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Reduction of separable Bethe-Salpeter equation kernels to quasipotential equations

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In view of relativistic three-body calculations we have investigated several quasipotential approximations to the Bethe-Salpeter equation assuming separable kernels with Yamaguchi-type form factors. In particular we have calculated π-N and N-N phase shifts with $l=0$, 1. As a result it is shown that the choice of a symmetric or unsymmetric reduction of the Bethe-Salpeter equation for πN or NN scattering is less important in comparison to the choice of the analytic form of the pole. The quasiparticle equation proposed by Erkelenz and Holinde has turned out to be superior to other quasiparticle equations which have also been considered. As a consequence we present parameters for a separable potential to determine π-N and N-N phase shifts.

NUCLEAR REACTIONS Separable Bethe-Salpeter kernels reduced to six different quasipotential equations; application to pion-nucleon and nucleon-nucleon phase shifts; $E=0$–300 MeV; $l=0,1$.

I. INTRODUCTION

The large amount of experimental values in three body systems, cf. N-d and π-d, calls for a correct theoretical treatment of such systems. Much work has been done in the nonrelativistic framework of the Faddeev equations,[1] where separable potentials are used to parametrize the two-body subsystem interactions. Simultaneously, it has been recognized that in describing the interaction of two hadrons, relativistic effects have to be taken into account[2] even at low energies. A natural consequence of these facts would be relativistic three body calculations. Freedman *et al.*[3] have derived a formalism in analog to the Faddeev one, with the difference that the two body subsystems are described via separable Bethe-Salpeter (BS) kernels. The reason why explicit calculations have not yet been performed with these equations is that until recently[4,5] no parametrizations of separable kernels for the BS equation have been given.

A first step in the direction of relativistic three-body calculations, cf. in the π-d system, is the use of relativistic kinematics at least for the pion.[6] Recent work on relativistic three body scattering[7] is based on the Blankenbecler-Sugar (BBS) reduction scheme,[8] which reduces the dimension of the BS equation from four to three. One main result of those calculations[9] is the correct incorporation of the process $\pi NN \to NN$ into a three body formalism. Since the reduction of the four dimensional (or partial wave decomposed two dimensional) BS equation into a three dimensional (one dimensional) integral equation is not unique,[10] it is not at all clear why the BBS formalism should be used in future three body calculations. In the present work we address ourselves to six widely used reductions of the BS equation, where a simple separable kernel is considered. Section II gives a short review of the results of Refs. 4 and 5, where N-N and π-N phase shifts have been calculated within a separable approach to the BS equation. The reduction procedure of the partial wave decomposed BS equation to one dimensional quasipotential (QP) equations for the nonequal mass case and the application to separable BS kernels is presented in Sec. III. In Sec. IV we show the effect of several QP reductions of the BS equation in terms of the πN and NN phase shifts, and summarize our investigations with some concluding remarks (Sec. V).

II. SEPARABLE BS-EQUATION KERNELS

The partial wave decomposed BS equation in momentum space is given by[4]

$$T_l(q_0,q,q_0',q';s)=V_l(q_0,q,q_0',q')+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,V_l(q_0,q,k_0,k)G(k_0,k;s)T_l(k_0,k,q_0',q';s) \tag{2.1}$$

with

$$G(k_0,k;s)=[(k_0+a\sqrt{s}\,)^2-E_1^{\,2}+i\epsilon]^{-1}[(k_0-b\sqrt{s}\,)^2-E_2^{\,2}+i\epsilon]^{-1}\,. \tag{2.2}$$

V_l is the kernel of this two dimensional integral equation. Using the abbreviations $a=m_1/(m_1+m_2)$ and $b=m_2/(m_1+m_2)$, the relative momenta are given by

$$\hat{q}=(q_0,q)=a\hat{q}_2-b\hat{q}_1\,; \tag{2.3}$$

m_1 and m_2 are the masses of the two particles with momenta $\hat{q}_1$ and $\hat{q}_2$, respectively; and $\hat{k}$ and $\hat{q}'$ are the intermediate and final relative momenta defined analogously. The total energy $\sqrt{s}$ is given by

$$\sqrt{s}=(p^2+m_1^{\,2})^{1/2}+(p^2+m_2^{\,2})^{1/2}\equiv\bar{E}_1+\bar{E}_2 \tag{2.4}$$

and

$$E_1=(k^2+m_1^{\,2})^{1/2},\quad E_2=(k^2+m_2^{\,2})^{1/2}\,.$$

The amplitude T_l with all four legs on the mass and energy shell is related to the phase shifts by

$$T_l(p_0,p,p_0,p;s)=T_l(p)=-\frac{8\pi\sqrt{s}}{p}e^{i\delta_l(p)}\sin\delta_l(p)\,. \tag{2.5}$$

The BS kernel consists of an infinite sum of irreducible Feynman graphs. Even in the ladder approximation of the BS equation, elaborate numerical calculations are necessary to determine, for example, N-N or π-N scattering phase shifts.[11] In view of using these results in tractable three-body calculations, we have introduced a separable approximation for the BS kernel in Refs. 4 and 5:

$$V_l(q_0,q,k_0,k)=v_l(q_0,q)\lambda v_l(k_0,k)\,. \tag{2.6}$$

As a consequence it was possible to solve the BS equation in closed form

$$T_l(q_0,q,q_0',q')=v_l(q_0,q)v_l(q_0',q')/D_l(s) \tag{2.7}$$

with

$$D_l(s)=\lambda^{-1}-\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,v_l^{\,2}(k_0,k)G(k_0,k;s)\,. \tag{2.8}$$

To keep three-body calculations as simple as possible, a relativistic generalization of the Yamaguchi form factor[4,5] for the covariant form factor

$$v_l(k_0,k)=\frac{k^l}{[(k_0^{\,2}-k^2-\beta^2)^2+\gamma^4]^{\frac{l+1}{2}}} \tag{2.9}$$

was chosen. The integration in the k_0 plane was performed analytically. The free parameters β and γ and the coupling parameter λ of the "separable potential" were determined by fitting $D_l(s)$ to the "experimental" N-N and πN phase shifts.

In the case of nucleon-nucleon scattering we have considered partial waves with $l=0$ and $l=1$.[4] Partial waves with no sign change in the phase shift and no pole (corresponding to a bound state or a resonance) were fitted with a constant coupling parameter λ. To fix the zero in the phase shifts, for example, in the N-N $^1S_0$ partial wave, an energy dependent $\lambda\to\lambda(s_0-s)$ was introduced.

