Entropy and hadrochemical composition in heavy ion collision

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The composite particle production in a heavy ion collision is calculated in the framework of a hadrochemical model. A critical comparison is performed between the produced entropy and the observables. The entropy production during the hadrochemical processes is found to be negligible.

[NUCLEAR REACTIONS Ne + NaF heavy ion collision, $E=0$, 4–2.1, calculated spectra p, d, π^+, van der Waals model, entropy.]

I. INTRODUCTION

In the search for exotic, in particular, quark-gluon, plasma states, the lepton pairs or the strange particles were pointed out as messengers from the early, hot, compressed state of the firecloud formed in heavy ion collisions.[1–5] These considerations were motivated by the fact that at lower temperatures (characteristic of the expansion and breakup phase of the fireball) the processes which could change the number of strange particles or the number and spectral distribution of leptons have very small probability.

On the other hand, the frequent interaction between hadrons was believed to destroy any possible signature of earlier states in the later thermal history of the firecloud. It has since been surmised, however, that even these hadrons may carry a signature of an earlier phase transition. Namely, the phase transition would show up in the total entropy of the firecloud and would be conserved during the later adiabatic expansion. On the other hand, the entropy can be read from the ratio of the number of composite particles to the number of nucleons. In fact, the entropy obtained from the experimental deuteron to proton ratio in heavy ion collisions seemed to show an excess over that calculated assuming a hot, hadronic, gaseous phase.[6] This observation led to lengthy discussions.

Our aim in the present work is to analyze carefully the role of entropy in heavy ion collisions. The relation between the specific entropy and observed spectra is discussed in Sec. II. A (more or less) consequent description of the expanding fireball in a vacuum is given in Sec. III. In Sec. IV we use these results in a hadrochemical model, and our results and conclusions are summarized in Sec. V. Throughout the paper we use $\hbar=k=c=1$.

II. ENTROPY AND THE OBSERVED SPECTRA

It was pointed out in Ref. 7 that one can calculate the entropy of the fireball formed in heavy ion collisions from the observed deuteron to proton ratio (R_{dp}). On the other hand, the authors of that work estimated the entropy of the fireball at the beginning of the expansion (supposed by them as being adiabatic). When comparing these two values they concluded that the experiments show an entropy excess during the adiabatic expansion. To avoid this contradiction they suggested some possible reason for this excess, e.g., the phase transition into the quark-gluon plasma or pion condensation, etc.

Because of the importance of such a conclusion, let us look again at these considerations. In Ref. 8 the nonrelativistic thermodynamical treatment of a one component ideal gas was applied to the fireball and it was found that the specific entropy depends only on the ratio R_{dp},

$$s_p/n_p = s_N/n_N = 3.95 - \ln R_{dp} \ . \tag{2.1}$$

But even at this low temperature limit, when the nonrelativistic treatment may be acceptable, the s_p/n_p ratio is not a conserved quantity which can be compared to its initial value. Only the total entropy of the expanding gas mixture and the total baryon number are conserved during the adiabatic expansion. Denoting by V the actual volume of the fireball one can construct the following conserved quantity:

$$\begin{aligned} S/N_b &= sV/n_b V \\ &= \frac{s_N + s_d}{s_N + 2n_d} \\ &= 3.95 - \ln R_{dp} - 1.25 R_{dp}/(1+R_{dp}) \ , \end{aligned} \tag{2.2}$$

which is somewhat lower (at the same observed R_{dp} ratio) than the one given by Eq. (2.1) and in Ref. 8. We mention, however, that Eq. (2.2) is valid only for a two component mixture of ideal gases.

Although Eq. (2.2) gives some entropy excess, one has to take into account, before drawing further conclusions, the entropy produced by the chemical equilibration process between nucleons and deuterons. If, after this correction, there remains some entropy excess, one may investigate for special sources of it. Some other mechanisms have been considered recently. The replacement of classical statistics by quantum statistics (Ref. 9) seems not to lower the calculated ratio R_{dp}. Stöcker (Ref. 10) proposed that the number of protons may increase after the breakup of the system by the decay of unstable particles or nuclei. We also should mention that the mesons (mainly pions) contribute to the total entropy but not to the number of baryons. One could also doubt the applicability of the nonrelativistic gas approximation and the assumption of pointlike deuterons. The large breakup cross section of the deuteron makes the assumption of the ideal deuteron gas doubtful, because it suggests that the deuteron cannot exist with nucleons being too close in its neighborhood. The more compact light fragments such as ^{3}He, t, and α might be more appropriate to "measure" the entropy. To summarize, we conclude that to predict the entropy related to the observed deuteron to proton ratio one must follow up the process of chemical equilibration in a rather relativistic treatment. We try to do this in the following sections.

