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Unified theory of nuclear reactions

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A theory of nuclear reactions and structures is proposed which unifies and generalizes several different approaches, such as the method of coupled reaction channels and the distorted wave Born approximation for rearrangement processes, the method of coupled channels array, the resonating group method, the Feshbach theory for the optical potential, and the multiple scattering formalism for high energy processes. The present theory employs an optical potential which can be systematically improved in a controlled way.

[NUCLEAR REACTIONS The resonating group method; the method of coupled reaction channels; antisymmetrized optical potential.]

The method of coupled reaction channels[1,2] (CRC) and a truncated version of the coupled channels array[3,4] (CCA), known as the bound state approximation[5,6] to the CCA (the BCCA), are supposed to be two powerful methods for treating nuclear reactions. The CRC has enjoyed widespread use[7] because of its phenomenological success in describing nuclear reactions, whereas the BCCA has its limited success.[6] The usual derivation[5,6] of the CRC employs an expansion of the wave function in an overcomplete and nonorthogonal set of basis states and a subsequent projection of the Schrödinger equation on these basis states. The use of such overcomplete and nonorthogonal basis states leads to serious numerical and mathematical problems and prohibits the introduction of correction terms.[5,6] The usual derivation of the BCCA, on the other hand, does not use such overcomplete and nonorthogonal basis states and indicates how one can, at least in principle, introduce[5] correction terms to the BCCA. It is because of this that various authors conjectured[5] mathematical superiority of the BCCA over the CRC. Yet the CRC and even a drastic approximation to it, the distorted wave Born approximation[1,7] (DWBA), have become very popular in treating various aspects of nuclear reactions. The success obviously is not purely accidental.

We also have the highly successful resonating group method[8] (RGM) for incorporating the Pauli principle for identical nucleons in the study of nuclear structure and reaction problems. The RGM (Ref. 8) is based on the same ansatz as the CRC (Refs. 1 and 2) and it also expands the antisymmetrized wave function in a set of overcomplete and nonorthogonal set of basis states, which again does not allow the introduction of correction terms. Yet the RGM and an approximate version of it, the orthogonality condition model[9] (OCM), have been very successful in explaining certain aspects of nuclear reactions. Again the success is not merely accidental.

The Feshbach unified theory of nuclear reaction[10] is the first work which puts the optical potential phenomenology in a solid theoretical framework. The Feshbach optical potential has some nice properties.[10] Its discrete singularity corresponds to compound resonance, and it is real below the inelastic threshold. But unfortunately, from the point of view of phenomenology, the Feshbach optical potential is not very useful, as it cannot treat the antisymmetrization of identical nucleons in a simple way, nor can it justify the success of methods such as the CRC,[1,2] the DWBA,[1] the RGM,[8] and the OCM.[9] It is desirable to have a generalization of the Feshbach theory which will yield the CRC and the RGM in the lowest order of an approximation scheme.

The multiple scattering (MS) theory[11] is a successful theory for treating high energy processes. It reduces the original many-body scattering equation to a two-body scattering equation with an optical potential and a MS series for the optical potential. Such a MS series converges at high energy and can be used to introduce correction terms in a controlled way.

In the present paper we propose a theory for nuclear reactions and bound states which enjoys the advantages of all the above-mentioned methods, but is not plagued by their disadvantages. We provide a unified derivation of the BCCA and the CRC and hence show that the CRC is in fact a special type of

BCCA. With this derivation any conjecture of theoretical superiority[5,6] of the BCCA over the CRC becomes baseless. Specifically, the present derivation uses orthogonal projection operators only in order to deduce the CRC and this allows one to use Feshbach's projection operator technique[10] for introducing correction terms to the CRC. At least this will give a measure of the error introduced in a CRC or a DWBA calculation[7] as compared to a complete solution of the Schrödinger equation.

As the CRC and the RGM are both based on the same ansatz, similar considerations for the case of identical particles lead to a new derivation of the RGM which will allow one to introduce correction terms to the RGM intercluster potential in a controlled way through a MS series. This generalizes the Feshbach theory to the case of identical particles.