The analytic form of λ in the case of the n-p $^3S_1$ partial wave was chosen to be

$$\lambda\to\lambda\frac{s_0-s}{s-m_0^{\,2}}\,,$$

whereby s_0 guaranteed the zero in the phase shift, as in the $^1S_0$ wave. $m_0^{\,2}$ was an additional free parameter, which was determined by a subtraction procedure, introducing a pole in the amplitude at the binding energy of the deuteron.

The π-N phase shifts, calculated within this formalism, were presented in Ref. 5. While the so-called small phases S_{11}, S_{31}, P_{13}, and P_{31} were fitted in analogy to the N-N case, the π-N phases P_{11} and P_{33} were treated separately and have to be discussed here. It is well known that the resonant P_{33} partial wave can be parametrized by a Breit-Wigner,[12] or at least a modified Breit-Wigner form,[13] with noticeable success. Two conclusions can be drawn from this fact: Firstly, the amplitude at the resonance energy can be taken to be separable, i.e., a separable potential should determine this partial wave very well. Secondly, the amplitude should have an analytic form which is at least similar to the Breit-

Wigner parametrization. When using a separable potential in nonrelativistic scattering theory, it is therefore necessary to choose an energy dependent coupling parameter λ. In the case of the BS equation, the energy dependence is explicitly inherent in the zeroth dimension of the integral equation. Therefore, it was not necessary to introduce an energy dependent λ (Ref. 5) to yield excellent agreement with the experimental phase shift.

Mizutani *et al.*[14] have shown explicitly that different contributions of the P_{11} partial wave are inherent in different places of three body calculations; therefore, a two potential formalism is necessary to describe this partial wave. The importance of the effect of the absorption, inherent in the πN-P_{11} partial wave, on the low energy π-d scattering was pointed out extensively in Ref. 9.

This fact has been taken into account in Ref. 5 to calculate the P_{11} phase within the BS framework. In addition we point out that the agreement between the N-N and π-N phase shifts presented in Refs. 4 and 5 and the experimental phase shifts is excellent.

III. REDUCTIONS OF THE BS EQUATION TO ONE DIMENSIONAL INTEGRAL EQUATIONS IN THE NONEQUAL MASS CASE AND THEIR APPLICATION TO SEPARABLE BS KERNELS

Three-body calculations with equations of the Faddeev-type, using relativistic kinematics, have been performed for the case of pion-deuteron scattering.[9] To keep these investigations as simple as possible, separable potentials with Yamaguchi form factors have been used. To take into account "relativistic" effects, the Blankenbecler-Sugar (BBS) equation was introduced to describe the two body subsystems. Since the reduction of the BS equation from a four (two) dimensional integral equation to a three (one) dimensional one (cf. the BBS equation) is not unique, it is not clear at all that the BBS equation is the best possible choice for use in connection with a Yamaguchi form factor for the separable potential.

The results of Refs. 4 and 5 now enable us to investigate several reductions of the BS equation, assuming in each case a Yamaguchi form factor for the separable kernel. We have chosen six well-known quasipotential (QP) equations denoted by (A)—(F). (A) is the Gross equation,[15] (B) has been proposed by Erkelenz and Holinde,[2] (C) has been derived by Kadyshevsky,[16] (D) is due to Thompson,[17] (E) is the so-called BBS equation,[8] and (F) has been presented by Woloshyn and Jackson.[18]

Equations (A)—(F) have already been investigated in view of the ladder approximated BS equation by Woloshyn and Jackson.[18] Another comparison between a BS approach and the QP approach [Eq. (A)] has been performed for N-N scattering by Zuilhof and Tjon.[19] Since these investigations[18,19] have been restricted to the case of equal masses of both particles, we derive formulas for the nonequal mass case and apply them to the separable BS kernel.

It is possible to write two coupled integral equations which are equivalent to the BS equation (2.1):

$$T_l(q_0,q,q_0',q';s)=W_l(q_0,q,q_0',q')+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,W_l(q_0,q,k_0,k)g(k_0,k;s)T_l(k_0,k,q_0',q';s)\,, \tag{3.1}$$

$$W_l(q_0,q,k_0,k)=V_l(q_0,q,k_0,k)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0'\int_0^{\infty}k'^2dk'V_l(q_0,q,k_0'k')[G(k_0',k';s)-g(k_0',k';s)]W_l(k_0',k';k_0,k)\,. \tag{3.2}$$

g is chosen in such a way that unitarity (i.e., the left-hand cut) is taken into account and that the two dimensional integral equation (3.1) is reduced to a one dimensional integral equation. The solution of (3.1) and (3.2) is equivalent to the solution of Eq. (2.1), although it is even more complicated to solve Eqs. (3.1) and (3.2) than to solve Eq. (2.1) directly. The approximation, which is inherent in all QP equations, consists of a series expansion of W in terms of V and a truncation of this series after the first or second term. In addition to rapid convergence of this series, g has to fulfill two additional requirements: (i) unitarity of the total amplitude and (ii) reduction of the dimensions of the BS equation from four (two) to three (one).

Unitarity is guaranteed by approximating the left-hand cut in the amplitude by a pole in g. This can be done by several analytic expressions: (C) and (F) have chosen the form

$$g\sim[\sqrt{s}-\sqrt{s'}+i\epsilon]^{-1}$$

with

TABLE I. Quasipotential propagators used to reduce the Bethe-Salpeter equation for two particles with different masses.