III. THE QUASIADIABATIC MODEL

For the description of the heavy ion collision (HIC) the adiabatically expanding gas model has been used in several papers.[11–13] The difference between these models and the present treatment is the replacement of the adiabacity hypothesis by a more precise one, which is valid for an arbitrary mixture of gases even if its components transform into one another. The evolution of such a multicomponent perfect fluid (i.e., the viscosity and the heat conductivity are neglected) will be called a quasiadiabatic one. In such systems entropy change may occur because of the chemical transmutations. In the following we shall derive the equations describing this system.

Let us take an infinitely small volume cell of an expanding fireball, which moves away with the four-velocity u^i, and fix our coordinate system to this. We can speak about thermodynamics in this local system after defining internal energy, pressure, particle number densities of the different components of the gas, temperature, chemical potentials; concisely, all the familiar thermodynamical quantities. The internal energy (e) and the pressure (p) of these cells depend on the number densities (n_a) of the components and the local temperature (T) or its inverse (β). For ideal gases they can be constructed as the sum of the quantities related to the components

$$e=\sum_a n_a e^a, \quad p=T\sum_a n_a \quad . \tag{3.1}$$

The energy density of a component in the case of the relativistic Boltzmann distribution is given as

$$e^a=m^a R\,(m^a/T),$$

where

$$R(x)=\frac{3}{x}+\frac{K_1(x)}{K_2(x)} \tag{3.2}$$

and $K_n(x)$ denotes the nth order modified Bessel function of the imaginary argument. Knowing the local parameters $\{n_a,T\}$ all the other thermodynamical quantities can be expressed, and we can write the energy-momentum tensor of the perfect fluid in the form

$$T^{ik}=(e+p)u^i u^k-pg^{ik} \quad . \tag{3.3}$$

Here g^{ik} is the metric tensor

$$(g_{00}=1, \quad g_{11}=g_{22}=g_{33}=-1) \quad .$$

We use only the energy-momentum conservation to describe the hydrodynamical features of the expansion in the vacuum. It is given in the form

$$\partial_i T^{ik}=0 \quad . \tag{3.4}$$

Supposing a spherically symmetric explosion of the fireball we need only two scalar equations. Let the first be the projection of T^{ik} onto the velocity field u_k:

$$u_k\partial_i T^{ik}=0 \quad . \tag{3.5}$$

For the second we shall use the energy conservation, i.e., the timelike component of Eq. (3.4):

$$\partial_i T^{i0}=0 \quad . \tag{3.6}$$

Besides these we have further equations describing chemical processes between the component gases in the familiar way,

$$\partial_i(n_a u^i)=\Psi_a \quad . \tag{3.7}$$

The source term, Ψ_a, depends on the temperature and the number densities of each or almost each component. In Sec. IV we will describe the "chemical reactions" (decays, collisions, etc.) included in our model, but here we need to know only the fact

that the number density of the ath component has a source. The set of equations (3.5)—(3.7) describes the expansion.

To recognize the physical meaning of Eq. (3.5) we transform it to obtain a form of a total four divergency plus another term. After that we substitute Eq. (3.3) into it. We get

$$u_k\partial_i T^{ik}=\partial_i(u_k T^{ik})-T^{ik}\partial_i u_k$$
$$=\partial_i(eu^i)+p\partial_i u^i=0 \ . \tag{3.8}$$

If we now apply the First Law of Thermodynamics for the local infinitely small volume cell of the fireball we get

$$\partial_i(eu^i)=T\partial_i(su^i)-p(\partial_i u^i)+\mu^a\partial_i(n_a u^i) \ .$$

Replacing this in Eq. (3.8) we get for the entropy the relation

$$\partial_i(su^i)=-\beta\mu^a\partial_i(n_a u^i)\neq 0 \ . \tag{3.9}$$

The physical meaning of this equation is to take into account the entropy produced by the chemical processes, i.e., there are possible exothermal and endothermal transmutations. Approximating the chemical equilibrium state the entropy increases according to Eq. (3.9).