In order to illustrate the present theory we consider a three-particle system with the Hamiltonian

$$H=H_0+V\equiv H_0+\sum_{i=1}^{3}V_i\ , \tag{1}$$

where H_0 is the kinetic energy operator and the V_i's, $i=1,2,3$, are the three pair potentials. We also introduce the potential $\bar{V}_i=V_j+V_k$ $(i\neq j\neq k\neq i)$. The simplest form of the CCA equations[4]—the Faddeev equation[3]—breaks the full wave function ψ into components ψ_i^F, $i=1,2,3$, which satisfy

$$(E-H_i)\psi_i^F=V_i\sum_{k\neq i}\psi_k^F,\quad k=1,2,3\ , \tag{2}$$

where $H_i=H_0+V_i$,

$$\psi=\sum_{i=1}^{3}\psi_i^F\ .$$

Equation (2) is commonly known as the differential form of the Faddeev equation. This is not the only possible way to break up ψ into three components.[4] Here we introduce another special way to break up ψ into three components ψ_i^C, $i=1,2,3$, which satisfy

$$(E-H_i)\psi_i^C=-(E-H_i)\sum_k\bar{\delta}_{ik}\psi_k^C + \bar{V}_i\sum_k\psi_k^C,\quad k=1,2,3\ , \tag{3}$$

where $\bar{\delta}_{ik}=(1-\delta_{ik})$ and where

$$\psi=\sum_{i=1}^{3}\psi_i^C\ .$$

Equations (3) are a very special type of CCA equation. It is easy to verify that though Eqs. (3) form a coupled set, each equation in this set, and hence also their sum, corresponds to the Schrödinger equation; whereas in the case of the CCA equations (2) only the sum corresponds to the Schrödinger equation. We shall see that the approximation which reduces Eq. (2) to the BCCA will reduce Eq. (3) to the CRC. Now it is easy to realize that one can rewrite Eqs. (2) and (3) in the form of the following matrix equation[5]:

$$(E\underline{1}-\underline{H}_0)\underline{\Psi}=\underline{V}\underline{\Psi}\ , \tag{4}$$

where $\underline{\Psi}$ is a column vector with components ψ_i^F, $i=1,2,3$, in the case of Eq. (2) and with components ψ_i^C, $i=1,2,3$, in the case of Eq. (3). $\underline{H}_0$ is a matrix operator with components $(\underline{H}_0)_{ik}=H_i\delta_{ik}$. $\underline{1}$ is the unit operator in the three-dimensional matrix space; and the matrix operator $\underline{V}$ is defined by

$$\underline{V}_{ik}=V_i\bar{\delta}_{ik} \tag{5}$$

in the case of Eq. (2), and by

$$\underline{V}_{ik}=-(E-H_i)\bar{\delta}_{ik}+\bar{V}_i \tag{6}$$

in the case of Eq. (3). Now one introduces the projection operator $\underline{P}$ by $\underline{P}_{ik}=P_i\delta_{ik}$, where the P_i's, $i=1,2,3$, denote the projectors onto the subspace of bound states of two-body subsystems, such that $P_i^2=P_i$ and $\underline{P}^2=\underline{P}$. Now the Feshbach projection operator technique when applied to Eq. (4) yields

$$(E\underline{1}-\underline{P}\,\underline{H}\,\underline{P})\underline{P}\underline{\Psi}=\underline{P}\,\underline{H}\underline{Q}\underline{\Psi}\ , \tag{7}$$

$$(E\underline{1}-\underline{Q}\underline{H}\underline{Q})\underline{Q}\underline{\Psi}=\underline{Q}\underline{H}\,\underline{P}\,\underline{\Psi}\ , \tag{8}$$

where $\underline{H}=\underline{H}_0+\underline{V}$ and $\underline{Q}=\underline{1}-\underline{P}$. The zeroth order approximation neglects the coupling between $\underline{P}$ and $\underline{Q}$ spaces and assumes $\underline{\Psi}=\underline{P}\underline{\Psi}$, $\underline{Q}\underline{\Psi}=0$. In this case Eqs. (7) and (8) reduce to

$$(E\underline{1}-\underline{P}\,\underline{H}\,\underline{P})\underline{P}\underline{\Psi}=0\ ,$$

which in explicit notation becomes

$$(E-P_iH_iP_i)P_i\psi_i^F=\sum_k\bar{\delta}_{ik}P_iV_iP_k\psi_k^F \tag{9}$$

in the case of Eq. (2), and

$$(E-P_iH_iP_i)P_i\psi_i^C=\sum_k P_i[\bar{V}_i-(E-H_i)\bar{\delta}_{ik}]P_k\psi_k^C \tag{10}$$

in the case of Eq. (3). Equation (9) is one type of BCCA equation. Explicitly Eq. (10) expands ψ in terms of bound states of two-body subsystems,

$$\psi=\sum_{i=1}^{3}P_i\psi_i^C$$

and it is easy to realize that Eq. (10) is the CRC equation for the expansion coefficients. In order to perform a complete calculation for $\underline{P}\underline{\Psi}$ one can eliminate the $\underline{Q}$ space between Eqs. (7) and (8) and arrive at

$$(E\underline{1}-\underline{P}\,\underline{H}_0\underline{P}-\underline{P}\,\underline{U}\,\underline{P})\underline{P}\underline{\Psi}=0\ , \tag{11}$$