Propagator	Arbitrary frame	c.m. frame
$g_{(A)}(k,s)$	$-2\pi i\int\frac{\sqrt{s'}}{\sqrt{s}}\frac{d\sqrt{s'}}{\sqrt{s}-\sqrt{s'}+i\epsilon}\delta^{(+)}[(aP+\hat{k})^2-m_1^2]\delta^{(+)}[(P'-aP-\hat{k})^2-m_2^2]$	$-\frac{\pi i(E_1+E_2)\delta[k_0+a\sqrt{s}-E_1]}{2\sqrt{s}E_1E_2[\sqrt{s}-(E_1+E_2)+i\epsilon]}$
$g_{(B)}(k,s)$	$-2\pi i\int\frac{ds'}{s-s'+i\epsilon}\delta^{(+)}[(aP+\hat{k})^2-m_1^2]\delta^{(+)}[(P'-aP-\hat{k})^2-m_2^2]$	$-\frac{\pi i(E_1+E_2)\delta[k_0+a\sqrt{s}-E_1]}{E_1E_2[s-(E_1+E_2)^2+i\epsilon]}$
$g_{(C)}(k,s)$	$-2\pi i\int\frac{d\sqrt{s'}}{\sqrt{s}-\sqrt{s'}+i\epsilon}\delta^{(+)}[(aP+\hat{k})^2-m_1^2]\delta^{(+)}[(P'-aP-\hat{k})^2-m_2^2]$	$-\frac{\pi i\delta[k_0+a\sqrt{s}-E_1]}{2E_1E_2[\sqrt{s}-(E_1+E_2)+i\epsilon]}$
$g_{(D)}(k,s)$	$-2\pi i\int\frac{\sqrt{s'}}{\sqrt{s}}\frac{d\sqrt{s'}}{\sqrt{s}-\sqrt{s'}+i\epsilon}\delta^{(+)}[(aP'+\hat{k})^2-m_1^2]\delta^{(+)}[(bP'-\hat{k})^2-m_2^2]$	$-\frac{\pi i(E_1+E_2)\delta[k_0-bE_1+aE_2]}{2\sqrt{s}E_1E_2[\sqrt{s}-(E_1+E_2)+i\epsilon]}$
$g_{(E)}(k,s)$	$-2\pi i\int\frac{ds'}{s-s'+i\epsilon}\delta^{(+)}[(aP'+\hat{k})^2-m_1^2]\delta^{(+)}[(bP'-\hat{k})^2-m_2^2]$	$-\frac{\pi i(E_1+E_2)\delta[k_0-bE_1+aE_2]}{E_1E_2[s-(E_1+E_2)^2+i\epsilon]}$
$g_{(F)}(k,s)$	$-2\pi i\int\frac{d\sqrt{s'}}{\sqrt{s}-\sqrt{s'}+i\epsilon}\delta^{(+)}[(aP'+\hat{k})^2-m_1^2]\delta^{(+)}[(bP'-\hat{k})^2-m_2^2]$	$-\frac{\pi i\delta[k_0-bE_1+aE_2]}{2E_1E_2[\sqrt{s}-(E_1+E_2)+i\epsilon]}$

$$s'=(E_1+E_2)^2 .$$

The other four QP equations differ by kinematical factors; i.e., in the case of (A) and (D) by the factor $\sqrt{s}/\sqrt{s'}$:

$$g\sim\frac{\sqrt{s'}}{\sqrt{s}}[\sqrt{s}-\sqrt{s'}+i\epsilon]^{-1}$$

and in the case of (B) and (F) by $[\sqrt{s}+\sqrt{s'}-i\epsilon]^{-1}$:

$$g\sim[s-s'+i\epsilon]^{-1} .$$

We would like to point out at this stage the importance of these different analytic expressions for our later results. To take into account the second requirement (ii), two types of δ functions were introduced for g. (A)–(C) made an unsymmetric reduction of the BS equation. For that purpose a product of two δ functions, which will put one particle on the mass shell and keep one off the mass shell, were considered. For the unequal mass case we set

$$\delta^{(+)}[(aP+\hat{k})^2-m_1^2]\delta^{(+)}[(P'-aP-\hat{k})^2-m_2^2] .$$

P is the four dimensional center of mass (c.m.) momentum $P=(s,\vec{0})$ and $P'=(\sqrt{s'}/\sqrt{s})P$; $\hat{k}$ denotes the four dimensional relative momentum $\hat{k}=(k_0,\vec{k})$. A different reduction of the BS equation, i.e., a so-called symmetric reduction of the BS equation, was introduced in (D)–(F). In these QP equations both particles are put on the mass shell by

$$\delta^{(+)}[(aP+\hat{k})^2-m_1^2]\delta^{(+)}[(bP-\hat{k})^2-m_2^2] .$$

The kinematical differences in the denominator of q combined with symmetric or unsymmetric reductions of the BS equation lead to the six QP equations (A)–(F). For the case of different masses of the two particles we present in Table I the appropriate propagators $g_{(A)}$–$g_{(F)}$ in an arbitrary frame and in the c.m. frame. The corresponding propagators for equal masses are easily derived from Table I and agree with the propagators given in Table I of Ref. 18.

In the case of a separable kernel (2.6) of the BS equation, the phase shifts (except for the π-p P_{11} partial wave) are related to the amplitude (2.7) via the denominator $D_l(s)$ (2.8) only:

$$\delta_l(p)=-\arctan\frac{\mathrm{Im}D_l(s)}{\mathrm{Re}D_l(s)} . \tag{3.3}$$

To study the influence of the reductions of the BS equations (A)–(F) in connection with a separable BS kernel we use the parameters λ, β, γ, and (s_0,m_0^2) of Refs. 4 and 5 and replace G by $g_{(A)-(F)}$ in (2.8):

TABLE II. Real parts of the denominator $D_l(s)$ of separable amplitudes, derived for different quasipotential equations.

Propagator	$\mathrm{Re}D_l(s)$ $(l=0,1)$
$g_{(A)}(k,s)$	$\lambda^{-1}-\frac{1}{8\pi^2}\int_0^\infty dk\frac{k^{2l+2}(E_1+E_2)}{\sqrt{s}E_1E_2[\sqrt{s}-(E_1+E_2)][((E_1-a\sqrt{s})^2-k^2-\beta^2)^2+\gamma^4]^{2l}}$
$g_{(B)}(k,s)$	$\lambda^{-1}-\frac{1}{4\pi^2}\int_0^\infty dk\frac{k^{2l+2}(E_1+E_2)}{E_1E_2[s-(E_1+E_2)^2][((E_1-a\sqrt{s})^2-k^2-\beta^2)^2+\gamma^4]^{2l}}$
$g_{(C)}(k,s)$	$\lambda^{-1}-\frac{1}{8\pi^2}\int_0^\infty dk\frac{k^{2l+2}}{E_1E_2[\sqrt{s}-(E_1+E_2)][((E_1-a\sqrt{s})^2-k^2-\beta^2)^2+\gamma^4]^{2l}}$
$g_{(D)}(k,s)$	$\lambda^{-1}-\frac{1}{8\pi^2}\int_0^\infty dk\frac{k^{2l+2}(E_1+E_2)}{\sqrt{s}E_1E_2[\sqrt{s}-(E_1+E_2)][((bE_1-aE_2)^2-k^2-\beta^2)^2+\gamma^4]^{2l}}$
$g_{(E)}(k,s)$	$\lambda^{-1}-\frac{1}{4\pi^2}\int_0^\infty dk\frac{k^{2l+2}(E_1+E_2)}{E_1E_2[s-(E_1+E_2)^2][((bE_1-aE_2)^2-b^2-\beta^2)^2+\gamma^4]^{2l}}$
$g_{(F)}(k,s)$	$\lambda^{-1}-\frac{1}{8\pi^2}\int_0^\infty dk\frac{k^{2l+2}}{E_1E_2[\sqrt{s}-(E_1+E_2)][((bE_1-aE_2)^2-k^2-\beta^2)^2+\gamma^4]^{2l}}$

$$D_l(s)=\lambda^{-1}-\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^\infty k^2dk\,v_l^2(k_0,k)g_{(A)-(F)}(k_0,k;s)\ . \tag{3.4}$$

