states with arbitrary numbers of hadrons, as long as all but n of the hadrons are in internal modes.

Figure 3(a) shows a typical diagram in the 1EH sector for the (N,π) process (nucleon in, pion out). As far as diagram structure (representing equation structure) is concerned, it is obviously useful to unify the hadron lines and use a solid line to represent both hadrons; each internal solid line must be summed over both of its possible hadron realizations. Then the diagram of Fig. 3(a) is one of the terms represented by the diagram of Fig. 3(b).

It is easy to see that all the diagrams in the 1EH sector can be expressed in terms of the SMS-irreducible Green's function G_I shown in Fig. 4; this Green's function satisfies the linear integral equation symbolized in Fig. 4(b). Figure 5 shows how the SMS-irreducible self-energy Σ is related to G_I. The bound-state eigenenergies in the 1EH sector are

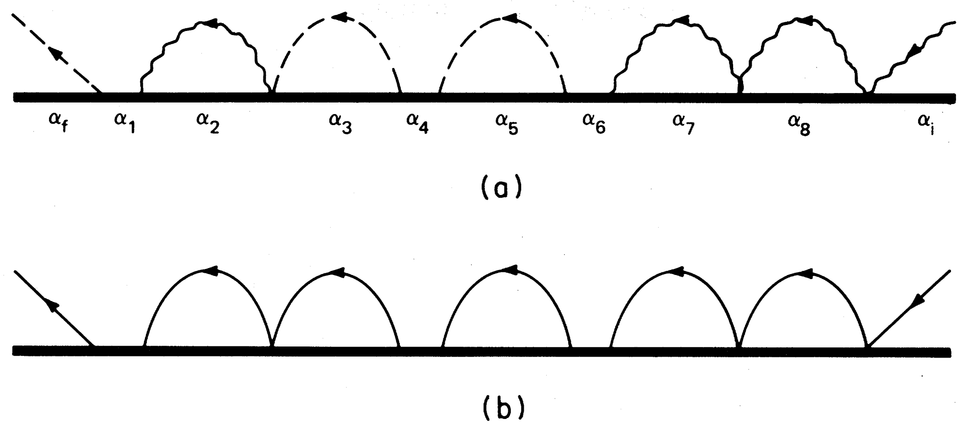

FIG. 3. A typical graph in the 1EH sector. (a) is one of the graphs represented by (b) in which the line represents both kinds of external hadron field.

the roots of the equation

$$\det[(\lambda-\epsilon_\alpha)\delta_{\alpha\beta}-\Sigma_{\alpha\beta}(\lambda)]=0\ . \tag{30}$$

Figure 6 gives the relation between the full Green's function in the 1EH sector and the SMS-irreducible Green's function G_I.

The diagrammatic manipulations of this section are all quite standard. The new features are the interpretation in terms of *external* hadron fields and the expansion in numbers of external hadron lines. There is also nothing new in the idea that all scattering processes that are related are related. However, the present work gives a concrete unitary realization of this elementary idea, in that the vertices and Green's functions symbolized in the diagrams are all explicitly determined in terms of the underlying Hamiltonian through the internal mode functions.

VI. REMARKS

In the past it has proved useful to separate the Hilbert space of the system in various ways. For example, there is the separation into "P" and "Q" subspaces developed by Feshbach.[8] As has been noted above, the separation of the field operators into internal and external parts leads to a decomposition of the Hilbert space into subspaces, each of which has a fixed number of external hadrons. By choosing the internal modes optimally, the terms that couple these subspaces are made small.

The idea of splitting the pion field into internal and external parts resembles in some ways the treatment of pion interactions in terms of delta resonances in nuclei, where a combination of a delta resonance and a nucleon hole plays a role like that of the internal pion field.

An obvious problem with the unified 1EH-sector scattering matrix described in Sec. V is that an incident or final nucleon is not accompanied by its own internal pion field. Corrections for this effect start to appear in the 2EH sector.

VII. SUMMARY

In the meson-nucleon model of nuclei described by a Hamiltonian like that of Eq. (5), the decompo-

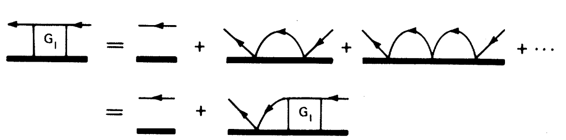

FIG. 4. The SMS-irreducible one-external-hadron Green's function in the 1EH sector G_I is shown (a) as an infinite sum and (b) in terms of the integral equation it satisfies.

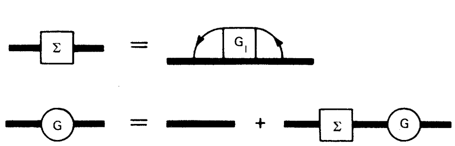

FIG. 5. The SMS-irreducible self-energy part in the 1EH sector Σ and its relation to the internal-field Green's function in the 1EH sector.

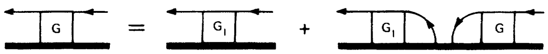

FIG. 6. The one-external-hadron Green's function in the 1EH sector.

sition of the meson and nucleon field operators into internal and external parts as in Eqs. (1)–(4) provides a way of treating orthogonality effects in the scattering of hadrons by a nucleus. Under this decomposition, the Hamiltonian of the system splits into an internal part and terms that couple external modes. The internal part of the Hamiltonian is $H_{||}$, the MNSM Hamiltonian whose eigenstates are approximations to the bound nuclear states. The rest of the Hamiltonian describes the external quanta of the fields and their interactions with the internal states.

The internal mode functions must be chosen so as to minimize the coupling of the external modes to the low-lying internal states. For the meson mode functions, the appropriate criterion has been given in Ref. 2; in the present work, analogous criteria for the nucleon single-particle functions have been given in Sec. III, together with a review of the conditions satisfied by the meson modes.

The Green's functions for the external fields and the diagrammatic representation of the perturbation series are given in Sec. IV; Sec. V uses these diagrams to describe the treatment of hadron scattering in the one-external-hadron sector.

More important than the details is the general idea of treating the external and internal fields as separate entities. This idea appears to have many possible applications.

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[7]Actually, the functions ϕ_i should be chosen to span the set $J^{\alpha,\beta}/(\omega+\lambda)$; here λ is a parameter that must be chosen so that the matrix element $\langle * \,|\, A_{i\nu}-\rho_{i\nu} \,|\, \text{g.s.} \rangle$ is zero, where $|\,*\,\rangle$ is the lowest eigenstate of $H_{||}$ connected to the ground state $|\,\text{g.s.}\rangle$ of $H_{||}$ by the operator $A_{i\nu}-\rho_{i\nu}$. Unless the ground state has $T=0$ or $J=0$, this gives $\lambda=0$. Even if λ is not zero, it is a nuclear excitation energy satisfying $|\lambda| \ll m$, so that it is adequate to set $\lambda=0$ in all nuclear cases. In the Lee model it is important that λ is not zero.

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