From Eq. (3.7) we can express the source of the entropy by the chemical sources and we get the following set of equations to describe the hadrochemistry of an expanding sphere:

$$\partial_i(su^i)=-\beta\mu^a\Psi_a \ , \tag{3.9'}$$

$$\partial_i T^{i0}=0 \ , \tag{3.6}$$

$$\partial_i(n_a u^i)=\Psi_a \ . \tag{3.7}$$

We would like to approximate the description of the evolution of the fireball by ordinary differential equations. For this purpose we shall average the spatial dependence of the variables. This averaging can be done in a given coordinate system: We choose the center of mass (c.m.) system of the fireball. In this system we suppose n_a and β to be homogeneous.

We need an assumption for the flow velocity field, too. We have chosen the form of the spatial dependence of it as follows Ref. 11:

$$\gamma(u)\vec{u}=\vec{r}\cdot\dot{R}(t)/R(t) \ , \tag{3.10}$$

where

$$\gamma(u)=(1-u^2)^{-1/2}$$

is the Lorentz factor; $R(t)$ is the radius of the fireball at time t, which is measured at the center of the fireball; and $\dot{R}(t)$ is its time derivative.

Using these assumptions we get the following set of averaged equations:

$$\frac{1}{V}\frac{d}{dt}(Vs\langle\gamma\rangle)=-\beta\mu^a\Psi_a \ , \tag{3.11}$$

$$\frac{1}{V}\frac{d}{dt}[V(e\langle\gamma^2\rangle+p\langle\gamma^2u^2\rangle)]=0 \ , \tag{3.12}$$

$$\frac{1}{V}\frac{d}{dt}[Vn_a\langle\gamma\rangle]=\Psi_a \ . \tag{3.13}$$

Here V denotes the actual (time-dependent) volume of the fireball, and the brackets $\langle\ \rangle$ mean a spatial averaging. In the present model we supposed n_a-s and β to be homogeneous, so we cannot avoid the spatial averaging of the expressions depending on the velocity field. In the following we will call them "kinematic factors." They are

$$\langle\gamma\rangle;\ \langle\gamma^2\rangle;\ \langle\gamma^2u^2\rangle=\langle\gamma^2\rangle-1 \ .$$

Using (3.10) we get

$$\langle\gamma\rangle=\frac{3}{\dot{R}^3}\int_0^{\dot{R}}x^2\sqrt{1+x^2}dx$$
$$=\frac{3}{8\dot{R}^3}[\dot{R}\sqrt{1+\dot{R}^2}(1+2\dot{R}^2)-Arsh\dot{R}] \ , \tag{3.14}$$

$$\langle\gamma^2\rangle=\frac{3}{\dot{R}^3}\int_0^{\dot{R}}x^2(1+x^2)dx=1+\tfrac{3}{5}\dot{R}^2 \ . \tag{3.15}$$

Finally, from trivial geometry we have the relation

$$\frac{\dot{V}}{V}=3\frac{\dot{R}}{R} \ . \tag{3.16}$$

The initial stage of Eqs. (3.10)—(3.16) is the total overlap of the colliding nuclei, when $V=V_0=r_0A^{1/3}$, $\dot{R}=0$, and $n_a(0)$, $\beta(0)$ were calculated in the familiar way as the final state of the ignition phase described in Ref. 13.

To predict the experimentally observed inclusive cross-section data we need to choose a time point (breakup time) when the expansion and hadrochemistry "freeze out," namely, the chemical composition of the mixture and momentum distribution of the components do not change from this moment until the detection of particles.

There are several criteria for the freeze out of fireball models.[11,14] In the present calculation we use the following criterion. If the average collision number per particle in the total volume during a characteristic cooling time is less than one, then the hydrodynamic description must lose its validity. Here we only check for the self-consistency of application of the hydrodynamic description in the present model: We calculate the averaged number

of collisions per baryon simultaneously with the prediction of the inclusive p, d, and π spectra.

We define the characteristic cooling time by the change of the thermal energy

$$\frac{1}{\tau}=\frac{|\partial_i(e_{\rm th}u^i)|}{e_{\rm th}} \ , \tag{3.17}$$

where

$$e_{\rm th}=e-\sum_a m^a n_a \ .$$

The averaged number of collisions per particle is calculated supposing collisions between nucleons to be independent:

$$a=\tau\langle\sigma_{pp}^{\rm tot}v_{\rm rel}\rangle n_{\rm baryon} \ . \tag{3.18}$$

Here the brackets mean the average over the momentum space supposing relativistic Boltzmann distribution. (See Sec. IV.)