Owing to the δ functions of $g_{(A)-(F)}$, the k_0 integration is trivial. The imaginary part of $D_l(s)$ is the same for all QP equations:

$$\mathrm{Im}D_l^{(A)-(F)}(s)=\frac{p^{2l+1}}{8\pi\sqrt{s}\,[((b\bar{E}_1-a\bar{E}_2)^2-p^2-\beta^2)^2+\gamma^4]^{2l}}\ , \tag{3.5}$$

$(l=0,1)$, and agrees with the result of the BS calculation.[4,5] This result reflects the fact that unitarity is not violated in replacing G by $g_{(A)-(F)}$. The real parts of $D_l^{(A)-(F)}(s)$ are given in Table II. To determine the π-p partial wave P_{11}, we have used a two potential formalism in Ref. 5; the derivation of the corresponding formulas for the QP equations (A)—(F) are given in the Appendix.

IV. THE INFLUENCE OF SEVERAL REDUCTIONS OF THE BS EQUATION ON N-N AND π-N PHASE SHIFTS

We present calculated π-N and N-N phase shifts with $l=0$ and $l=1$ in Figs. 1—10. The dashed lines in Figs. 1—9 represent the BS calculations of Refs. 4 and 5; the agreement of these phase shifts with the experimental phases[20,21] is satisfactory. In addition,

$$I(s)=\mathrm{Re}\left[\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^\infty k^2dk\,v_l^2(k_0,k)G(k_0,k;s)\right]\ , \tag{4.2}$$

we have found that

$$I_{BS}(s)<I_{(A)-(F)}(s) \tag{4.3}$$

for all QP Eqs. (A)—(F). Inequality (4.3) can be interpreted as an increasing potential strength of the QP approach in comparison to the BS approach.

Figs. 1—9 show results calculated with the QP equations (A)—(F). In all these calculations the values of the parameters λ, β, and γ are fixed at the values of the BS approach.

A. Partial waves with $l=0$

1. The π-p S_{31} phase shift. This is shown in Fig. 1. We have calculated this phase shift, which is due to a repulsive force, in the energy range between $T_\pi^{\mathrm{lab}}=0$ and 300 MeV. The QP equations produce less repulsion than the BS equation. Considering the real part of Eq. (2.8), which can be written as

$$\mathrm{Re}D_l(s)=\lambda^{-1}-I(s)\ , \tag{4.1}$$

with

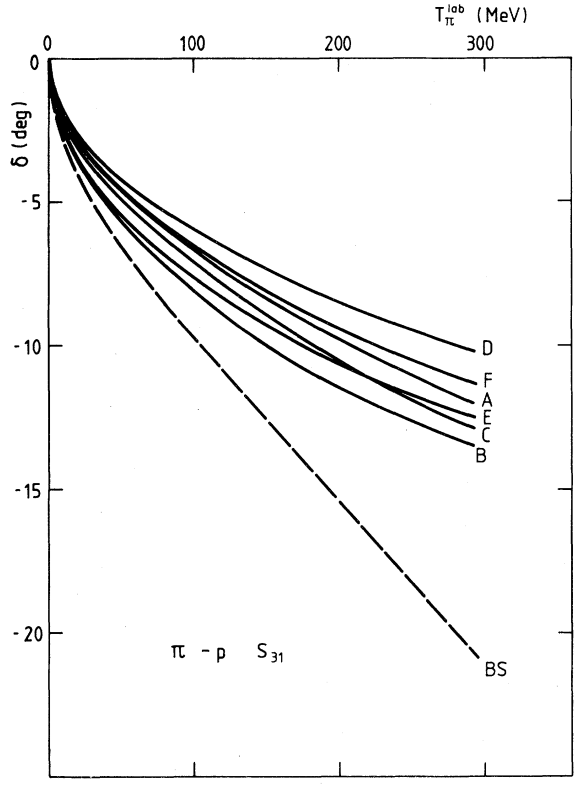

FIG. 1. π-N and N-N phase shifts for $l=0,1$. The dashed lines correspond to calculations using the Bethe-Salpeter (BS) equation (Refs. 4 and 5). The solid lines represent the results of several quasipotential equation (A)—(F) reductions of the BS equation.

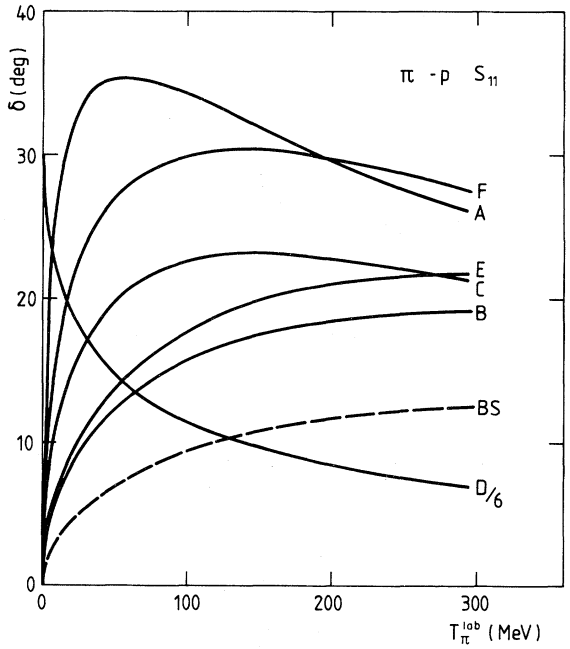

FIG. 2. See the caption to Fig. 1.