where

$$\underline{P}\,\underline{U}\,\underline{P}=\underline{P}\,\underline{V}\,\underline{P}+\underline{P}\,\underline{V}\underline{Q}(E\underline{1}-\underline{Q}\underline{H}\underline{Q}+i\epsilon)^{-1}\underline{Q}\underline{V}\,\underline{P} \tag{12}$$

is the optical potential to be used in Eq. (11). The lowest order approximation to the optical potential given by $\underline{P}\,\underline{U}\,\underline{P}=\underline{P}\,\underline{V}\,\underline{P}$ will correspond to Eq. (9) in the case of Eq. (2) and to Eq. (10) in the case of Eq. (3). It is easy to realize that $\underline{U}$ given by Eq. (12) satisfies the dynamical equation

$$\underline{U}=\underline{V}+\underline{V}\underline{Q}(E\underline{1}-\underline{Q}\underline{H}_0\underline{Q}+i\epsilon)^{-1}\underline{Q}\underline{U}\ . \tag{13}$$

One should remember that as Eqs. (2) and (3) are differential equations, their solutions need a specification of the usual boundary condition, e.g., for a two-cluster initial state in channel i, asymptotically ψ_k^F and ψ_k^C have outgoing waves for all k, and, in addition, an incoming wave for $k=i$. This boundary condition should be contained in Eqs. (12) and (13). In the case of a self-adjoint Hamiltonian such boundary conditions are maintained by the $+i\epsilon$ prescription. Equation (6) yields a self-adjoint Hamiltonian $\underline{H}$, and hence in this case boundary conditions are maintained by the $+i\epsilon$ prescription. In the case of Eq. (5) the Hamiltonian $\underline{H}$ is not self-adjoint, but Eq. (13) has a connected kernel, and the boundary condition is automatically taken care of. Of course, it does not mean that for any $\underline{Q}$ Eq. (13) is readily solvable numerically, because then Eq. (13) may have a disconnected kernel. However, in the case of a realistic many-body problem with $\underline{V}$ defined by Eq. (6), Eq. (13) may have a connected kernel for various approximations to $\underline{Q}$, but in the limit as $\underline{Q}\to\underline{1}-\underline{P}$ its kernel is disconnected. Our objective is, however, not to find the complete solution $\underline{\Psi}$ of the problem, but to use Eq. (11) to introduce breakup corrections to the CRC amplitudes by using connected approximations to $\underline{PUP}$ through Eq. (12), such that Eq. (11) is a meaningful equation with well-defined solutions.

We have presented a theory of nuclear reactions which treats the CRC and the BCCA on the same footing. When the lowest order approximation to $\underline{U}$ given by $\underline{P}\,\underline{U}\,\underline{P}=\underline{P}\,\underline{V}\,\underline{P}$ is a good approximation to the full solution of Eq. (13) (or to the complete solution of the Schrödinger equation) it is expected that the contribution of the last term on the right-hand side of Eq. (12) will be small. In this case, one can hopefully construct a convergent MS (or iterative) series for Eq. (13) through which one can systematically improve on the BCCA or the CRC in a controlled way. So we have been able to combine the attractive features of the CRC, the BCCA, the Feshbach theory, and the MS formalism in the present theory.

As the CRC and the RGM are based on the same ansatz, the above derivation of the CRC is easily extendable to the case of the RGM. In the case of three identical particles one starts with the Schrödinger equation $(E-H)\psi^a=0$, where ψ^a is fully antisymmetrized (symmetrized) for three fermions (bosons). Here $\psi^a\equiv\mathscr{A}_i\psi_i$, where $\mathscr{A}_i$ is the antisymmetrizer[12] (symmetrizer) for fermions (bosons), and where ψ_i describes the physical scattering process of three particles when particle i is incident on the bound state of particles j and k ($j\neq k\neq i\neq j$). ψ^a is an appropriate linear combination of ψ_i's, e.g.,

$$\psi^a\equiv\mathscr{A}_i\psi_i=\psi_i+\psi_j+\psi_k$$

for bosons and

$$\psi^a\equiv\mathscr{A}_i\psi_i=\psi_i-\psi_j-\psi_k$$

for fermions ($i\neq j\neq k\neq i$). Then the Schrödinger equation for ψ^a can be rewritten as

$$(E-H_i)\psi_i=M_i\psi_i\ , \tag{14}$$

where

$$M_i=\bar{V}_i\mathscr{A}_i+(E-H_i)(1-\mathscr{A}_i)\ . \tag{15}$$

It is easy to realize that for three identical bosons Eq. (14) is identical to Eq. (3) and for three fermions Eqs. (14) and (3) are very similar, though the meaning of ψ^C in Eq. (3) is different from that of ψ_i in Eq. (14). Now one can proceed as in the discussion related to the derivation of CRC equations and introduce the matrix notation as in Eq. (4). But that is not necessary, as Eq. (14) is an uncoupled equation for a single function ψ_i. Now the matrix notation is redundant and one can apply the Feshbach projection operator[10] technique, with the operator P_i, directly to Eq. (14), and one arrives at