IV. HADROCHEMISTRY

In this section we concentrate on the right-hand side of Eq. (3.7), viz., on the Ψ_a chemical source term. The different processes of chemical reactions are grouped into the following types:

(1) "one to two," $A\rightleftarrows B+C$, e.g, $\Delta\rightleftarrows N$;
(2) "two to two," $A+B\rightleftarrows C+D$, e.g., $NN\rightleftarrows D$;
(3) "two to three," $A+B\rightleftarrows A+C+D$, e.g., $ND\rightleftarrows NNN$.

The first type, the well-known decay, changes the number density of the ath component as

$$\dot{n}_a=-\Gamma_a n_a \ , \tag{4.1}$$

where Γ_a is the width of the decaying resonance.

In the co-moving frame of a fluid cell the source term gives the change of n_a in the proper time, so

$$\Psi_a(1)=-\Gamma_a n_a \ .$$

But since the energy and momentum distribution of the decaying particles is Boltzmannian, the average can be written as

$$\Psi_a(1)=-\langle\Gamma_a\rangle n_a=-\Gamma_a n_a\langle 1/\gamma_{\rm th}\rangle=-\Gamma_a n_a\frac{K_1(\beta m_a)}{K_2(\beta m_a)} \ . \tag{4.2}$$

Here $K_n(x)$ denotes the nth order Bessel function of the imaginary argument and $\gamma_{\rm th}$ is the Lorentz factor corresponding to the thermal motion.

The second type is the two-body collision. If each colliding set of particles has Boltzmann distribution in the momentum space, then the averaged rate factor is

$$\langle\sigma v_{\rm rel}\rangle=\frac{\beta\int_{(m_3+m_4)^2}^{\infty}\sigma(s)\lambda(s)K_1(\beta\sqrt{s})\frac{1}{2\sqrt{s}}ds}{4m_1^2m_2^2K_2(\beta m_1)K_2(\beta m_2)} \tag{4.3}$$

where

$$\lambda(s)=(s-m_1^2-m_1^2)^2-4m_1^2m_2^2 \ ;$$

$s=(p_1+p_2)^2$ represents the invariant c.m. energy squared; $m_1\cdots m_4$ are the rest masses of particles $A\cdots D$; $\sigma(s)$ is the cross section of the process investigated; and $\beta=1/T$ is the inverse temperature. In the case of the process $A+B\rightleftarrows C+B$ the chemical source term is proportional to the rate factor and the number densities of the colliding particles:

$$\Psi_A(2)=-\langle\sigma_{AB}v_{\rm rel}\rangle n_A n_B \ . \tag{4.4}$$

The third type of hadrochemical processes is very similar to the second one: Only the difference is shown by the reverse process, which is a three-body collision. But we generally take into account the reverse processes by means of the equilibrium ratio. For example, for $A+B\rightleftarrows A+C+D$ the total change of B is

$$\Psi_B(3)=-\langle\sigma_{AB}v_{\rm rel}\rangle n_A(n_B-\rho_B^{CD}n_C n_D) \ . \tag{4.5}$$

Similar expressions are written for the reverse processes of the first and second types:

$$\Psi_B(1)=-\langle\Gamma_A\rangle(n_A-\rho n_B n_C) \ , \tag{4.6}$$

$$\Psi_B(2)=-\langle\sigma_{AB}v_{\rm rel}\rangle(n_A n_B-\rho n_C n_D) \ . \tag{4.7}$$

The equilibrium ratios are determined by the following restriction: In chemical equilibrium the source term has to be equal to zero. On the other hand, we know from the Boltzmann distribution that

$$n_a=e^{\beta\mu^a}Q(\beta,m_a) \ ,$$

where

$$Q(\beta,m_a)=\frac{d_a}{2\pi^2\beta}m_a{}^2K_2(\beta m_a)$$

is the familiar canonical partition function, and d_a denotes the spin and isospin degeneracy factor of the particle of type a. Hence the equilibrium ratio ρ is given as

$$\rho=\left[\frac{n_An_B}{n_Cn_D}\right]_{\rm eq}=\frac{e^{-\beta(\mu_A+\mu_B)}Q_AQ_B}{e^{-\beta(\mu_C+\mu_D)}Q_CQ_D}=\frac{Q_AQ_B}{Q_CQ_D}\ . \tag{4.8}$$

One can get the equilibrium ratios for each type of process in a similar way.