For the case of the π-p S_{31} wave this results in "less repulsive forces" in the QP equations. It can be seen from Fig. 1 that the QP equations (B) and (E) produce phase shifts which are nearer the BS result than do the QP Eqs. (A), (C), (D), and (F). The differences between (A)—(F) are mainly due to the different choices of the denominators of the propagators $g_{(A)}$—$g_{(F)}$. We can see that the propagators using a denominator

$$[s-(E_1+E_2)^2+i\epsilon]$$

give results closer to the BS results than do propagators with

$$\frac{\sqrt{s'}}{\sqrt{s}}[\sqrt{s}-(E_1+E_2)+i\epsilon]^{-1}$$

or

$$[\sqrt{s}-(E_1+E_2)+i\epsilon]^{-1}\,.$$

In addition, we can deduce from Fig. 1 that the choice of the denominator is more important than the choice of an unsymmetric or symmetric reduction of the zeroth component in the BS kernel.

2. The π-p S_{11} phase shift. Since this phase shift arises from an attractive force, one might expect from the S_{31} discussion that the QP equations lead to phase shifts showing more attraction than the BS result. The conclusions drawn for the S_{31} partial wave can be directly used as explanations of the QP equation results for the S_{11} partial wave in Fig. 2. While (B) and (E), and with some restrictions (C) and (F), show a similar behavior to that of the BS result over the energy range $T_\pi^{lab}=0$—300 MeV (although more attractive), the kinematical factors in the QP Eqs. (A) and (D) lead to totally different forms for the phase shifts. Especially for the QP Eq. (D) the "attraction" due to λ is so large that the phase shift starts at 180° instead of 0°.

3. The n-p $^1S_0$ phase shift. We have plotted $^1S_0$ phase shifts, calculated with the BS formalism and with the QP Eqs. (A)—(F), in Fig. 3. While the results of the QP Eqs. (B) and (E) reflect the behavior of the BS phase, all other phase shifts start at 180°. This fact can be illuminated by a statement from Ref. 22: "As the potential weakens the bound state moves down the imaginary axes and produces an antibound state or a resonance." The opposite holds for the $^1S_0$ QP results. Owing to the choice of $\lambda\to\lambda$ (s_0-s), all $^1S_0$ phase shifts change their sign at the same energy.

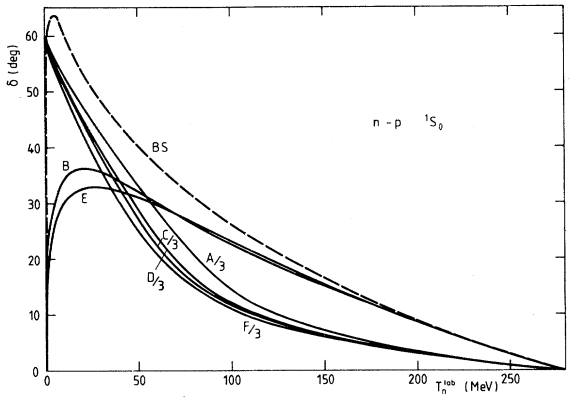

FIG. 3. See the caption to Fig. 1.

4. The n-p $^3S_1$ phase shift. The BS result was calculated using an energy dependent coupling parameter λ of the form

$$\lambda \frac{s_0 - s}{s - m_0^2}$$

and is presented together with the results of the QP Eqs. (A)—(F) in Fig. 4. The form of λ was chosen in such a way that a pole in the amplitude at the bound state energy and a zero in the amplitude at the zero of the phase shift are guaranteed. These two constraints allow no large deviations between the QP results and the BS calculation. As in all partial waves mentioned above, (B) and (E) are nearest to the BS result.

To summarize the conclusions to be drawn from consideration of π-N and N-N partial waves with $l=0$: (a) QP equations using a propagator of the form

$$[s-(E_1+E_2)^2+i\epsilon]^{-1}$$

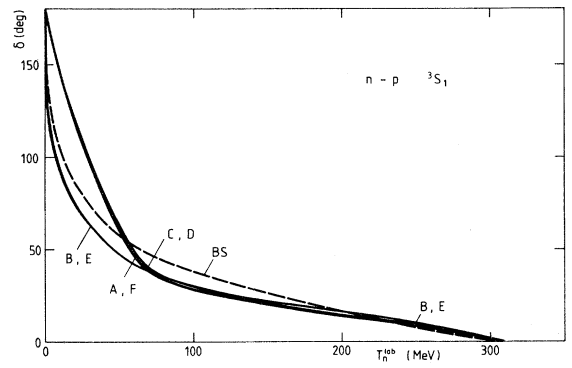

FIG. 4. See the caption to Fig. 1.

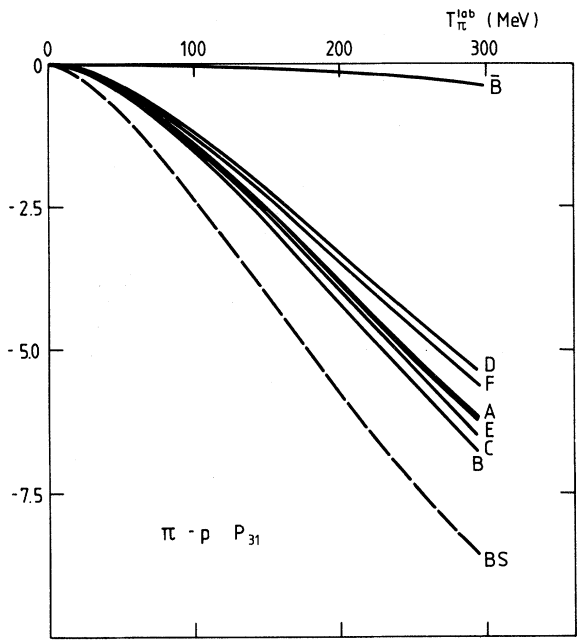

FIG. 5. See the caption to Fig. 1.

are superior over QP equations with different denominators (provided the same kernel is used); (b) the choice of the reduction (symmetric or unsymmetric) of the dimension of the BS equation has less influence on the results.

We want to point out that these statements are definitely true only for the special separable kernel (with Yamaguchi-type form factors).

B. Partial waves with $l=1$

1. The π-$p+P_{31}$, π-p P_{13}, and n-p $^1P_1$ phase shifts. What we have said for partial waves with $l=0$ is also true for the small π-N phases P_{31} and P_{13} [cf. the n-p $^1P_1$ phase shift]. We have displayed our results for those partial waves in Figs. 5—7. It is worthwhile to point out that the "symmetric" QP Eq. (E) is superior to the "unsymmetric" QP Eq. (B) for equal masses and conversely, as might be expected. Nevertheless the difference between (B) and (E) is as small as for the $l=0$ case. For illustration only we have plotted in Fig. 5, in addition to phase shifts which are calculated from the BS and QP Eqs. (A)—(E), a phase shift denoted by $\bar{B}$. This is the result of the QP Eq. (B), where the lighter particle is put on the mass shell instead of the more correct treatment which puts the heavier particle on the mass shell.