$$(E-P_iH_iP_i-P_iM_iP_i)P_i\psi_i=P_iM_iQ_i\psi_i\ , \tag{16}$$

$$(E-Q_iH_iQ_i-Q_iM_iQ_i)Q_i\psi_i=Q_iM_iP_i\psi_i\ , \tag{17}$$

where $Q_i=1-P_i$. The lowest order approximation again neglects the coupling between P and Q spaces and assumes $\psi_i=P_i\psi_i$ and $Q_i\psi_i=0$, and one has

$$(E-P_iH_iP_i-P_iM_iP_i)P_i\psi_i=0\ . \tag{18}$$

Equation (18) is the RGM equation[8] and the potential $P_iM_iP_i$ is the RGM intercluster interaction.[12] Again in order to perform a complete calculation for $P_i\psi_i$ one can eliminate the Q space between Eqs. (16) and (17) and one arrives at

$$(E-P_iH_iP_i-P_iU_iP_i)P_i\psi_i=0\ , \tag{19}$$

where

$$P_i U_i P_i = P_i M_i P_i + P_i M_i Q_i \times (E - H_i - Q_i M_i Q_i + i\epsilon)^{-1} Q_i M_i P_i \quad (20)$$

is the optical potential in this case. Equation (20) suggests the following dynamical equation for U_i:

$$U_i = M_i + M_i Q_i (E - H_i + i\epsilon)^{-1} Q_i U_i \ , \quad (21)$$

which has been derived in Ref. 12 using other methods. Remembering that the antisymmetrizer (symmetrizer) $\mathscr{A}_i$ is Hermitian and that $[\mathscr{A}_i, H] = 0$ it follows that M_i is Hermitian.[12] Using this property of M_i in the Feshbach unified theory[10] one can easily show[12] that the optical potential defined by Eqs. (20) and (21) is real below the first inelastic threshold and that the discrete singularities of the optical potential correspond to physical resonances. Although the effective Hamiltonian $H_i + M_i$ of Eq. (14) is Hermitian, as $[H, \mathscr{A}_i] = 0$ and as $\mathscr{A}_i$ is Hermitian, it is interesting to note that Eq. (19) does not appear to be invariant under the action of the permutation group but rather the label i in it may change to j, $j \neq i$, under such an operation. But this causes no problem as the operators H_i and U_i are label transforming operators in the language of Bencze and Redish[13] and hence $P_i H_i P_i$ and $P_i U_i P_i$ are independent[13,14] of the label i. This label transforming property of U_i is used in Ref. 12 to show that the optical potential has no elastic unitarity cuts (associated with partitions $j \neq i$ of the equivalence class) and that the physical resonances correspond to discrete singularities of the optical potential.

The present antisymmetrized optical potential is different from that obtained by Kowalski *et al.* in some recent works on the subject.[14,15] Although all these antisymmetrized optical potentials are supposed to yield the same exact on-shell elastic scattering amplitudes for two-fragment scattering, the off-shell results in various approaches are expected to be different. However, one is not usually interested in an exact solution of the problem but only in approximate solutions. Such approximate solutions using these two approaches are supposed to be different. There are two interesting differences between the present approach and that of Kowalski *et al.* First, the approach of Kowalski *et al.* deals with non-self-adjoint operators, whereas the present approach deals with self-adjoint operators only. Second, in the approach of Kowalski *et al.*[15] the analog of the present M_i contains exchange effects nonlinearly and is thus not easily amenable to numerical calculations. In contrast, the present M_i contains exchange effects linearly, and hence appears to be more attractive for developing simple models for the optical potential for two-fragment scattering.

When the lowest order approximation to U_i given by $U_i = M_i$ is a good approximation to the full solution of Eqs. (20) and (21), one may again employ a convergent MS (or iterative) series solution of Eq. (21) through which one can systematically improve on the RGM optical potential. So we find that the present theory can be easily extended to combine the virtues of the RGM, the CCA, the Feshbach theory, and the MS formalism.

In conclusion, we have achieved a generalization of the Feshbach theory[10] to include the CRC (Refs. 1 and 2) and the DWBA (Refs. 1 and 7) for rearrangement collisions, and the RGM (Ref. 8) and the OCM (Ref. 9) for identical nucleons at the lowest level of an approximation scheme. As a result, we eliminate the mathematical objections[5,6] against the previous derivations of the CRC and the RGM, and show how to include correction terms to the RGM and CRC in a controlled way through a presumably convergent MS series.

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