In a central heavy ion collision a thermalization process goes on. This can be thought of as a sort of chemical reaction when the stages of this "reaction chain" are the more and more Maxwell-Boltzmann-type distributed particles. In the present model we describe only two stages of this thermalization process: the original component having sharp energy and momentum distribution (named "cold" nucleons, N_0), and the Boltzmannian components ("hot" particles, N, Δ, $\pi\cdots$). It has been pointed out that the Boltzmann distribution is almost reached after 2–3 collisions per particle.[15] Now we model the thermalization as a one step process: $N_0+N_0\to A+B$ or $N_0+X\to N+X$.

The source terms related to these processes contain the rate factor for the elastic nucleon-nucleon scattering averaged over the momentum distribution of the cold nucleons. For an N_0+N_0 collision it is trivial:

$$\langle\sigma v\rangle_{00}=\sigma(s_0)\frac{\lambda^{1/2}(s_0)}{2E_0{}^2}$$
$$=\sigma(m_0\sinh\zeta_0)2\tanh\frac{\zeta_0}{2}\ . \tag{4.9}$$

For an N_0+X collision the rate factor is

$$\langle\sigma v\rangle_{01}=\frac{\int_{(m_3+m_4)^2}^{\infty}\sigma(s)\lambda^{1/2}(s)\left\{\exp\left[-\beta\left[\frac{E_0h^{1/2}(s)+P_0\lambda^{1/2}(s)}{2m_0{}^2}\right]\right]-\exp\left[-\beta\left[\frac{E_0h^{1/2}(s)-P_0\lambda^{1/2}(s)}{2m_0{}^2}\right]\right]\right\}ds}{8m_1{}^2P_0E_0K_2(\beta m_1)} \tag{4.10}$$

In these equations ζ_0 is the rapidity of the cold nucleon; E_0 and P_0 are its energy and momentum, respectively; $s=(p_0+p_1)^2$ is the invariant c.m. energy squared; s_0 is the same for cold nucleons; and the functions $h(s)$ and $\lambda(s)$ are given as

$$h(s)=(s-m_0{}^2-m_1{}^2)^2\ , \tag{4.11}$$

$$\lambda(s)=h(s)-4m_0{}^2m_1{}^2\ . \tag{4.12}$$

Finally, we list the hadrochemical processes included in the present model:

First type:

$\Delta\rightleftarrows N\pi$, $\rho\rightleftarrows\pi\pi$;

Second type:

$N_0N_0\to NN$ or $N\Delta$, $N_0N\to N\Delta$,

$N_0X\to NX(X=N,D,\pi,\Delta)$,

$NN\rightleftarrows N\Delta$, $NN\rightleftarrows D\pi$;

Third type:

$N_0D\to NNN$, $ND\rightleftarrows NNN$.

The cross section for these processes is taken from experimental data (Ref. 16).

V. RESULTS AND DISCUSSIONS

In this paper we calculate such symmetric collisions where the total baryon number is 80 or 40, mostly at 0.8 GeV/nucleon bombarding energy. This choice describes a situation very similar to the experiments on KCl + Ar; nevertheless, slight differences in the final chemical composition can be expected because there is a 6% neutron excess in the experimental situation.

Regarding the hadrochemical processes, Fig. 1 shows that at $E/A=0.8$ GeV the final composition is as follows:

$$N_0=28,\ N=40,\ D=6,\ \pi=8\ .$$

The break on the N curve is an artifact from a simplification in the model, namely that the $N_0+N_0\to N+N$ process is neglected after the total overlap. Nevertheless, one can conclude that there remains a cold subsystem. The equilibration of the deuteron number is very rapid, in accordance with Kapusta's calculation, while the pion and delta numbers have monotonous trends because the equilibrium ratios in the $\Delta\rightleftarrows N\pi$ process depend on the density. Observe, however, that the sum of the number of deltas and