2. The π-p P_{33} phase shift. Concerning the resonant partial wave P_{33} two investigations have been performed. In Ref. 5 we used a constant coupling parameter λ for the separable BS kernel. This

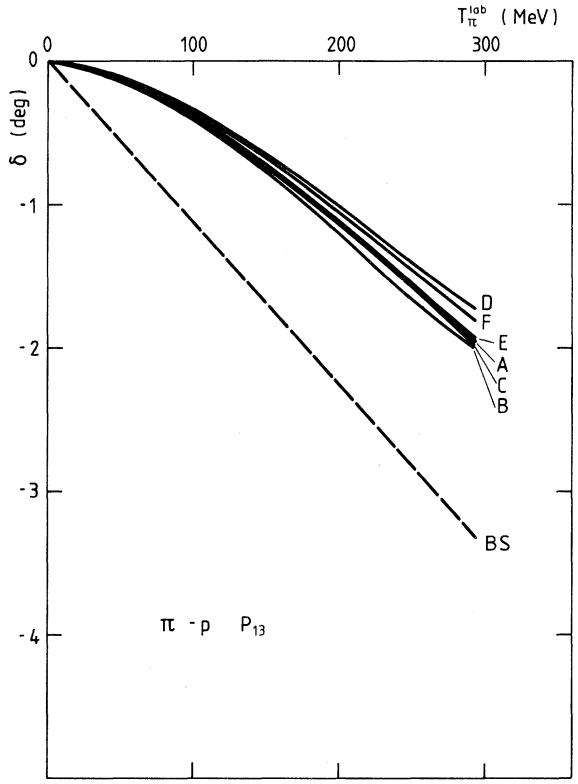

FIG. 6. See the caption to Fig. 1.

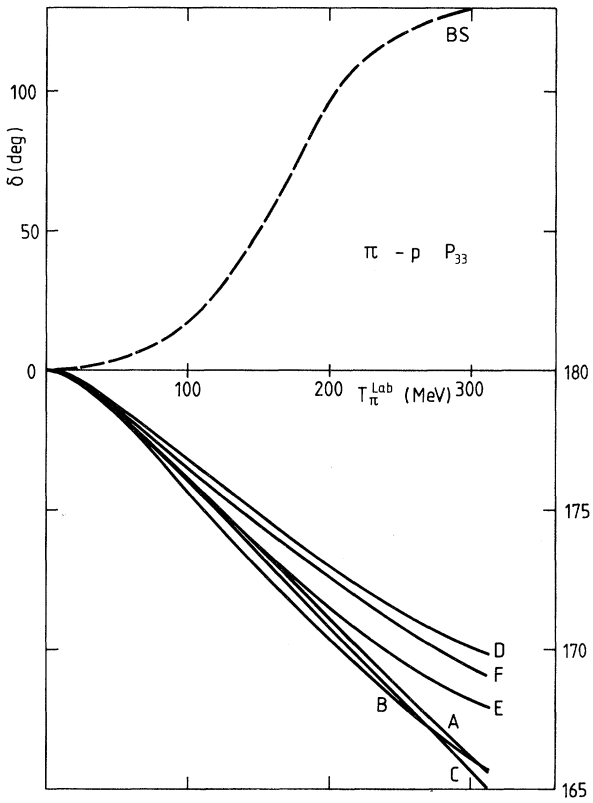

FIG. 8. See the caption to Fig. 1.

result as well as the QP equation calculations are shown in Fig. 8. We have mentioned the fact that in each partial wave the reduction of the BS equation to a QP equation leads to more attraction. The same is true for the P_{33} wave; the resonance switches over to a bound state with "increasing potential strength." Considering a type of a Breit-Wigner parametrization for the P_{33} partial wave it is well known[13] that an energy dependent coupling parameter has to be used to incorporate the correct threshold behavior and the different contributions of cuts which are not far from the Δ resonance.

Accordingly, we have performed a second fit to the experimental phase shift, where we have used an energy dependent λ of the form $\lambda(s-m_0^2)^{-1}$ for the separable BS kernel. The parameters λ, β, and γ are given in Table III; the BS result and the QP approximations (which start now correctly at 0°) for the P_{33} phase shift are displayed in Fig. 9. As in all other partial waves, the QP Eq. (B) produces a result which is nearer to the BS calculations.

3. The π-p P_{11} phase shift. We used a two potential formalism in Ref. 9 to determine the π-p P_{11} partial wave within the BS approach. The explicit equations as well as the transition to QP equations are given in the Appendix. The scattering phase

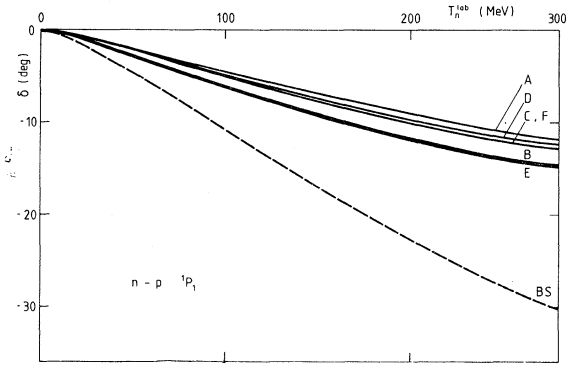

FIG. 7. See the caption to Fig. 1.

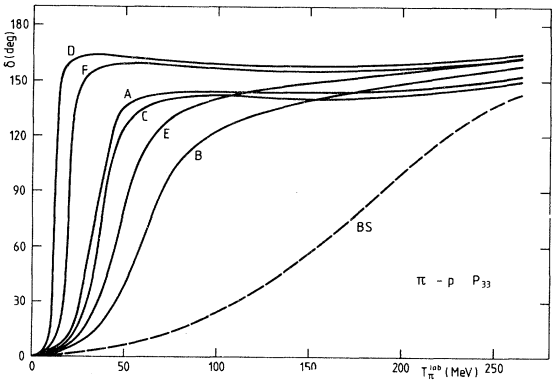

FIG. 9. See the caption to Fig. 1.

TABLE III. Parameters λ, β, and $\gamma(s_0)$ fitted to π-N and N-N phase shifts calculated using the BS equation and the quasipotential equation of Erkelenz and Holinde (Ref. 2).