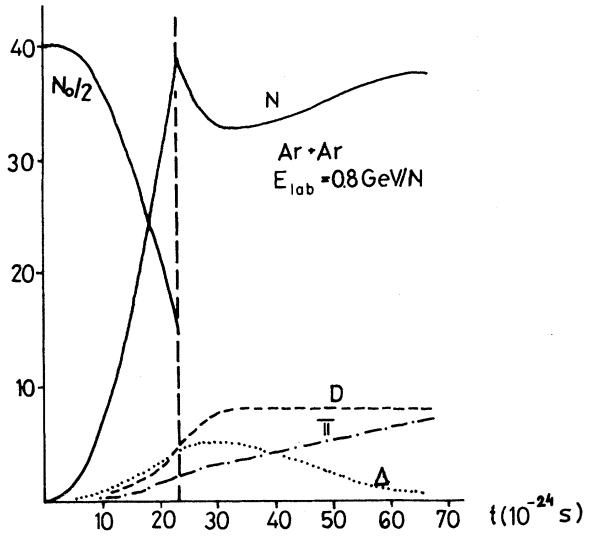

FIG. 1. The number of different kinds of particles is shown as a function of time for the reaction Ar + KCl at $E_{\text{lab}}=800$ MeV/nucleon. The dashed vertical line marks the total overlap of the colliding heavy ions. Observe that the $\Delta+\pi$ number, i.e., the number of the detectable pions, very soon reaches the final value.

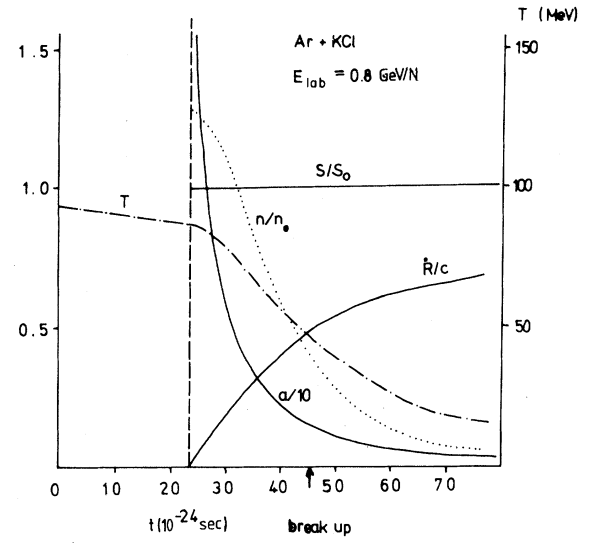

FIG. 2. Thermodynamical quantities versus time for the heavy ion collision Ar + KCl at $E_{\text{lab}}=800$ MeV /nucleon. S denotes the total entropy; S_0 is its value at the total overlap; T and n are the temperature and the number density, respectively; n_0 is the normal nuclear matter density; R is the expansion velocity of the surface; and a is the average collision number per particle during a characteristic cooling time [Eq. (3.18)]. The broken line marks the total overlap. The arrow denotes the breakup time at which the effective proton temperature fits the experimental one. This corresponds to $a_{\text{cr}}=1.6$.

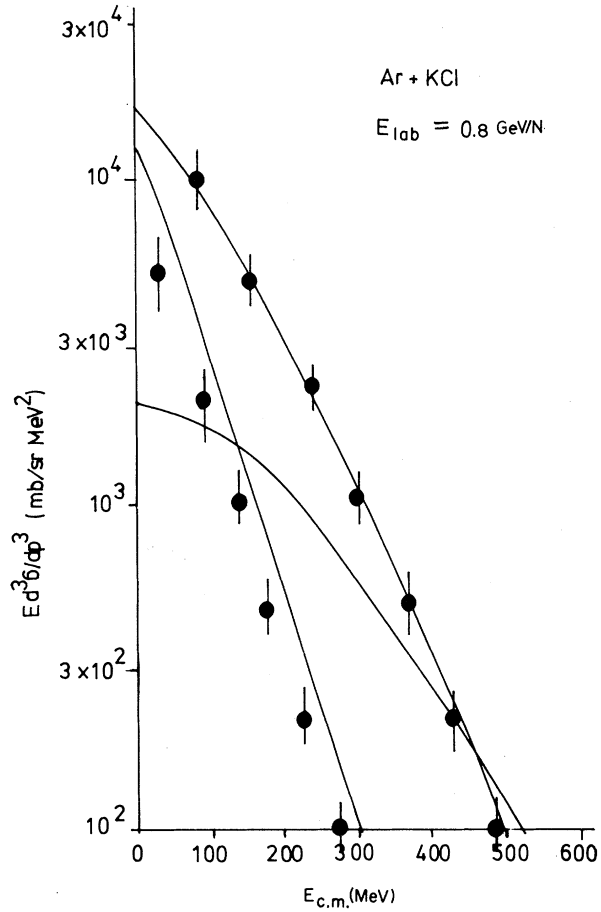

FIG. 3. Invariant p, π, and d cross sections calculated for the Ar + Ar reaction at $E_{\text{lab}}=800$ MeV/nucleon. The effective temperatures correspond to the slope factors. In parenthesis the experimental values (Ref. 18) are given.