Partial waves	Potential parameters	QP Eq. (B)	BS Eq.
π-p S_{31}	λ	1 350.259 fm^{-4}	931.819 fm^{-4}
	β	0.610 fm^{-1}	0.610 fm^{-1}
	γ	2.298 fm^{-1}	2.298 fm^{-1}
π-p S_{11}	λ	−2 441.251 fm^{-4}	−3 647.086 fm^{-4}
	β	2.151 fm^{-1}	2.151 fm^{-1}
	γ	2.916 fm^{-1}	2.916 fm^{-1}
n-p $^1S_0$	λ	−783.123 fm^{-2}	−644.796 fm^{-2}
	β	2.819 fm^{-1}	2.820 fm^{-1}
	γ	0.293 fm^{-1}	0.528 fm^{-1}
	S_0	104.104 fm^{-2}	104.104 fm^{-2}
n-p $^3S_1$	λ	1 512.416 fm^{-4}	1 052.500 fm^{-4}
	β	1.055 fm^{-1}	1.055 fm^{-1}
	γ	0.560 fm^{-1}	0.560 fm^{-1}
	S_0	105.598 fm^{-2}	105.598 fm^{-2}
π-p P_{31}	λ	3 466.034 fm^{-6}	2 468.943 fm^{-6}
	β	0.547 fm^{-1}	0.547 fm^{-1}
	γ	2.006 fm^{-1}	2.006 fm^{-1}
π-p P_{13}	λ	22 052.010 fm^{-6}	11 914.030 fm^{-6}
	β	1.304 fm^{-1}	1.304 fm^{-1}
	γ	2.872 fm^{-1}	2.872 fm^{-1}
n-p $^1P_1$	λ	214 701.500 fm^{-6}	429 284.031 fm^{-6}
	β	1.569 fm^{-1}	3.188 fm^{-1}
	γ	2.485 fm^{-1}	0.424 fm^{-1}
π-p P_{33}	λ		3 864.839 fm^{-6}
	β		1.256 fm^{-1}
	γ		1.670 fm^{-1}
π-p P_{33}	λ	8 105.780 fm^{-8}	1 456.500 fm^{-8}
	β	0.840 fm^{-1}	0.360 fm^{-1}
	γ	1.320 fm^{-1}	1.050 fm^{-1}
	m_0^2	39.056 fm^{-2}	35.106 fm^{-2}
π-p P_{11}	λ	6 578.256 fm^{-8}	148.020 fm^{-6}
	β	0.360 fm^{-1}	1.317 fm^{-1}
	γ	1.712 fm^{-1}	0.697 fm^{-1}
	m_0^2	48.744 fm^{-2}	
	Λ_P	2 087.918 fm^{-8}	13 682.701 fm^{-4}
	β_P	0.760 fm^{-1}	0.870 fm^{-1}
	γ	1.230 fm^{-1}	1.370 fm^{-1}

shifts δ_p^{BS} and δ_{NP}^{BS} (shown in Fig. 10) are results of Ref. 5 and represent pion absorption and σ and ρ exchange, respectively. The phases δ_P^B and δ_{NP}^B are results of the reduction of the BS approach[5] to QP Eq. (B). It is important to notice that the so-called Roper resonance around $T^{lab}=400$ MeV, created via δ_{NP}^{BS} with an energy independent coupling parameter λ,[5] leads to QP results which are similar to the P_{33} case (Fig. 8). To be specific: δ_{NP}^{QP} starts at 180° instead of 0° and produces a bound state instead of a

resonance. Since the QP results of (B) are representative for all other reductions of the BS approach, we have given for clarity δ_P^B and δ_{NP}^B only in Fig. 10. The introduction of an energy dependent coupling parameter λ in the BS approach results in attractive phase shifts δ_{NP}^{QP}, although they differ very much from δ_{NP}^{BS} in analogy to Fig. 9. A detailed study of the P_{11} partial wave, including refits of the QP approaches (A)—(F), is under investigation.

V. CONCLUDING REMARKS

It is obvious from Figs. 1—10 that the difference between phases, calculated with the BS equation and phases, derived by a QP approximation, is large. We have therefore investigated the effect of the second order of V in Eq. (3.2):

$$W_l(q_0,q,k_0,k)=V_l(q_0,q,k_0,k)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0'\int_0^{\infty}k'^2dk'V_l(q_0,q,k_0',k')[G(k_0',k';s)-g_{(A)-(F)}(k_0',k';s)]V_l(k_0',k',k_0,k) \tag{5.1}$$

on phase shifts calculated by QP Eqs. (A)—(F). Equation (5.1) effects the real part of $D_l(s)$ only and can be written as a combination of $\mathrm{Re}D_l^{BS}$ calculated from the BS equation and $\mathrm{Re}D_l^{(A)-(F)}$ derived from the "first order" QP approximation ($W\sim V$):

$$\mathrm{Re}D_l(s)=\lambda^{-1}\{1+\lambda[\mathrm{Re}D_l^{(A)-(F)}(s)-\mathrm{Re}D_l^{BS}(s)]\}^{-1}-\lambda^{-1}+\mathrm{Re}D_l^{(A)-(F)}(s)\ . \tag{5.2}$$

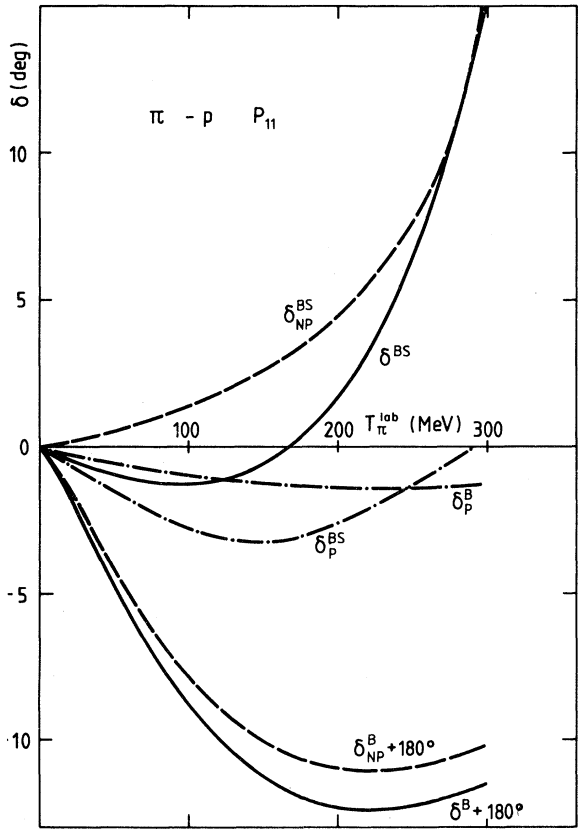

FIG. 10. The solid lines $\delta^{BS}(\delta^B)$ represent π-p P_{11} phase shifts, calculated using a Bethe-Salpeter [quasipotential (B)] formalism, which can be decomposed into a phase shift $\delta_P^{BS}(\delta_P^B)$ corresponding to the nucleon pole and a phase shift $\delta_{NP}^{BS}(\delta_{NP}^B)$ due to σ and ρ meson exchanges.