pions, i.e., the number of pions which will be detected, soon reaches a constant value. In view of this, the prediction of the measurable pion to nucleon ratio is largely insensitive to the choice of the breakup time. With increasing bombarding energy the final pion number increases and the deuteron number decreases, because of the higher temperature.

After the total overlap a one-fluid model was used. The evolution of some hydrodynamic and thermodynamic parameters is displayed in Fig. 2. Obviously, the thermal equilibrium, cannot, however, be valid when the cooling is too fast compared with the equilibrating processes. The quantity "a" defined by Eq. (3.18) shows the relation between the characteristic time of these processes. Obviously the critical value of a is about 1, and in this model we assume that for $a<a_{\text{cr}}$ the particles are free. Nevertheless, the $a(t)$ curve is very flat at $a=1$; thus the breakup time is not well defined. For this

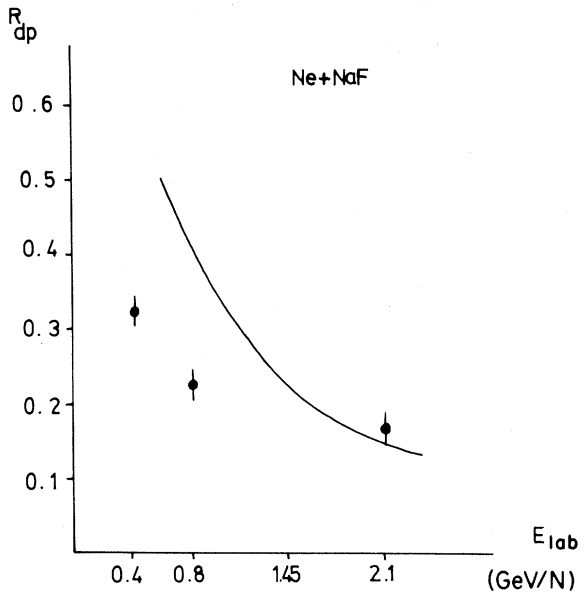

FIG. 4. The d/p ratios are calculated at different bombarding energies. The dots are the experimental values (Ref. 18) for the Ne + NaF reaction.

reason we regard a_{cr} as a free parameter to be fitted to the slope of the proton spectrum. However, Fig. 2 shows that the breakup density is about $\frac{1}{4}$ normal nuclear density, $T_{br} \sim 40$ MeV, and the velocity of the surface is about $c/2$. The entropy produced by the chemical processes is about 1%, definitely less than the neglected contribution of the viscosity.

The calculated p, π, and d spectra are shown in Fig. 3. It is expected that the slopes should be different because of the different masses. The breakup time has been chosen within the breakup interval in such a way that the high energy proton slope be correct (79 MeV). Then even the initial low energy part of the proton spectrum is correctly given, and similarly the π slope is near the experimental value (64 MeV instead of 66 MeV). The calculated deuteron spectrum definitely differs from the exponential one in the displayed energy region due to the larger rest mass of the deuteron.

We can conclude that the chemical composition of the reaction products in a central heavy ion collision can be understood within the framework of the present hadrochemical model for the energies $E \approx 2$ GeV/nucleon. At lower energies (0.4—2.0 GeV/nucleon) the predicted number of deuterons is greater than the experimentally observed value (cf. Fig. 4), so some physical mechanisms have to be looked for which suppress the number of deuterons in a HIC. Such suppression in the dense nuclear matter is understandable. Namely, the low nucleon momentum components of the deuteron wave function are excluded by the Pauli principle, while, due to the large extension, these low energy components contribute a substantial part of the deuteron wave function.[17] At the same time it is important to realize that, due to the contribution of pions and other hadrons to the entropy, the deuteron number is not the proper quantity for measuring the entropy produced in the collision, i.e., for finding a piece of evidence of the exotic states of matter.

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