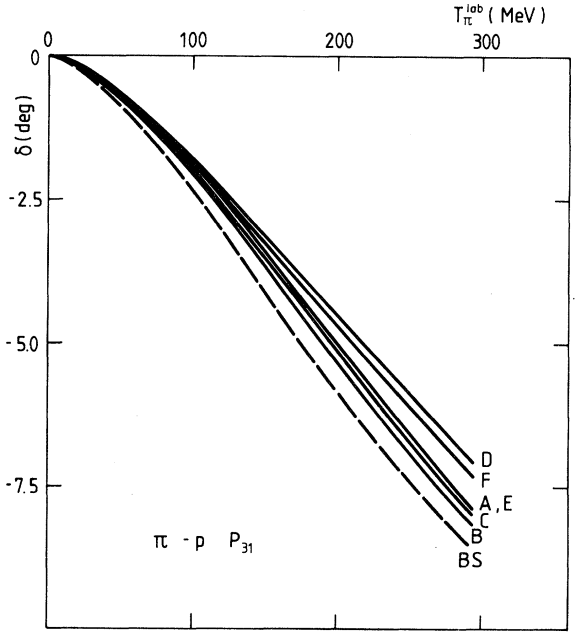

FIG. 11. π-N and N-N phase shifts. The dashed lines correspond to calculations using the Bethe-Salpeter equation (Refs. 4 and 5). The solid lines (A)—(F) display results calculated using "second order" quasipotential equation on (A)—(F).

The results for the n-p $^1S_0$ and π-p P_{31} are shown in Figs. 11 and 12. As expected, these QP results are nearer but not equal to the BS results.

We may summarize our investigations as follows: Using a separable potential with a Yamaguchi-type form factor (which is the most widely used form factor for π-d and N-d three-body Faddeev calculations), it is very important to choose a QP equation which replaced the two body Green's function by a propagator of the form

$$[s-(E_1+E_2)^2+i\epsilon]^{-1} ,$$

cf. (B) or (E). It can be seen from Figs. 1—12 that πN and NN phase shifts derived from QP Eqs. (B) and (E) are closer to "exact" results than are phases calculated from (A), (C), (D), or (F). As a consequence, we might state that a symmetric or unsymmetric reduction of the BS equation to a QP equation is less important in comparison to the proper choice of the denominator of the propagator. For the same denominators an unsymmetric reduction is superior for the case of π-N while the converse is true for N-N scattering. Considering the overall agreement of πN and NN phase shifts with the BS results (Figs. 1—10) we would tend to favor the QP Eq. (B) for further three body calculations. For that purpose we have refitted all πN and NN phase shifts in the QP formalism of Ref. 2 [(B)]. The parameters λ, β, γ, and (s_0) for the QP Eq. (B) and for the BS equation are given in Table III, the agreement of the phase shifts [refitted with (B)] with the experimental phases is satisfactory and of the quality of the BS results.

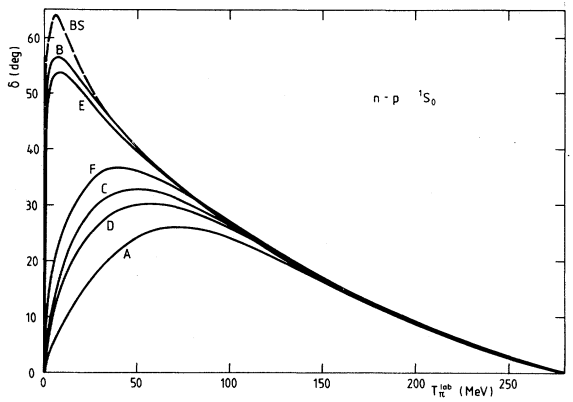

FIG. 12. See the caption to Fig. 11.

ACKNOWLEDGMENTS

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APPENDIX

It was pointed out in Ref. 14 that a two potential formalism is necessary to describe the π-p P_{11} partial wave. In terms of the Feynman diagrams one has the exchange of σ and ρ mesons in the t channel on one hand, and pion absorption, i.e., the s-channel nucleon pole, on the other. Therefore, we decompose the total T operator (the dependence on $l=1$ is suppressed)

$$T=T_{\mathrm{NP}}+T_P , \tag{A1}$$

where T_P contains the s-channel nucleon pole and correspondingly, T_{NP} is s-channel one particle irreducible.

Using a separable ansatz for T_{NP} [analogous to (2.7)], the one particle irreducible part has to be unitary, which is ensured by

$$D_{\mathrm{NP}}(s)=\lambda^{-1}-\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,v'^2(k_0,k)G(k_0,k;s) . \tag{A2}$$

The second term in T will generate the nucleon pole term T_P through

$$T_P(q_0,q,q_0',q';s)\equiv\frac{h(q_0,q;s)h(q_0',q';s)}{D_P(s)} \tag{A3}$$

with

$$D_P(s)=\Lambda^{-1}(s)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k^2dk\,v(k_0,k)G(k_0,k;s)h(k_0,k;s) \tag{A4}$$

and

$$h(q_0,q;s)=v(q_0,q)+\frac{i}{4\pi^3}\int_{-\infty}^{\infty}dk_0\int_0^{\infty}k'^2dk'T_{\mathrm{NP}}(q_0,q,k'_0,k';s)G(k'_0,k';s)v(k'_0,k')\ ; \tag{A5}$$

v' and v are defined as in (2.9). It is obvious that Eq. (3.3) has to be modified to

$$\delta(p)=\arctan\frac{\mathrm{Im}T_P(p)}{\mathrm{Re}T_P(p)}+\arctan\frac{\mathrm{Im}D_{\mathrm{NP}}(s)}{\mathrm{Re}D_{\mathrm{NP}}(s)}\ . \tag{A6}$$

The phase shifts δ_{NP} and δ_P corresponding to the potentials V_{NP} and V_P, calculated in Ref. 5 are given in Fig. 10. To study the effect of reductions of the BS approach to QP equations, we only have to replace G by $g_{(\mathrm{A})-(\mathrm{F})}$ in Eqs. (A4) and (A5).

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